Prediction of Resistance and Propulsion Power of Ships

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Project no. 2016-108: Update of decision support system for calculation of exhaust gas emissions
Report no. 04
June 2017

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Resistance and Propulsion power – Full-scale Prediction

Introduction

To calculate the propulsion power for a ship, the resistance and the total propulsive efficiency have to be determined with the highest possible accuracy. As empirical methods are normally used for these calculations, it is worthwhile at least to know the accuracy of the different elements in the calculation procedures such that the propulsive power can be predicted in combination with an estimate of the uncertainty of the result. In the following the calculation procedures used for the present project will be described in detail

Main Dimensions and other Definitions

Following parameters are used in calculation procedure of the ship resistance R_T:

L_{wl}	Length of waterline of ship
L_{pp}	Length between perpendiculars
B	Breadth, moulded of ship
T	Draught, moulded amidships (mean draught)
W_L	Lightship weight
D_w	Deadweight of ship
Δ	Displacement mass of ship ($\rho \cdot \nabla = W_L + D_W$)
∇	Displacement volume of ship
S	The wetted surface of immersed hull
A_M	Immersed midship section area
A_{wl}	Area of water plane at a given draught)
D_{prop}	Propeller diameter
V	Speed of ship
g	gravitational constant (9.81 m/s^2)
Fn	Froude number ($Fn = \frac{V}{\sqrt{g \cdot Lpp}}$)
C_B	Block coefficient ($C_B = \frac{\nabla}{Lpp \cdot B \cdot T}$)
C_M	Midship section coefficient ($C_M = \frac{A_M}{B \cdot T}$)
C_p	Prismatic coefficient ($C_P = \frac{C_B}{C_M}$)
C_w	Water plane area coefficient ($C_w = \frac{A_{wl}}{L \cdot B}$)
M	Length displacement ratio or slenderness ratio, $M = \frac{L}{\nabla^{1/3}}$
ρ	Mass density of water
t	Water temperature
Rn	Reynolds number
υ	The kinematic viscosity of water
C_T	Total resistance coefficient

C_R	Residuary resistance coefficient
C_F	Frictional resistance coefficient
C_A	Incremental resistance coefficient
C_{AA}	Air resistance coefficient

Fixed values

Design values: *L, B, T, Δ, V*
Calculated values (using design values): C_B, C_p, *M, Fn, Rn*
Calculated values using approximations: *S*
Environmental constants: Water density, temperature, kinematic viscosity

Parameters assumed or calculated based on empirical methods/data

C_T	Total resistance coefficient
C_R	Residuary resistance coefficient
C_F	Frictional resistance coefficient
C_A	Incremental resistance coefficient
C_{AA}	Air resistance coefficient
D_{prop}	Propeller diameter
w	Wake fraction
t	Thrust deduction fraction
η_o	Propeller efficiency,
η_R	Relative rotative efficiency
η_S	Transmission efficiency (shaft line and gearbox losses)
S	Wetted surface

Total Resistance Coefficient

The total resistance coefficient, C_T, of a ship can be defined by:

$$C_T = C_F + C_A + C_{AA} + C_R = \frac{R_T}{\frac{1}{2} \cdot \rho \cdot S \cdot V^2}$$

This is the originally ITTC1957 method from the International Towing Tank Committee (ITTC).

All parameters in the above equation will be described in the present section.

Wetted Surface

The wetted surface is normally calculated by hydrostatic programs. However for a quick and fairly accurate estimation of the wetted surface many different methods and formulas exist based on only few ship main dimensions, as example Mumford´s formula below:

$$S = 1.025 \cdot L_{pp} \cdot (C_B \cdot B + 1.7 \cdot T) = 1.025 \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{pp} \cdot T\right)$$

In the present project an analysis of the wetted surface data of 125 different newer ships (of different type as well as size) shows that the wetted surface according to the above mentioned version of Mumford´s formula can be up to 7 % too small or too large for container ships, bulk carriers and tankers and up to 15 % for Ro-Ro ships. Therefore it has been analysed if the formula (i.e. the constants in the formula) can be adjusted in order to increase the accuracy. The results of the analysis for the wetted surface for bulk carriers, tankers, container ships, Ro-Ro twin screw ships and Ro-Ro twin skeg ships can be seen in Appendix B.

The equations for the wetted surface, which have been deducted from the present analysis, are shown in the table below:

Bulk carriers and tankers	$S = 0.99 \cdot \left(\frac{\nabla}{T} + 1.9 \cdot L_{wl} \cdot T\right)$
Container vessels (single screw)	$S = 0.995 \cdot \left(\frac{\nabla}{T} + 1.9 \cdot L_{wl} \cdot T\right)$
Single screw Ro-Ro ships	$S = 0.87 \cdot \left(\frac{\nabla}{T} + 2.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin screw ships (Ro-Ro ships) with open shaft lines (and twin rudders)	$S = 1.21 \cdot \left(\frac{\nabla}{T} + 1.3 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin skeg ships (Container ships and Ro-Ro ships with twin rudders)	$S = 1.13 \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.31 \cdot C_{BW})$

The formulas for calculation of the wetted surface include the area of rudder(s) skegs and shaft lines. However any additional surfaces, *S',* from appendages such as bilge keels, stabilizers etc. shall be taken into account by adding the area of these surfaces to the wetted surface of the main hull.

If the wetted surface, S_1, is given for a given draught, T_1, the wetted surface, S_2, for another draught, T_2, can be calculated by using following formulas, which have been deducted based on an analysis of data for container ships, tankers, bulk carriers and Ro-Ro ships:

Container ships (single screw): $S_2 = S_1 - 2.4 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$

Tankers and bulk carriers: $S_2 = S_1 - 2.0 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$

Single screw Ro-Ro ships: $S_2 = S_1 - 3.0 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$

Conventional twin screw Ro-Ro ships: $S_2 = S_1 - 2.5 \cdot (T_1 - T_2) \cdot (L_{wl} + B)$

Twin-skeg container and Ro-Ro ships: $S_2 = S_1 – 3.0·(T_1 – T_2)·(L_{wl} + B)$

Also based on a statistical analysis of container ships, tankers, bulk carriers and Ro-Ro ships following empirical relations between L_{wl} and L_{pp} have been found:

Container ships: $L_{wl} = 1.01·L_{pp}$

Tankers and bulk carriers: $L_{wl} = 1.02·L_{pp}$

Single screw Ro-Ro ships: $L_{wl} = 1.01·L_{pp}$

Conventional twin screw Ro-Ro ships: $Lwl = 1.035·L_{pp}$

Twin-skeg Ro-Ro ships: $L_{wl} = 1.04·L_{pp}$

Frictional Resistance Coefficient

The frictional resistance coefficient, C_F, in accordance with the ITTC-57 formula is defined by:

$$C_F = \frac{0.075}{(\log R_n - 2)^2} = \frac{R_F}{½ \cdot \rho\, S \cdot V^2}$$

where the frictional resistance, R_F, is sum of tangential stresses along the wetted surface in the direction of the motion.

