Reverse Engineering

For a given plane (e.g. B747-400), the following parameters are known: s_{LFL}, s_{TOFL}, V_{APP}, R @ max payload, b, m_{MTO}/S, m_L/m_{MTO}, m_{OE}, m_{MTO}, m_{OE}/m_{MTO}, T_{TO}, $T_{TO}/(m_{MTO} \cdot g)$, V_{CR}, h_{CR}, M, $n_{PAX,std}$, $n_{PAX,max}$, m_{PL}, BPR.

The following values can be easily calculated:

$S = 541,2\ m^2$

$A = \frac{b^2}{S}$

Other values are unknown, the companies secrets:

- SFC
- $E = \frac{L}{D}$
- $C_{L,max,L}$

These parameters can be found by reverse engineering.

PLAN:

- reverse the excel-file
- make a list of the known parameters; add the calculated values – calculate the percentage of deviation for each parameter
  - make the squared sum of the deviation
- make a list that contains the parameters we can change – this includes an upper and lower limit (according to skybrary)

=> Make a solver that minimises the squared sum by changing the parameters in between the limits from skybrary.

Normale gang van zaken:

* E_{max} wordt berekend volgens loftin of raymer

$$E_{max} = k_E \cdot \sqrt{\frac{A}{S_{wet}/S_W}} \quad \text{met} \quad k_E = \frac{1}{2} \cdot \sqrt{\frac{\pi \cdot e}{C_f}} = 14,9 \quad \text{(loftin)}$$

$$k_E = 15,8 \quad \text{(Raymer)}$$

* $$E = \frac{2 \cdot E_{max}}{\frac{C_{L,m}}{C_L} + \frac{C_L}{C_{L,m}}}$$

* $$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{\frac{T_{CR}}{T_{TO}} \cdot E} \longrightarrow \left(\frac{T_{CR}}{T_{TO}}\right)_{CR} = \frac{1}{\frac{T_{TO}}{m_{MTO} \cdot g} \cdot E}$$

waarbij de thrust-to-weight ratio gekend is na de wing loading.

* $$\frac{T_{CR}}{T_{TO}} = (0,0013\mu - 0,0397)\frac{1}{km} \cdot h_{CR} - 0,0248\mu + 0,7125$$

$$h_{CR} = \frac{\frac{T_{CR}}{T_{TO}} + 0,0248\mu - 0,7125}{0,0013\mu - 0,0397}$$

* Sinds de temperatuur i.f. kan gesteld worden van de hoogte:

$$\left.\begin{aligned} T_{Troposfeer} &= 288,15 - 0,0065 \cdot h_{CR} \\ T_{Stratosfeer} &= 216,5\ k \end{aligned}\right\} \text{als } T_T < T_S \Rightarrow T_S \text{ geldt.}$$

* Snelheid vh geluid is ook een functie van de temperatuur

$$a = 20,5 \cdot \sqrt{T_x}$$

* $V_{CR} = M \cdot a$ met M is gekend.

Reverse Engineering

* V_{CR} en Π zijn gekend $\Rightarrow a = \frac{V_{CR}}{\Pi}$

↳ Data ⇒ uitwerp! → Breguet!

* $T_x = \left(\frac{a}{20{,}05}\right)^2 \longrightarrow$ als $T_x < 216{,}65\,k \Rightarrow T_x = T_S$

anders $T_x = T_T$

The temperature namely has a value that is included by the troposphere.

* $h_{CR} = \frac{288{,}15 - T_{Troposphere}}{0{,}0065}$

* $\frac{T_{CR}}{T_{TO}} = (0{,}0013\,\mu - 0{,}0397) \cdot h_{CR} - 0{,}0248\,\mu + 0{,}7125$

* $E = \left(\frac{T_{TO}}{m_{MTO} \cdot g} \cdot \frac{T_{CR}}{T_{TO}}\right)^{-1}$

* $E_{max} = \frac{E \cdot \left(\frac{C_{L,m}}{C_L} + \frac{C_L}{C_{L,m}}\right)}{2}$

Assume that $\frac{T}{W}$ and $\frac{W}{S}$ is known.

