Memo

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Maximum Glide Ratio of Box Wing Aicraft – Fundamental Considerations

Abstract

In the ideal case of an infinite height to span ratio (*h/b* ratio) the induced drag of the box wing aircraft is only 50 % of that of the reference aircraft. Based on this, the effect on the glide ratio can be calculated: a) With optimum conditions for the reference aircraft (condition that $D_{i,ref} = D_{0,BW}$) the glide ratio of the box wing aircraft is 33 % higher than that of the reference aircraft. b) With optimum conditions for the box wing aircraft (condition that $D_{i,BW} = D_{0,BW}$) the glide ratio of the box wing aircraft is 50 % higher than that of the reference aircraft. c) With the condition that both the box wing and the reference aircraft fly at their individual maximum glide ratio, the glide ratio of the box wing aircraft is 41 % higher than that of the reference aircraft if resulting differences in the aspect ratio are not considered (as they should be following the derivation). Taking a realistic ratio $h/b = 0.25$ yields an induced drag only 64 % of that of the reference aircraft and a 25 % higher glide ratio of the box wing aircraft compared to a conventional reference aircraft. In a practical design solution this may reduce to a glide ratio of the box wing aircraft still 18 % to 20 % higher.

Contents

Symbols

a	speed of sound
A	aspect ratio
b	wing span
C	coefficient
D	drag
e	span efficiency factor (also called: Oswald factor)
E	glide ratio
h	height
L	lift
m	mass
M	Mach number
q	dynamic pressure
S	wing reference area

Greek

ρ	density of air

Indices

0	zero lift
BW	box wing aircraft
D	drag
i	induced
iso	isolated wing of the box wing aircraft
L	lift
max	maximum
ref	reference aircraft

1 Introduction

The most important feature of the box wing configuration is its induced drag which is significantly lower than that of conventional aircraft. Because of these drag savings its performance is superior to the performance of conventional aircraft.

1.1 Aerodynamic Basics

One key indicator of performance is the glide ratio E

$$E = \frac{L}{D} = \frac{C_L}{C_D} \tag{1.1}$$

where L is lift, D is drag, C_L is the lift coefficient and C_D is the drag coefficient.

$$D = D_0 + D_i = \frac{1}{2} \rho V^2 C_D S \quad . \tag{1.2}$$

The drag D is composed of the zero lift drag D_0 and the induced drag D_i . The zero lift drag mainly depends on the shape and the wetted area of the aircraft components, whereas the induced drag depends on lift, the wing planform and the spanwise lift distribution. The zero lift drag D_0 can be expressed with the zero lift drag coefficient $C_{D,0}$

$$D_0 = \frac{1}{2}\rho V^2 C_{D,0} S \quad . \tag{1.3}$$

The induced drag D_i can be expressed with the induced drag coefficient $C_{D,i}$

$$D_i = \frac{1}{2}\rho V^2 C_{D,i} S \quad . \tag{1.4}$$

$$q = \frac{1}{2}\rho V^2 \tag{1.5}$$

is called dynamic pressure.

$$C_{D,i} = \frac{C_L^{\ 2}}{\pi \cdot A \cdot e} \quad . \tag{1.6}$$

where e is the span efficiency factor (also called Oswald factor). The aspect ratio is called A and is defined as

$$A = \frac{b^2}{S} \tag{1.7}$$

with b the wing span. Lift L is expressed with the lift coefficient C_L:

$$L = \frac{1}{2}\rho V^2 C_L S \quad . \tag{1.8}$$

From (1.4) with (1.5), (1.6), (1.7) and (1.8) we get

$$D_i = \frac{L^2}{q \cdot \pi \cdot b^2 \cdot e} \quad . \tag{1.9}$$

The derivations in this memo assume a simple drag polar. An influence of Mach number on the drag polar is neglected.