R_n is the Reynolds number: $R_n = \frac{V \cdot L_{wl}}{\upsilon}$

υ is the kinematic viscosity of water: $\upsilon = ((43.4233 - 31.38 \cdot \rho) \cdot (t + 20)^{1.72 \cdot \rho - 2.202} + 4.7478 - 5.779 \cdot \rho) \cdot 10^{-6}$

t is water temperature in degrees Celcius.

As in the original resistance calculation method by Harvald (called “Ship Resistance”), it is here decided to leave out a form factor in the C_F part, but include a correction for special hull forms having U or V shape in the fore or after body, as suggested by Harvald. The influence of a bulbous bow on the resistance is included in a bulb correction, see section regarding this topic.

Incremental Resistance Coefficient

The frictional resistance coefficient is related to the surface roughness of the hull. However the surface roughness of the model will be different from the roughness of the ship hull. Therefore, when extrapolating to ship size, an incremental resistance coefficient C_A is added in order to include the effect of the roughness of the surface of the ship. This incremental resistance coefficient for model-ship has very often been fixed at C_A = 0.0004. However experience has shown that C_A decreases with increasing ship size and following roughness correction coefficient is proposed according to Harvald:

Δ = 1000t	$10^3 \cdot C_A$ =	0.6
Δ = 10000t	$10^3 \cdot C_A$ =	0.4
Δ = 100000t	$10^3 \cdot C_A$ =	0.0
Δ = 1000000t	$10^3 \cdot C_A$ =	-0.6

The C_A values in the table can be estimated using the following expression by Harvald (1983):

$$1000 \cdot C_A = 0.5 \cdot log(\Delta) - 0.1 \cdot \left(log(\Delta)\right)^2$$

Using the above mentioned correction formula results in too low resistance values for larger vessels (displacement more than 160000 t). Therefore following revised equation is used for calculation of C_A:

$$1000 \cdot C_A = Maximum\left(-0.1;\ 0.5 \cdot log(\Delta) - 0.1 \cdot \left(log(\Delta)\right)^2\right)$$

The minimum C_A value of -0.1 x 10^{-3} has been found by using the 'trial and error' principle until reasonable correlations between empirical calculated propulsion power data and full scale power values were obtained.

Air Resistance Coefficient

Air resistance caused by the movement of the ship through the air, shall be included in the resistance calculation procedure. The air resistance X can be calculated by following formula:

$$R_{air} = X = \frac{1}{2} \cdot C_X \cdot \rho_{air} \cdot A_{VT} \cdot V^2$$

where:

C_X	Wind resistance coefficient
ρ_{air}	Density of air
A_{VT}	Front area of ship

The air resistance coefficient C_{AA} is defined as follows:

$$C_{AA} = \frac{X}{\frac{1}{2} \cdot \rho_w \cdot V^2 \cdot S}$$

As the ratio between air and water density is 825 the air resistance coefficient becomes:

$$C_{AA} \approx C_X \cdot \frac{A_{VT}}{825 \cdot S}$$

See Appendix A for analysis of this factor. Based on this analysis the following air resistance coefficient; C_{AA} values, are recommended.

Tankers and Bulk Carriers

	C_{AA}·1000
Small tankers	0.07
Handysize tankers	0.07
Handymax tankers	0.07
Panamax tankers	0.05
Aframax tankers	0.05
Suezmax tankers	0.05
VLCC	0.04

Container Vessels

$C_{AA} \cdot 1000 = 0.28 \cdot TEU^{-0.126}$ but newer less than 0.09

Steering Resistance

It is here decided not to include a correction for added steering resistance.

Residual Resistance Coefficient C_R – Harvald (1983)

The residual resistance coefficient, C_R, is defined as the total model resistance coefficient minus the model friction resistance coefficient, i.e:

$$C_{Rm} = C_{Tm} - C_{Fm}$$

The residual resistance includes wave resistance, the viscous pressure resistance, and the additional resistance due to the form or curvature of the hull.

As the residual resistance coefficient of the ship model is identical with the residual resistance coefficient of the ship, C_R is normally determined by model tests, where the resistance in model scale is measured and converted to full scale values according to methods agreed upon by the International Towing Tank Committee (ITTC) as example by using the resistance correction factors, C_A and C_{AA} as described earlier. Alternatively the residuary resistance can be predicted by empirical calculation methods, which are based on analysis of many model tests results.

One of the most well known methods has been developed by Holtrop and Mennen [Holtrop and Mennen, 1978] from the model tank in Holland (MARIN). This method is very flexible, but many details are needed as input for the calculation procedure, and the calculation model by Holtrop and Mennen is therefore not suitable when a quick calculation procedure is needed.

In 1965 - 1974 Guldhammer and Harvald developed an empirical method ("Ship Resistance") based on an extensive analysis of many published model tests. The method depends on relatively few parameters and is used for residual resistance prediction in the present analyses. Harvald presents curves (see Appendix H and I) for C_R ($C_{R,Diagram}$) as function of three parameters: 1) The length-displacement ratio (M), 2) the prismatic coefficient (Cp) and finally 3) the Froude number (Fn).