① $\frac{m_{ML}}{S_w} = k_L \cdot \sigma \cdot C_{L,max,L} \cdot s_{LFL}$ where $s_{LFL} = \left(\frac{V_{APP}}{k_{APP}}\right)^2$

and $\frac{m_{MTO}}{S_w} = \frac{m_{ML}}{S_w} \cdot \frac{m_{MTO}}{m_{ML}}$

$$\Longrightarrow \boxed{C_{L,max,L} = \frac{\frac{m_{MTO}}{S_w} \cdot \frac{m_{ML}}{m_{MTO}}}{k_L \cdot \sigma \cdot s_{LFL}}}$$

TO $\frac{T_{TO}}{m_{MTO} \cdot g} = a \cdot \frac{m_{MTO}}{S_w} = \frac{k_{TO}}{s_{TOFL} \cdot \sigma \cdot C_{L,max,TO}} \cdot \frac{m_{MTO}}{S_w}$

$$\Longrightarrow \boxed{C_{L,max,TO} = \frac{k_{TO}}{s_{TOFL} \cdot \sigma} \cdot \frac{m_{MTO}/S_w}{T_{TO}/m_{MTO} \cdot g}}$$

2nd $\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{n_E}{n_E - 1} \cdot \left(\frac{1}{E_{TO}} + \sin\gamma\right)$

$$\overset{?}{\Longrightarrow} \boxed{E_{TO} = \left(\frac{T_{TO}}{m_{MTO} \cdot g} \cdot \frac{n_E - 1}{n_E} - \sin\gamma\right)^{-1} = \frac{C_{L,TO}}{C_{D,P} + \frac{C_{L,TO}^2}{\pi \cdot e \cdot A}}}$$

→ zijn ze gelijk in excel ?

MA $\frac{T_{TO}}{m_{ML} \cdot g} = \frac{n_E}{n_E - 1} \cdot \left(\frac{1}{E_L} + \sin\gamma\right)$

$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{n_E}{n_E - 1} \cdot \left(\frac{1}{E_L} + \sin\gamma\right) \cdot \frac{m_{ML}}{m_{MTO}}$

$$\overset{?}{\Longrightarrow} \boxed{E_L = \left(\frac{T_{TO}}{m_{MTO} \cdot g} \cdot \frac{n_E - 1}{n_E} \cdot \frac{m_{MTO}}{m_{ML}} - \sin\gamma\right)^{-1} = \frac{C_{L,L}}{C_{D,P} + \frac{C_{L,L}^2}{\pi \cdot e \cdot A}}}$$

→ komt dit overeen in exc.

(c) Formulas which involve E_{max}:

$$C_{D0} = \frac{\pi \cdot A \cdot e}{4 \cdot E_{max}^2} \longrightarrow C_{L,md} = \sqrt{C_{D,0} \cdot A \cdot e \cdot \pi}$$

$$\longrightarrow C_L = \frac{C_{Lmd}}{(V/V_{md})^2}$$

$$E = \frac{2 \cdot E_{max}}{\frac{1}{C_L/C_{L,md}} + \frac{C_L}{C_{L,md}}}$$

Relation with wing loading and thrust to weight ratio:

$$\frac{m_{MTO}}{S_w} = \frac{C_L \cdot M^2}{g} \cdot \frac{\gamma}{2} \cdot p_0 \cdot (1 - 0,02256 \cdot h)^{5,258}$$

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{(T_{CR}/T_{TO}) \cdot E} \longrightarrow \frac{T_{CR}}{T_{TO}} = (0,0013\mu - 0,0397) \cdot h - 0,0248\mu + 0,7125$$