1.2 Box Wing and Conventional Reference Aircraft

For assessing the performance of the box wing aircraft it is useful to define the parameters of the box wing aircraft with respect to a conventional reference aircraft. Box wing and reference aircraft are supposed to have the same total wing area and the same wing span, thus

$$S_{BW} = S_{ref} \tag{1.10}$$

and

$$b_{BW} = b_{ref} \quad . \tag{1.11}$$

Consequently both aircraft have also the same aspect ratio A. Compare with (1.7). It follows that

$$A_{BW} = A_{ref} \quad . \tag{1.12}$$

Note: The aspect ratio of the box wing aircraft is defined as the aspect ratio of the whole aircraft with its total wing area. If only a single wing of the box wing aircraft would be considered the aspect ratio of this single wing would be

$$A_{BW,iso} = 2\,A_{BW} \quad . \tag{1.13}$$

Additionally both the box wing aircraft and the reference aircraft are assumed to have the same mass

$$m_{BW} = m_{ref} \quad . \tag{1.14}$$

For comparing aircraft performance it is important to define a reference speed. Considering aircraft design and the definition of design missions, Mach number is usually specified instead

$$M_{BW} = M_{ref} \quad . \tag{1.15}$$

For making a generalized comparison of the box wing aircraft with a reference aircraft it makes sense to assume that both aircraft have the same zero lift drag coefficient

$$C_{D,0,BW} = C_{D,0,ref} = C_{D,0} \quad . \tag{1.16}$$

This is a reasonable assumption, because both aircraft show a very similar wetted area.

1.3 Induced Drag of a Box Wing Aircraft

As initially stated in **Prandtl 1924** the savings of induced drag of a non planar wing configuration compared to a conventional one depend on its *h*/*b* ratio, which is the height to span ratio. **Durand 1935** provided information on the lift distribution for minimum induced drag of a box wing.

1.3.1 Infinite *h*/*b* Ratio

Ideally both wings of the box wing aircraft do not interfere with each other, which leads to the highest possible saving of induced drag. Theoretically this means that the vertical distance between both wings is infinite. At this condition the loading of the winglets becomes zero. Otherwise, if there would be a loading of the winglets, it would lead to an infinite force generated by the winglets because of the infinite winglet dimension. Consequently the box wing passes into a simple biplane for an infinite *h*/*b* ratio.

With the help of a simple approach the saving for the box wing with infinite *h*/*b* ratio can be derived. Consider the wing area of the reference wing to be split along the wing span into two wings with equal wing area. The lift L generated by the reference wing is consequently distributed onto these two smaller wings, so that each of the smaller wings generates $L/2$.

For calculating the induced drag of the conventional wing (1.9) can be used. The same equation also applies for the wings of the box wing aircraft. Building the ratio of the induced drag of one isolated wing of the box wing aircraft $D_{i,iso}$ and the induced drag of the reference wing $D_{i,ref}$ leads to

$$\frac{D_{i,iso}}{D_{i,ref}} = \frac{(L/2)^2}{q_{iso} \cdot \pi \cdot b_{iso}^2 \cdot e_{iso}} \cdot \frac{q_{ref} \cdot \pi \cdot b_{ref}^2 \cdot e_{ref}}{L^2} \quad . \tag{1.17}$$

Considering the conditions from Section 1.2 and assuming that both wings operate at the **same dynamic pressure** and have **equal span efficiencies** results in

$$\frac{D_{i,iso}}{D_{i,ref}} = \frac{1}{4} \quad . \tag{1.18}$$

Since the box wing aircraft has two lifting surfaces the final ratio is

$$\frac{D_{i,BW}}{D_{i,ref}} = \frac{1}{2} \quad . \tag{1.19}$$

So, in the ideal case of an infinite *h/b* ratio the induced drag of the box wing aircraft is only half of that of the reference aircraft. We will see in the next Section that the result calculated with (1.19) is exactly what has been calculated for biplanes with an infinite *h*/*b* ratio.

1.3.2 Real *h*/*b* Ratio

In **Prandtl 1924** a formula is derived for the ratio $D_{i,BW}/D_{i,ref}$ depending on the *h*/*b* ratio

$$\frac{D_{i,BW}}{D_{i,ref}} = \frac{1+0.45\,h/b}{1.04+2.81\,h/b}; \quad \frac{1}{15} < h/b < \frac{1}{2} \quad . \tag{1.20}$$

(1.20) is only applicable when both the box wing and the reference wing have the same wing span and generate the same amount of lift. With the help of a formal mathematical approach **Frediani 2009** showed that (1.20) gives realistic results within the defined range. Figure 1.1 compares the results from **Frediani 2009** with those from (1.20) coming from **Prandtl 1924**. **Frediani 2005** confirms the limit value of $D_{i,BW}/D_{i,ref}$ to be 0.5. This was also calculated at the end of Section 1.3.1 with Equation (1.19).