By an extensive regression analysis of the original C_R curves (shown in Appendix H) following expressions have been developed by Guldhammer in 1978:

$C_R = f(M, C_P, Fn)$

$10^3 \cdot C_R = E + G + H + K$

where:

$E = \left(A_o + 1.5 \cdot Fn^{1.8} + A_1 \cdot Fn^{N_1}\right) \cdot \left(0.98 + \frac{2.5}{(M-2)^4}\right) + (M-5)^4 \cdot (Fn - 0.1)^4$

$A_o = 1.35 - 0.23 \cdot M + 0.012 \cdot M^2$

$A_1 = 0.0011 \cdot M^{9.1}$

$N_1 = 2 \cdot M - 3.7$

$G = \frac{B_1 \cdot B_2}{B_3}$

$B_1 = 7 - 0.09 \cdot M^2$

$B_2 = (5 \cdot C_P - 2.5)^2$

$B_3 = (600 \cdot (Fn - 0.315)^2 + 1)^{1.5}$

$H = EXP(80 \cdot \left(Fn - (0.04 + 0.59 \cdot C_P) - 0.015 \cdot (M-5)\right))$

$K = 180 \cdot Fn^{3.7} \cdot EXP(20 \cdot C_P - 16)$

The formula for C_R is valid for $Fn <= 0.33$

The resistance coefficient C_R calculated according to the formulas above is given without correction for hull form, bulbous bow or position of *LCB*. Harvald gives additional corrections for these parameters.

The residual resistance coefficient curves must be corrected for

- Position of *LCB* ($\Delta C_{R,LCB}$)
- Shape / hull form ($\Delta C_{R,form}$)
- *B/T* deviation from 2.5 (C_R curves are all given a breadth-draft ratio equal 2.5) ($\Delta C_{R,B/T\neq2.5}$)
- Bulbous bow shape and size ($\Delta C_{R,bulb}$)

$$C_R = C_{R,Diagram} + \Delta C_{R,B/T\neq2.5} + \Delta C_{R,LCB} + \Delta C_{R,form} + \Delta C_{R,bulb}$$

A proposal for corrections for *LCB* not placed amidships in the vessel is given. Harvald allows only *LCB* forward of amidships and the correction will always be positive, which gives an increased resistance.

➔ In the present analysis the LCB correction will be ignored

The correction for both the hull form and the *B/T* correction are used as described by Harvald. These factors are assumed not to have changed since the method was developed by Harvald; the correction must be the same disregarding age of vessel.

➔ Correction of form and B/T is in the present project taken as Harvald recommends: No correction for B/T equal 2.5, else $\Delta C_{R,bulb} = 0.16 \cdot \left(\frac{B}{T} - 2.5\right) \cdot 10^{-3}$

➔ Hullform
A hull shape correction to C_R is applied if the aft or fore body is either extremely U og V shaped

Fore body	Extreme U: $- 0.1 \cdot 10^{-3}$	Extreme V: $+ 0.1 \cdot 10^{-3}$
After body	Extreme U: $+ 0.1 \cdot 10^{-3}$	Extreme V: $- 0.1 \cdot 10^{-3}$

Bulbous bow forms have been optimised and bulbs developed in the recent years can reduce the resistance quite considerably. Earlier non-projecting bulbous bows decreased resistance at best by some 5 – 10 %. Modern bulbs can decrease resistance by up to 15 - 20% [Schneekluth and Bertram 1998]

➔ New analyses and equations for bulbous bow corrections will be included in the present analyses.

<u>Draft dependency (Tankers and bulk carriers):</u>

Assuming C_M constant equals 0.995, the prismatic coefficient can approximately be set to C_B, which is near constant for each vessels size. The coefficient, C_M, will for most vessels be constant or slightly decrease for decreasing draft. As M is both length and displacement dependent, this value will also be draft dependent. The Froude number is independent of the draft.

Bulbous Bow Correction for Bulk Carriers and Tankers

In the method by Harvald it is assumed that the ship has a standard non bulbous bow. The method includes corrections for a bulbous bow having a cross section area of at least 10 % of the midship section area of the ship. There has been written much about the influence of a bulbous bow on the ship resistance. Many details have an influence, as example the transverse and longitudinal shape of a bulbous bow including its height compared to the actual operational draught.

The bulb correction might, as C_R, be function of the of three parameters, 1) the length-displacement ratio (M), 2) the prismatic coefficient (C_P) and 3) the Froude number (Fn).

In Appendix C it is shown that M and C_P vary within a limited range for tankers and bulk carriers as follows:

M:	4.4 – 5.2
Cp:	0.78 – 0.87

For a given condition/draught the wave pattern and therefore the residual resistance varies mainly with the speed. The bulbous bow correction will therefore mainly be a function of the Froude number.

$$\Delta C_{R,bulb} = \Delta C_{R,bulb}(Fn)$$

The bulb correction will also be draft and trim dependent, but this dependency can be very complex. Therefore in this analysis for bulk carriers and tankers, the bulb correction has been assumed to be independent of these two parameters and only dependent on the Froude number.

In the present project, the bulb correction is determined by analysis of several model tests results for ships having bulbous bows. The total resistance coefficient of each individual ship has been calculated by Harvalds method without any corrections for bulbous bow. Subtracting this value from the total resistance coefficient found by model tests gives the bulbous bow correction which is needed for updating of the method. See Appendix D.

For tankers and bulk carriers the correction thus found can be approximated by following formula:

$$\Delta C_{R,bulb} = \mathrm{Max}(-0.4; -0.1 - 1.6 \cdot Fn)$$

For all ship sizes the bulb correction is calculated by both Harvalds method and the new proposal for tankers and bulk carriers, see Figure 1.