Altitude as a function, expressed in 'km':

$$h = \frac{1}{0,02256} \cdot \left[1 - \left(\frac{2 \cdot g \cdot m_{MTO}/S_w}{C_L \cdot M^2 \cdot \gamma \cdot p_0} \right)^{\frac{1}{5,258}} \right]$$

$$h = \frac{\frac{T_{CR}}{T_{TO}} + 0,0248\mu - 0,7125}{0,0013\mu - 0,0397}$$

Composing equation for E_{max}:

$$E = \frac{2 \cdot E_{max}}{\frac{1}{C_L/C_{L,md}} + \frac{C_L}{C_{L,md}}} = \frac{1}{\frac{T_{CR}}{T_{TO}} \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}$$

$$E_{max} \cdot \frac{2 \cdot \frac{T_{CR}}{T_{TO}} \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{1}{C_L/C_{L,md}} + \frac{C_L}{C_{L,md}}} - 1 = 0$$

$$\text{with} \quad \frac{2 \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{1}{C_L / C_{L,md}} + \frac{C_L}{C_{L,md}}} = Z$$

$$E_{max} \cdot Z \cdot \frac{T_{CR}}{T_{TO}} - 1 = 0$$

$$Z \cdot E_{max} \cdot \left((0{,}0013\mu - 0{,}0397)\, h - 0{,}0248\mu + 0{,}7125 \right) - 1 = 0$$

$$Z \cdot E_{max} \cdot \left[\underbrace{\frac{0{,}0013\mu - 0{,}0397}{0{,}02256}}_{} \cdot \left(1 - \left(\frac{2 \cdot g \cdot m_{MTO}/S_W}{C_L \cdot \Pi^2 \cdot \gamma \cdot p_0} \right)^{\frac{1}{5{,}258}} \right) \underbrace{- 0{,}0248\mu + 0{,}7125}_{} \right] - 1 = 0$$

$$Z \cdot E_{max} \cdot \left[0{,}0328\mu - 1{,}05 - \left(\frac{2 \cdot E_{max}}{\frac{C_L}{C_{L,md}} \cdot \pi \cdot A \cdot e} \right)^{\frac{1}{5{,}258}} \cdot \left(\frac{2 \cdot g \cdot m_{MTO}/S_W}{\Pi^2 \cdot \gamma \cdot p_0} \right)^{\frac{1}{5{,}258}} \cdot (0{,}0576\mu - 1{,}76) \right] - 1 = 0$$

$$E_{max}^{\frac{6{,}258}{5{,}258}} \cdot \frac{2 \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{1}{C_L / C_{L,md}} + \frac{C_L}{C_{L,md}}} \cdot \left[\frac{4 \cdot g \cdot \frac{m_{MTO}}{S_W}}{\frac{C_L}{C_{L,md}} \cdot \pi \cdot A \cdot e \cdot \Pi^2 \cdot \gamma \cdot p_0} \right]^{\frac{1}{5{,}258}} \cdot (0{,}0576\mu - 1{,}76) - E_{max} \cdot \frac{2 \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{1}{C_L / C_{L,md}} + \frac{C_L}{C_{L,md}}} \cdot (0{,}0328\mu - 1{,}05) + 1 = 0$$

$$\frac{{,}258}{258} \cdot E_{max}^{\frac{1}{5{,}258}} \cdot \frac{2 \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{1}{C_L / C_{L,md}} + \frac{C_L}{C_{L,md}}} \cdot \left[\frac{4 \cdot g \cdot \frac{m_{MTO}}{S_W}}{\frac{C_L}{C_{L,md}} \cdot \pi \cdot A \cdot e \cdot \Pi^2 \cdot \gamma \cdot p_0} \right]^{\frac{1}{5{,}258}} (0{,}0576\mu - 1{,}76) - \frac{2 \cdot \frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{1}{C_L / C_{L,md}} + \frac{C_L}{C_{L,md}}} \cdot (0{,}0328\mu - 1{,}05) = 0$$