For actual box wing applications the *h*/*b* ratio is supposed to be within the range of 0.1 … 0.25.

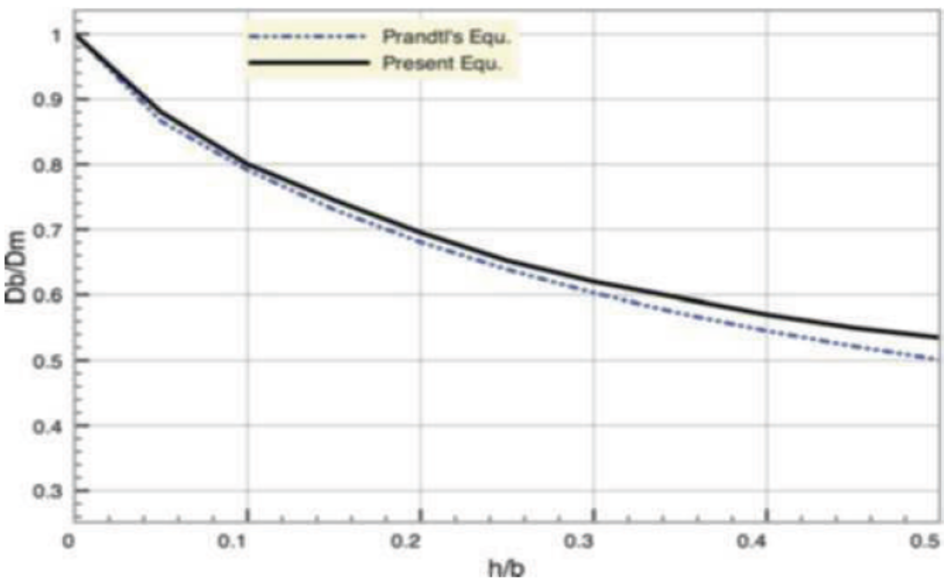

Figure 1.1 Induced drag savings of a box wing aircraft according to **Prandtl 1924** and **Frediani 2009.** Source: **Frediani 2009**

2 Maximum Glide Ratio of a Box Wing Aircraft

As it was shown in Section 1.3 it is very convenient to work with ratios relative to a reference aircraft other than with absolute values when it comes to the induced drag of box wing aircraft. This is why this approach is also applied for assessing the glide ratio of a box wing aircraft. In this manner it is possible to produce significant results with the help of simple calculations.

2.1 Two "Unfair" Comparisons between Box Wing and Reference Aircraft

2.1.1 Approach for the "Unfair" Comparisons

The first approach to build the ratio of glide ratios applies (1.1) and (1.2) and yields

$$\frac{E_{BW}}{E_{ref}} = \frac{L_{BW}}{D_{0,BW} + D_{i,BW}} \cdot \frac{D_{0,ref} + D_{i,ref}}{L_{ref}} \quad . \tag{2.1}$$

Since both aircraft are supposed to have the same mass (see Section 1.2), they generate the same amount of lift. Consequently (2.1) can be simplified to

$$\frac{E_{BW}}{E_{ref}} = \frac{D_{0,ref} + D_{i,ref}}{D_{0,BW} + D_{i,BW}} \quad . \tag{2.2}$$

Assuming that both aircraft fly at the same altitude h and with the same Mach number M, the dynamic pressure q is the same since

$$q = \frac{1}{2} \rho (M \cdot a)^2 \quad , \tag{2.3}$$

so

$$q_{BW} = q_{ref} \quad . \tag{2.4}$$

Assuming that both aircraft have the same wing area S (condition (1.10)) and the same zero lift drag coefficient $C_{D,0}$ (condition (1.16)), also the zero lift drag D_0 of both aircraft is the same since

$$D_0 = C_{D,0} \cdot S \cdot q \quad . \tag{2.5}$$

Now (2.2) simplifies (what is necessary for the further derivation in (2.7) and (2.9)) to

$$\frac{E_{BW}}{E_{ref}} = \frac{D_0 + D_{i,ref}}{D_0 + D_{i,BW}} \quad . \tag{2.6}$$