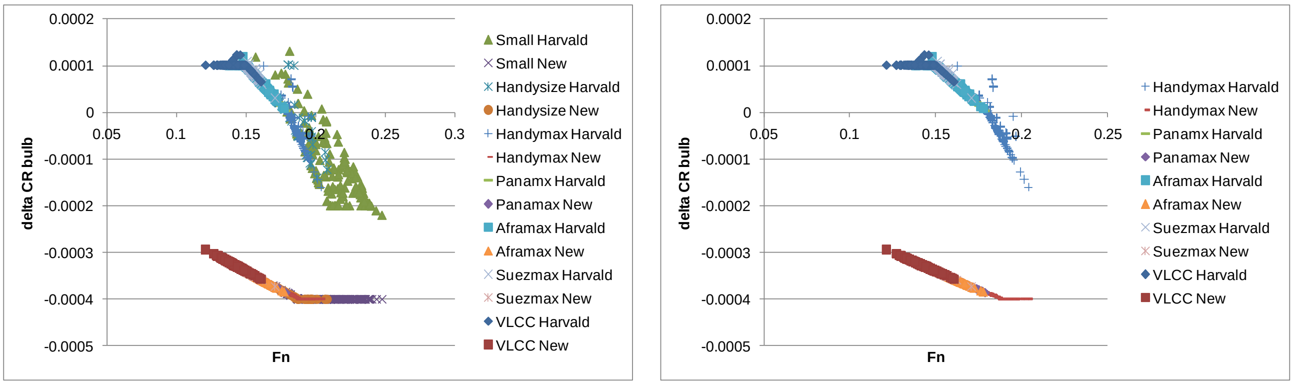


Figure 1. The bulb correction calculated using Harvalds original bulb correction and the new correction proposal. (Tankers – standard vessels).

For all vessels and for all values of *Fn*, the new bulb correction will be negative, meaning that the bulb will decrease the total resistance on all vessel sizes. A relatively large scatter is seen for small and handysize vessels for Harvalds method, this is due to the large standard deviation in C_P for these vessels.

Bulbous Bow Correction for Container Ships

The bulbous bow correction for container ships will also be a function of the Froude number. Also for this ship type, the bulb correction is determined by analysis of several model tests results for ships having bulbous bows and having a block coefficient in the range 0.5 – 0.7. The total resistance coefficient of each individual ship has been calculated by Harvald´s method without any corrections for bulbous bow. Subtracting this value from the total resistance coefficient found by model tests gives the bulbous bow correction which is needed for updating of the method. See Appendix E.

For container ships the correction found is different from the bulb correction found for tankers and bulk carriers as it is expressed as a percentage of the residual resistance found by Harvald´s method without bulb correction (see Fig. 2). The correction is still a function of the Froude number and it is still negative in the normal speed range. The new bulbous bow correction can be approximated by following formula (detailed description is given in Appendix E):

$$\Delta C_{R,bulb} = (250 \cdot Fn - 90) \cdot \frac{C_{R\ Harvald\ \ NO\ bulbous\ bow}}{100}$$

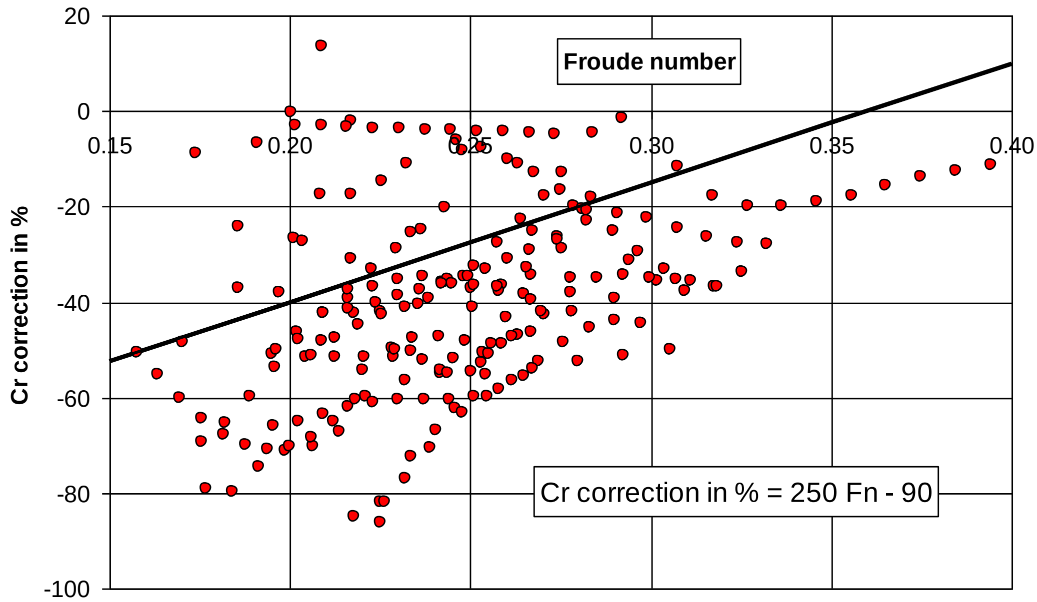


Fig. 2 Residual resistance coefficient correction due to the influence of a bulbous bow found by model tests

Bulbous bow correction for twin-skeg container ships

Very few container ships are built as twin-skeg container ships (as example Maersk Line Triple E). Model tests for some twin-skeg ships, mostly Ro-Ro ships but also for 3 twin-skeg ships, have been analysed in order to obtain a correction factor for the residual resistance coefficient for twin-skeg container Ro-Ro ships.

The twin-skeg correction is still a function of the Froude number and it is still negative in the normal speed range. The new bulbous bow C_R correction for twin-skeg container ships is -0.2 10-3 as can be seen from Fig. 3