As it is shown in Appendix A, E_{max} is reached when $C_{D,i} = C_{D,0}$ and because of (2.5) $D_i = D_0$. $C_{D,0}$ is considered a fixed value. However, $C_{D,i}$ can be adapted with the help of selecting a suitable C_L from (1.6). A selection of a lift coefficient C_L means the selection of a cruise altitude as it follows from the lift equation (1.9). If now one aircraft flies at its optimum altitude and the comparison requires the other aircraft to fly at the same altitude, it will not fly at its optimum conditions. Therefore two cases have to be differentiated:

1) $D_{i,ref} = D_0$, which means that the reference aircraft flies at its optimum altitude and at E_{max} (the box wing aircraft is not flying at E_{max})
2) $D_{i,box} = D_0$, which means that the box wing aircraft flies at its optimum altitude and at E_{max} (the reference aircraft is not flying at E_{max}).

Both comparisons are "unfair". In each case one aircraft has an advantage and the other one a disadvantage. In search of a "fair" comparison Section 2.2 defines these "fair" conditions and calculates the ratio of E_{max} based upon it. But as a first and simple exercise we start with the "unfair" comparisons as given above in 1) and 2).

2.1.2 Optimum Conditions for the Reference Aircraft: $D_{i,ref} = D_0$

Here the reference aircraft flies at its maximum glide ratio while the box wing flies at the same altitude and the same airspeed as the reference aircraft. The box wing aircraft is not able to reach its maximum glide ratio under these conditions.

For this case (2.6) can be solved as follows:

$$\frac{E_{BW}}{E_{ref}} = \frac{D_0 + D_{i,ref}}{D_0 + D_{i,BW}} = \frac{2\,D_{i,ref}}{D_{i,ref} + D_{i,BW}} = \frac{2}{1 + \dfrac{D_{i,BW}}{D_{i,ref}}} \quad . \tag{2.7}$$

Considering the ideal case where $D_{i,BW}/D_{i,ref} = 0{,}5$ results in

$$\frac{E_{BW}}{E_{ref}} = \frac{4}{3} = 1.33 \quad . \tag{2.8}$$

So with an infinite h/b ratio and the condition that $D_{i,ref} = D_{0,BW}$ the glide ratio of the box wing aircraft is 33 % higher than that of the reference aircraft. The result for actually applicable h/b ratios can be determined with the help of (1.20).

2.1.3 Optimum Conditions for the Box Wing Aircraft: $D_{i,BW} = D_0$

Here the box wing aircraft flies at its maximum glide ratio while the reference aircraft flies at the same altitude and the same airspeed, and thus with a glide ratio smaller than the optimum.

For this case (2.6) can be solved as follows:

$$\frac{E_{BW}}{E_{ref}} = \frac{D_0 + D_{i,ref}}{D_0 + D_{i,BW}} = \frac{D_{i,BW} + D_{i,ref}}{2\,D_{i,BW}} = \frac{1 + \frac{D_{i,ref}}{D_{i,BW}}}{2} \quad . \tag{2.9}$$

Considering the ideal case where $D_{i,ref} / D_{i,BW} = 2$ results in

$$\frac{E_{BW}}{E_{ref}} = \frac{3}{2} = 1.5 \quad . \tag{2.10}$$

So with an infinite h/b ratio and the condition that $D_{i,BW} = D_{0,BW}$ the glide ratio of the box wing aircraft is 50 % higher than that of the reference aircraft. The result for actually applicable h/b ratios can be determined with the help of (1.20).