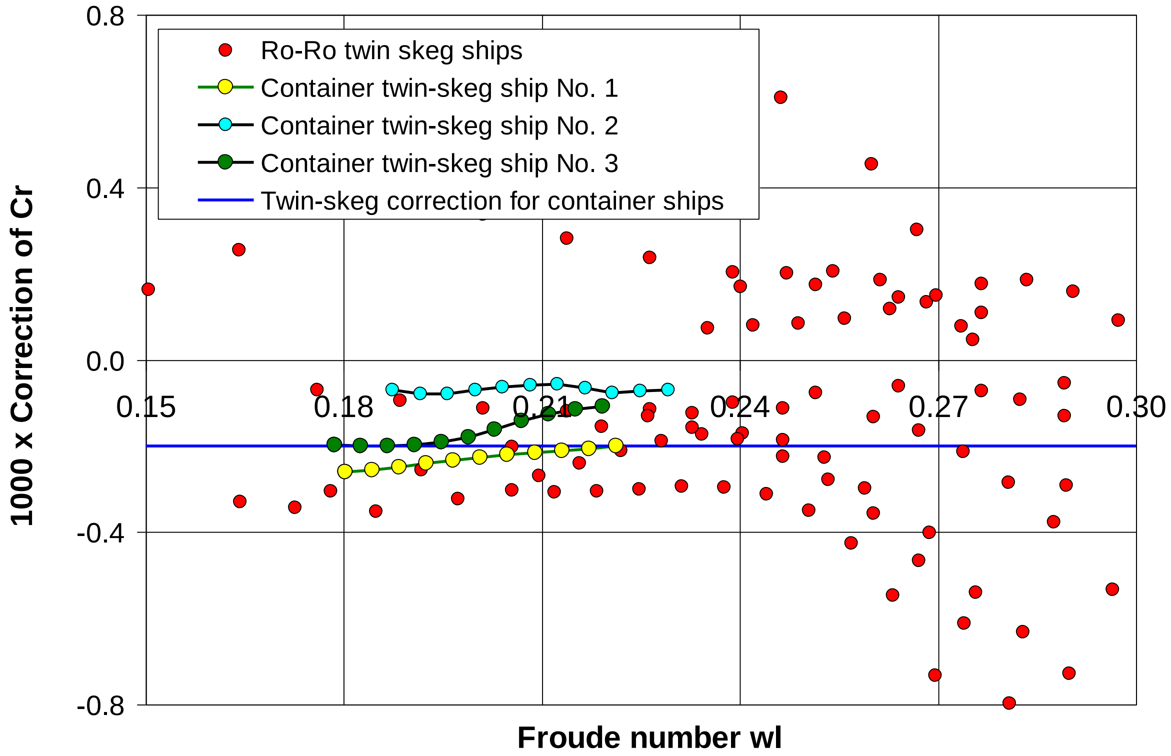


Fig. 3 Residual resistance coefficient correction due to the influence of a bulbous bow found by model tests for twin-skeg ships

Total Ship Resistance

$$R_T = \frac{1}{2} \cdot C_T \cdot \rho \cdot S \cdot V^2$$

Effective Power

$$P_E = R_T \cdot V$$

Service allowance

The service allowance is used for determination of the installed main engine power, which means that it shall be determined based on the expected service area. Harvald suggests following service allowances:

North Atlanctic route, westbound	25 – 35 %
North Atlantic, eastbound	20 – 25 %
Europe Australia	20 – 25 %
Europe – Eastern Asia	20 – 25 %
The Pacific routes	20 – 30 %

The above figures are only rough figures, which can be used for guidance. For more accurate predictions, the size of the ship shall be taken into account, as the service allowance will be relatively higher for small ships compared to large ships. Furthermore the hull form will also have an influence on the necessary service allowance. The more slender hull form, the less service allowance is needed.

$$P_E service = R_T \cdot V \cdot (1 + \frac{sercice\ allowance\ in\ \%}{100})$$

Propulsive Efficiencies

Total efficiency: $\eta_T = \eta_H \cdot \eta_O \cdot \eta_R \cdot \eta_S$

η_H Hull efficiency
η_o Propeller in open water condition
η_R Relative rotative efficiency
η_S Transmission efficiency (shaft line and gearbox)

<u>Hull efficiency</u>

η_H The hull efficiency is a function of the wake fraction, w, and the thrust deduction fraction, t, [Harvald 1983]

$$\eta_H = \frac{1-t}{1-w}$$

Wake fraction: $w = w_1\left(\frac{B}{L}, C_B\right) + w_2(\text{form}, C_B) + w_3\left(\frac{D_{prop}}{L}\right)$

Thrust deduction fraction: $t = t_1\left(\frac{B}{L}, C_B\right) + t_2(\text{form}) + t_3\left(\frac{D_{prop}}{L}\right)$

For normal N-shaped hull forms, w_2 and t_2 will be equal 0, which means that both the wake fraction and the thrust deduction is a function of the breadth-length ratio, the ratio of the propeller diameter and the length and finally the block coefficient.

The form in the aft body (F_a) can be described by factors: [-2, 0, +2], negative values for U-shape, positive for V-shape and zero for N-shaped hull form.
The approximations given by Harvald are used in the present work. In [Harvald 1983] are all values given in diagrams. These values are approximated by simple regression formulas as follows.

The wake fraction:

$$w = w_1 + w_2 + w_3$$

$$w_1 = a + \frac{b}{c \cdot (0.98 - C_B)^3 + 1}$$

$$w_2 = \frac{0.025 \cdot F_a}{100 \cdot (C_B - 0.7)^2 + 1}$$

$$w_3 = -0.18 + \frac{0.00756}{\frac{D_{Prop}}{L} + 0.002}$$ and $w_3 \leq 0.1$,

$$a = \frac{0.1 \cdot B}{L} + 0.149$$

$$b = \frac{0.05 \cdot B}{L} + 0.449$$

$$c = 585 - \frac{5027 \cdot B}{L} + 11700 \cdot \left(\frac{B}{L}\right)^2$$

D_{prop} is the propeller diameter. If not known the following approximations can be used to calculate D_{Prop} as function of the maximum draught (see Appendix F for statistical analysis):

Tankers and bulk carriers: $D_{prop} = 0.395 \cdot \text{max. draught} + 1.30$

Container ships: $D_{prop} = 0.623 \cdot \text{max. draught} - 0.16$

Ro Ro ships: $D_{prop} = 0.713 \cdot \text{max. draught} - 0.08$

For trial trip conditions with clean hull the wake fraction shall be reduced by 30% for single screw ships. For twin screw vessels no reduction is to be applied.

The trust deduction fraction:

$$t = t_1 + t_2 + t_3$$

$$t_1 = d + \frac{e}{f \cdot (0.98 - C_B)^3 + 1}$$

$$t_2 = -0.01 \cdot F_a$$

$$t_3 = 2 \cdot \left(\frac{D_{Prop}}{L} - 0.04\right)$$

$$d = \frac{0.625 \cdot B}{L} + 0.08$$
$$e = 0.165 - \frac{0.25 \cdot B}{L}$$
$$f = 525 - \frac{8060 \cdot B}{L} + 20300 \cdot \left(\frac{B}{L}\right)^2$$

The wake fraction and thrust deduction fraction have been calculated by Harvalds method for the same ships which have been used for deduction of the residual resistance correction mentioned earlier. The results of the analysis are shown in Appendix G, which show that the wake fraction according to Harvald is slightly higher than obtained from model tests. The same is also valid for the thrust deduction fraction.

I order to obtain more correct values of *w* and *t* (which corresponds with the model test values), the difference between the values obtained by model tests and calculated by Harvald´s method were plotted as function of the length displacement ratio, M. These results are shown in Appendix G. It is seen that the difference depends on the length displacement ratio such that the difference is highest for the lowest length displacement ratios.

Based on the analysis in Appendix G, following corrected formulas for calculation of the wake fraction and the thrust deduction fraction for tankers and bulk carriers have been derived:

$$w_{Corrected} = 0.7 \cdot w_{Harvald} - 0.45 + 0.08 \cdot M$$

$$t_{Corrected} = t_{Harvald} - 0.26 + 0.04 \cdot M$$

The updated values of the hull efficiency according to the new formulas are also shown in Appendix G. The mean value of model test generated hull efficiencies is identical with the mean value of the corresponding hull efficiency calculated by using the corrected w and t formulas.

Propeller efficiency

η_o In Breslin and Andersen [1994] are curves for efficiencies of various propulsion devises given. The efficiency is presented as function of the thrust loading coefficient C_{Th}.

The trust loading coefficient:

$$C_{Th} = \frac{T}{\frac{1}{2} \cdot \rho \cdot A_{disk} \cdot {V_A}^2} \quad \text{and} \quad C_{Th} = \frac{8}{\pi} \cdot \frac{R}{(1-t) \cdot \rho \cdot (V_A \cdot D_{prop})^2} \quad \text{as}$$

$$C_{Th} = \frac{8}{\pi} \cdot \frac{K_T}{J^2} \qquad J = \frac{V_A}{n \cdot D} \qquad K_T = \frac{R}{(1-t) \cdot \rho \cdot n^2 \cdot D_{prop}^4}$$

$$R = (1-t) \cdot T \qquad V_A = (1-w) \cdot V$$

Breslin and Andersen [1994] shows curves for approximated values of η_o for the conventional Wageningen B – series propellers. The values taken from this curve will here be denoted as $\eta_{o,Wag}$

As the propeller efficiency is primary a function of the thrust loading coefficient C_{Th} it is the intention is to determine a function, f, so $\eta_{o,Wag} = \eta_{o\ ideal} \cdot f(C_{Th})$ where $\eta_{o\ ideal}$ is the co-called ideal efficiency defined by:

$$\eta_{o\ ideal} = \frac{2}{1+\sqrt{\frac{T}{\frac{1}{2}\rho \cdot A_{disk} \cdot V_A^2}+1}} = \frac{2}{1+\sqrt{C_{Th}+1}}$$

When dividing $\eta_{o,Wag}$ with $\eta_{o\ ideal}$ it is found that $f(C_{Th})$ can be expressed by a linear function: $f(C_{Th}) = 0.81 - 0.014 \cdot C_{Th}$ however not lower than 0.69 resulting in following equation:

$$\eta_{o,Wag} = \frac{2}{1+\sqrt{C_{Th}+1}} \mathrm{Max}(0.69; (0.81 - 0.014 \cdot C_{Th}))$$

In Fig. 4 are shown comparisons between the Wageningen efficiency values form Andersen and Breslin (Fig. 7) and the above mentioned approximate equation and some additional results from Wageningen B-series calculations. These additional calculated results were prepared to cover a larger C_{Th} range than obtained from Andersen and Breslin.

The efficiency calculated by the approximated propeller efficiency equation is compared with some open water efficiencies found from model tests with different ship types (Fig. 5). From this comparison it is observed that the model tests results are 3 – 5 % lower than the approximated Wageningen efficiency.

Experience (by model tanks and propeller manufacturers) from comparisons of efficiencies from model tests with full-scale efficiencies shows that model test values are normally 3 – 5 % lower than full-scale values. This means that the propeller efficiency obtained by the above mentioned expression represents the full scale efficiency. In the efficiency diagram by Andersen and Breslin is also shown an efficiency curve for a ducted propeller solution (denoted "Kort nozzle"). Using the same principles as for the Wageningen propeller curves following equation has been derived for the ducted propeller efficiency $\eta_{o,nozzle}$:

$$\eta_{o,nozzle} = \eta_{o\ ideal} \cdot g(C_{Th})$$

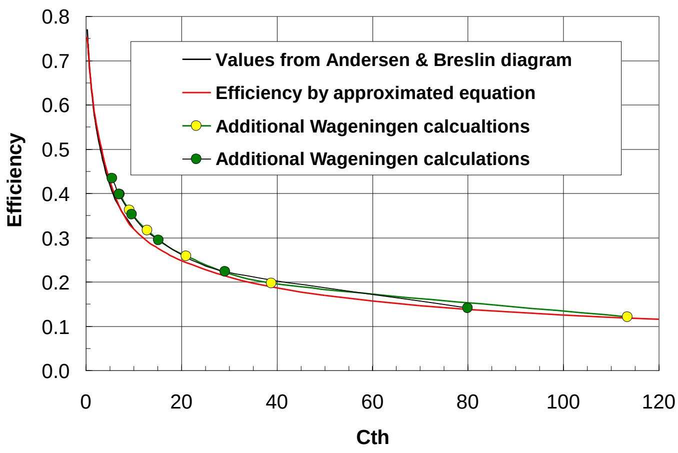


Fig. 4 Efficiencies for a Wageningen B-series propeller based on Andersen and Breslin and numerical approximation

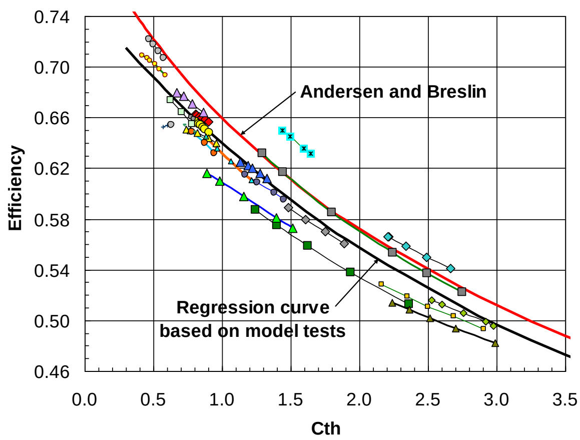


Fig. 5 Propeller Wageningen B series efficiencies from Andersen and Breslin compared with efficiencies obtained from

model tests

Up to a C_{Th} value of 7 the function $g(C_{Th})$ can be approximated by a forth degree polynomial of C_{Th}, as shown below:

$$g = 0.59 + 0.177 \cdot C_{Th} - 0.0462 \cdot C_{Th}^2 + 0.00518 \cdot C_{Th}^3 - 0.000205 \cdot C_{Th}^4 \text{ for } C_{Th} < 7$$

and for $C_{Th} \geq 7$: $g = 0.85$

In Fig. 6 are shown comparisons between the nozzle efficiency values from Andersen and Breslin and the above mentioned approximate equation for a nozzle propeller.