2.2 The "Fair" Comparison between Box Wing and Reference Aircraft

In this case both the box wing and the reference aircraft fly at their own and individual optimum altitude and and at their maximum glide ratio E_{max}. This is the case which should give a "fair" comparison of both aircraft. However, the approach in Section 2.1.1 is not applicable any more. The following equation derived in Appendix B for calculating the maximum glide ratio is used instead:

$$E_{max} = \frac{1}{2}\sqrt{\frac{\pi \cdot A \cdot e}{C_{D,0}}} \quad . \tag{2.11}$$

Building the ratio of maximum glide ratios yields

$$\frac{E_{max,BW}}{E_{max,ref}} = \sqrt{\frac{A_{BW} \cdot e_{BW}}{C_{D,0,BW}} \cdot \frac{C_{D,0,ref}}{A_{ref} \cdot e_{ref}}} \quad . \tag{2.12}$$

Condition (1.16) states that both aircraft are assumed to have the same zero lift drag coefficient, so (2.12) can be simplified to

$$\frac{E_{max,BW}}{E_{max,ref}} = \sqrt{\frac{A_{BW} \cdot e_{BW}}{A_{ref} \cdot e_{ref}}} \quad . \tag{2.13}$$

It has to be noted that in (2.13) the span efficiencies e and the aspect ratios A refer to the whole aircraft and not to individual and isolated wings, as it was the case in Section 1.3.1.

It is possible to get the ratio of span efficiencies with the help of (1.9) and (1.20)

$$\frac{e_{BW}}{e_{ref}} = \frac{D_{i,ref}}{D_{i,BW}} \tag{2.14}$$

For this it is necessary to assume that both aircraft have the same lift L and hence mass m (condition (1.14)), the same span b (condition (1.11)) and are exposed to the same dynamic pressure q (condition (2.4)). The same q means that they fly at the same altitude considering all previous assumptions. Both aircraft can now only fly at their individual optimum C_L and maximum glide ratio E_{max} if condition (1.10) (equal wing areas S) and as a consequence (1.12) (equal aspect ratio A) are given up. So again: **At this point it is necessary to violate the condition of equal wing reference areas *S* and equal aspect ratio *A*. However, the wing span *b* is considered the same for both aircraft** otherwise (2.14) would not be valid. Furthermore span is the limiting parameter at airports with its important influence on cruise drag. In this way it is possible to adjust the lift coefficient of each aircraft so that the crucial condition for E_{max}, namely $C_{D,0} = C_{D,i}$ or $D_0 = D_i$, can be satisfied for each aircraft individually.

So

$$\frac{E_{max,BW}}{E_{max,ref}} = \sqrt{\frac{A_{BW} \cdot D_{i,ref}}{A_{ref} \cdot D_{i,BW}}} \quad \text{with } A_{BW} \neq A_{ref} \tag{2.15}$$

Considering the ideal case for (2.15), where $D_{i,BW} / D_{i,ref} = 0.5$ leads to

$$\frac{E_{max,BW}}{E_{max,ref}} = \sqrt{2} \cdot \sqrt{\frac{A_{BW}}{A_{ref}}} = 1.4142 \cdot \sqrt{\frac{A_{BW}}{A_{ref}}} \quad \text{with } A_{BW} \neq A_{ref} \quad . \tag{2.16}$$

So with an infinite h/b ratio and the condition that both the box wing and the reference aircraft fly at their individual maximum glide ratio, the glide ratio of the box wing aircraft is 41 % higher than that of the reference aircraft if resulting differences in the aspect ratio are not considered (as they should be following the derivation). The result for actually applicable h/b ratios can be determined with the help of (1.20).

By the way: Taking the average of the results from 2.1.2 and 2.1.3 yields the same value

$$\frac{4/3+3/2}{2} = 1.4167 \tag{2.17}$$

with a deviation of only 0.2 % compared to the results from this Section.

Even without going into details, the total effect including aspect ratio can be easily understood: The box wing aircraft has a higher Oswald factor e and requires due to (1.6) a larger lift coefficient C_L . This leads to a smaller wing area S and with a constant span b to a higher aspect ratio A. (2.16) shows that the glide ratio of the box wing aircraft is even larger including the effect of a resulting higher aspect ratio of the box wing aircraft. The benefit in glide ratio compared to a conventional reference aircraft and including the aspect ratio effect is further elaborated in Section 2.3.