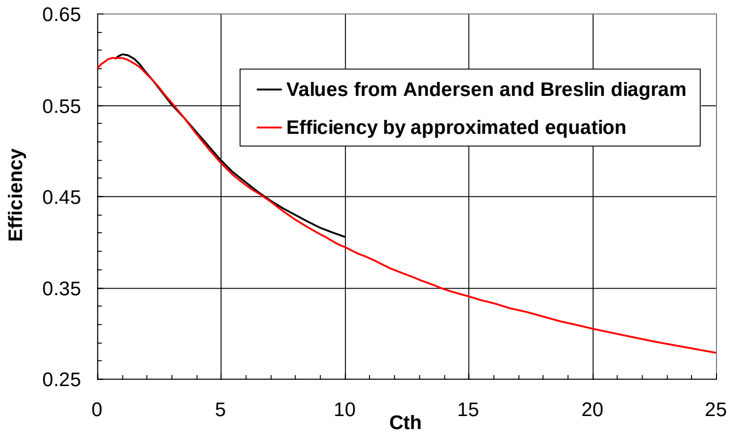


Fig. 6 Efficiencies for a nozzle propeller based on Andersen and Breslin and numerical approximation. Normally C_{Th} is less than 10, but the efficiency approximation has been extended in order to cover more extreme bollard pull conditions where C_{Th} is higher than 10.

By expressing the open water efficiency as function of the thrust loading coefficient, it is possible to obtain a relatively accurate efficiency without a detailed propeller optimization procedure. As the thrust loading depends on the propeller diameter and the resistance, these two parameters are automatically included in the efficiency calculation.

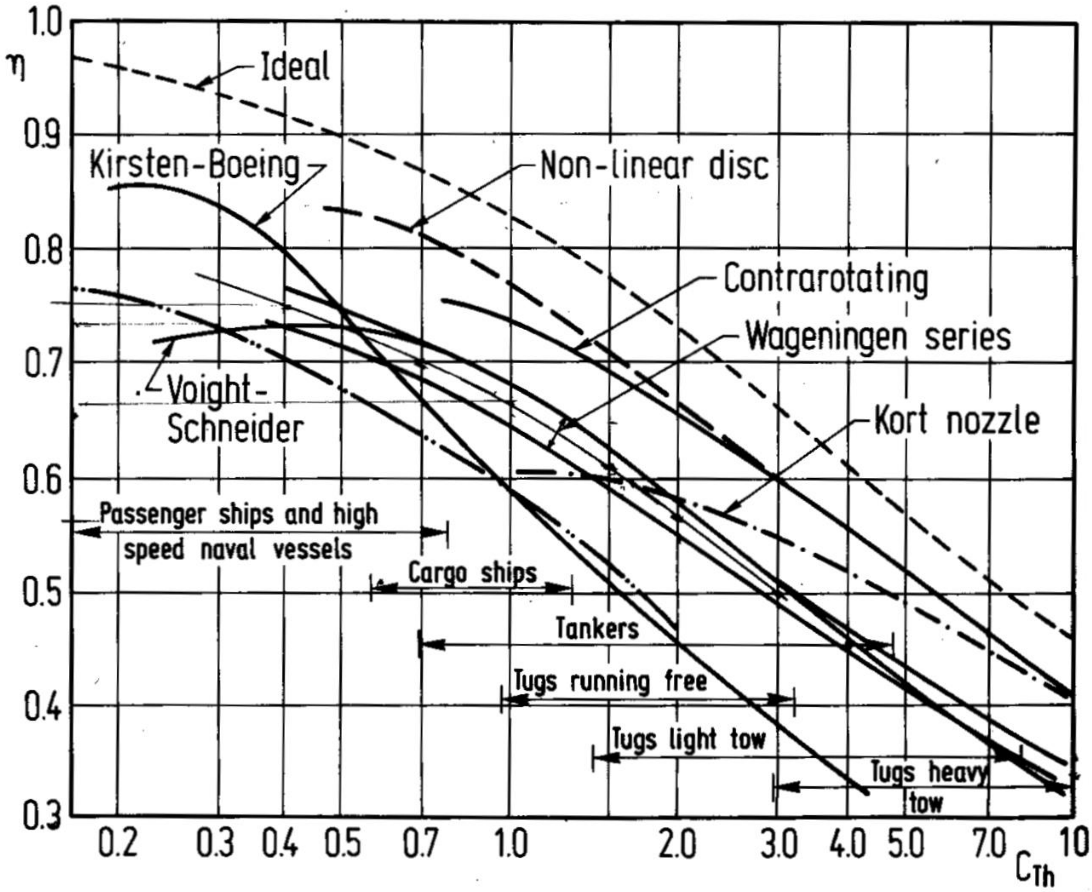


Fig. 7 Efficiencies of various propulsion devices and C_{Th} for different ship types (Andersen and Breslin)

<u>Relative rotative efficiency and shaft efficiency</u>

η_o, η_R — Behind propeller efficiency $\eta_B = \eta_O \cdot \eta_R \sim \eta_O$ as the relative rotative efficiency in average is close to one (it normally varies between 0.95 and 1.05)

η_S — The size of this value depends of propeller shaft length, number of bearings and the gearbox. For a shaft line with directly mounted propeller η_s is approximately 0.98, while it is 0.96 – 0.97 for a shaft system including a gearbox solution.