2.3 The "Ultimate" Comparison between Box Wing and Reference Aircraft

In (2.15) there is the unknown ratio A_{BW}/A_{ref} which does not allow to conclude a final statement regarding the maximum glide ratio of a box wing aircraft. In this Section it is shown how a final expression taking account of the effects of aspect ratio can be derived.

Starting point for the derivation is the lift equation (1.8). As shown in Appendix B it is necessary to fly at the lift coefficient for minimum drag $C_{L,md}$ for attaining the maximum glide ratio E_{max}. Taking account of this condition as well as (1.5) and (1.7) the lift equation can be written as

$$L = q \cdot C_{L,md} \cdot \frac{b^2}{A} \quad . \tag{2.18}$$

According to condition (1.14) both aircraft have the same mass and hence the same lift. With the help of this condition and (2.18) it is possible to write

$$q_{BW} \cdot C_{L,md,BW} \cdot \frac{b_{BW}^{\ 2}}{A_{BW}} = q_{ref} \cdot C_{L,md,ref} \cdot \frac{b_{ref}^{\ 2}}{A_{ref}} \quad . \tag{2.19}$$

With the conditions (2.4) (equal dynamic pressure q) and (1.11) (equal wing span b) it follows that

$$\frac{C_{L,md,BW}}{C_{L,md,ref}} = \frac{A_{BW}}{A_{ref}} \quad . \tag{2.20}$$

As derived in Appendix B the lift coefficient for minimum drag $C_{L,md}$ is given as

$$C_{L,md} = \sqrt{C_{D,0} \pi A e} \quad . \tag{2.21}$$

Using condition (1.16) (equal zero lift drag coefficient $C_{D,0}$) and (2.21), (2.20) changes to

$$\sqrt{\frac{A_{BW}\, e_{BW}}{A_{ref}\, e_{ref}}} = \frac{A_{BW}}{A_{ref}} \quad , \tag{2.22}$$

which is simplified to

$$\frac{A_{BW}}{A_{ref}} = \frac{e_{BW}}{e_{ref}} \quad . \tag{2.23}$$

Inserting (2.23) into (2.13) finally yields

$$\frac{E_{BW}}{E_{ref}} = \frac{e_{BW}}{e_{ref}} \quad . \tag{2.24}$$

Using (2.14)

$$\frac{E_{BW}}{E_{ref}} = \frac{D_{i,ref}}{D_{i,BW}} \quad . \tag{2.25}$$

Assuming an infinite h/b ratio so that $D_{i,BW}/D_{i,ref} = 0.5$ leads to

$$\frac{E_{BW}}{E_{ref}} = 2 \quad . \tag{2.26}$$

The result for actually applicable h/b ratios can be determined with the help of (1.20).

Keep in mind that (2.25) is only valid when (2.23) is applied, which means that for the case of an infinite h/b ratio the aspect ratio of the box wing aircraft is twice the aspect ratio of the reference aircraft. Because of (1.13) this means that the aspect ratio of a single wing of the box wing aircraft is four times the aspect ratio of the reference wing. This is impracticable for many reasons. Depending on the point of view, either the resulting wing area of the reference aircraft would be impracticably high or the resulting wing area of the box wing aircraft would be impracticably low, since

$$\frac{A_{BW}}{A_{ref}} = \frac{S_{ref}}{S_{BW}} \tag{2.27}$$

according to (1.7) and taking account of condition (1.11) (equal wing span *b)*. For (2.26) it is also assumed that both aircraft have the same zero lift drag coefficient $C_{D,0}$ (condition (1.16)). Considering the dependency of the zero lift drag coefficient on the wetted area, to which the wing area contributes in large part, condition (1.16) may only be satisfied when S_{ref}/S_{BW} is close to unity.

The conclusion is that the results of this Section are merely of theoretic nature. Problems for a box wing aircraft based on the reference aircraft would come from e.g.: wing mass, fuel tank volume, wing loading, landing distance, just to name the most important points.

For getting a simple and fair comparison of the glide ratios between box wing and reference aircraft the "ultimate" comparison does not help much further.

3 Conclusion

As last resort it is seen necessary to size and conceptually design a box wing aircraft in more detail and to compare its performance with that of the reference aircraft. This is done in the research project Airport2030 at Hamburg University of Applied Sciences.

For a simple answer, (2.15) is probably getting the closest to the question of box wing and reference aircraft comparison on glide ratio. Ignoring limitations on aspect ratio (for simplicity)

$$\frac{E_{max,BW}}{E_{max,ref}} = \sqrt{\frac{D_{i,ref}}{D_{i,BW}}} \quad . \tag{3.1}$$

Taking a realistic ratio $h/b = 0.25$ yields with (1.20) $D_{i,BW}/D_{i,ref} = 0.6385$ and finally

$$\frac{E_{max,BW}}{E_{max,ref}} = 1.25 \quad .$$

So, the box wing aircraft may show a 25 % higher glide ratio. In a practical design solution this may reduce to glide ratio still 18 % to 20 % higher compared to a conventional reference aircraft.

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Appendix A

Derivation of the Drag Condition at Maximum Glide Ratio

The glide ratio E is defined as

$$E=\frac{L}{D}=\frac{C_L}{C_D} \quad . \tag{A.1}$$

As idealization it is assumed that the total drag coefficient is composed of the zero lift drag coefficient $C_{D,0}$ and the induced drag coefficient $C_{D,i}$, thus

$$C_D = C_{D,0}+C_{D,i} \quad . \tag{A.2}$$

So (A.1) can now be written as

$$E = \frac{C_L}{C_{D,0}+C_{D,i}} \quad . \tag{A.3}$$

The induced drag coefficient is given as

$$C_{D,i} = \frac{C_L^{\,2}}{\pi A e} \quad . \tag{A.4}$$

Finally the glide ratio can be expressed as a function of the lift coefficient C_L:

$$E = \mathrm{f}\left(C_L\right) = \frac{C_L}{C_{D,0}+\dfrac{C_L^{\,2}}{\pi A e}} \quad . \tag{A.5}$$

For getting the conditions for E_{max} it is necessary to find the extremum of (A.5). This is done by equating the first derivative of (A.5) with regard to C_L to zero:

$$\frac{\mathrm{d}E}{\mathrm{d}C_L} = \frac{C_{D,0}+\dfrac{C_L^{\,2}}{\pi A e} - C_L\dfrac{2C_L}{\pi A e}}{\left(C_{D,0}+\dfrac{C_L^{\,2}}{\pi A e}\right)^2} = 0 \quad . \tag{A.6}$$

Simplifying (A.6) gives the expression

$$C_{D,0} - \frac{C_L^{\ 2}}{\pi A e} = 0 \quad . \tag{A.7}$$

Applying (A.4) finally yields

$$C_{D,0} = C_{D,i} \quad , \tag{A.8}$$

which means that for flying at the maximum glide ratio E_{max} it is necessary that the zero lift drag coefficient $C_{D,0}$ equals the induced drag coefficient $C_{D,i}$.

Appendix B

Derivation of the Equation for the Maximum Glide Ratio

Combining (A.8) and (A.4) yields

$$C_{D,0} = \frac{C_L^2}{\pi A e} \quad , \tag{B.1}$$

which is only applicable when flying at the maximum glide ratio E_{max}. The only parameter which can be varied to satisfy (B.1) is the lift coefficient C_L, since all of the other parameters are for now assumed to only depend on the geometry of the aircraft. The lift coefficient C_L which satisfies (B.1) is called the lift coefficient for minimum drag $C_{L,md}$, since flying at the maximum glide ratio also means flying with minimum drag, and is expressed as

$$C_{L,md} = \sqrt{C_{D,0} \pi A e} \quad . \tag{B.2}$$

Combining (A.7) and (A.2) it is possible to write

$$C_D = 2 C_{D,0} \quad , \tag{B.3}$$

which is only valid when flying at the maximum glide ratio E_{max}. Inserting (B.2) and (B.3) into (A.1) finally yields

$$E_{max} = \frac{1}{2} \sqrt{\frac{\pi A e}{C_{D,0}}} \quad . \tag{B.4}$$