Propulsion Power, P_P

$$P_P = P_E \cdot \eta_T$$

References

Harvald S. A. (1983), *Resistance and Propulsion of Ships,* Wiley 1983, ISBN 0-89464-754-7

Holtrop, J. and Mennen, G. G. (1978), *A Statistical Power Prediction Method*, International Shipbuilding, Progress

Schneekluth, H. and Bertram, V. (1998), *Ship Design for Efficiency and Economy*, Second edition, Butterworth-Heinemann, ISBN 0 7506 4133 9

Breslin, S. and Andersen, P. (1994), *Hydrodynamics of Ship Propellers*, Cambridge: Cambridge University Press

Blendermann, W. (1986), *Die Windkräfte am Shiff*, Institut of Naval Architecture, University of Hamburg

Appendix A – Air Resistance

The axial wind force coefficient: $C_X = \frac{X}{\frac{1}{2} \cdot \rho_{air} \cdot V^2 \cdot A_{VT}}$

The air resistance coefficient: $C_{AA} = \frac{X}{\frac{1}{2} \cdot \rho_w \cdot V^2 \cdot S}$

The relation between C_{AA} and C_{xx}: $C_{AA} = C_X \cdot \frac{\rho_{air}}{\rho_w} \cdot \frac{A_{VT}}{S} \approx C_X \cdot \frac{A_{VT}}{800 \cdot S}$

The value of C_x [Blendermann 1986]:

Bulk carriers and tankers	0.85
Container vessels	0.8
Ro Ro ships (cargo and passenger)	0.8

Wetted surface: Se Appendix B.

Tankers and Bulk Carriers

Estimation of front area A_{VT}: $A_{VT} = B \cdot (D - T + h)$

Accommodation height: h
The accommodation height is defined by the number of floors and floor height. Based on photo observations the floor number is estimated. A floor height of 3 m is used. An additional height of 2 m is added counting for equipment at top of vessel.

	Number of floors	C_{AA} (mean) ·1000	C_{AA} (standard dev.) ·1000
Small	3	0.074	0.010
Handysize	4	0.069	0.007
Handymax	5	0.069	0.003
PanaMax	5	0.049	0.002
Aframax	5	0.052	0.002
Suezmax	5	0.052	0.002
V.L.C.C.	5	0.040	0.002

From the above analyses are the following C_{AA} values recommended:

	C_{AA} ·1000
Small	0.07
Handysize	0.07
Handymax	0.07
Panamax	0.05
Aframax	0.05
Suezmax	0.05
V.L.C.C.	0.04

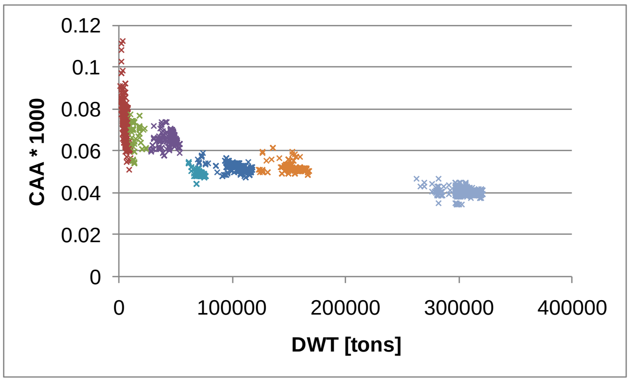


Fig. A1 The air resistance coefficient as function DWT – all tankers and bulk carriers.

Container Vessels

Estimation of front area A_{VT}: $A_{VT} = B \cdot (D - T + h)$

Accommodation height: h

The accommodation height is a function of the number container tiers on deck as can be seen from Fig. A2, showing the number tiers of containers (8.5 feet high) for different vessel sizes. In addition to the container stack some tiers of houses are extend above the containers as shown in Fig. A3. The breadth of these houses is often approximately a half ship breadth.

With the tiers shown in Fig. A2, a hatch height of 2 m and with wheelhouse and equipment at top of vessel (according to Fig. A3) following heights above the main deck have been calculated:

Feeder vessels:	11 - 20.6 m
Panamax vessels:	24.2 m
Post Panamax vessels:	24.2 – 26.8 m

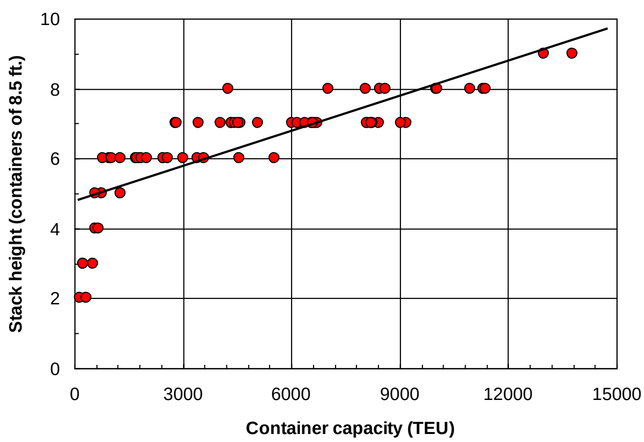


Fig. A2 Stack height of containers (of 8.5 feet each) on container ships (Significant Ships)

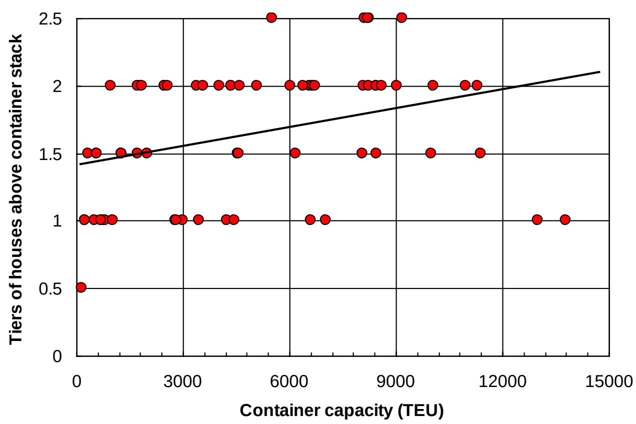


Fig. A3 Tiers of houses above the container stack (Significant Ships)

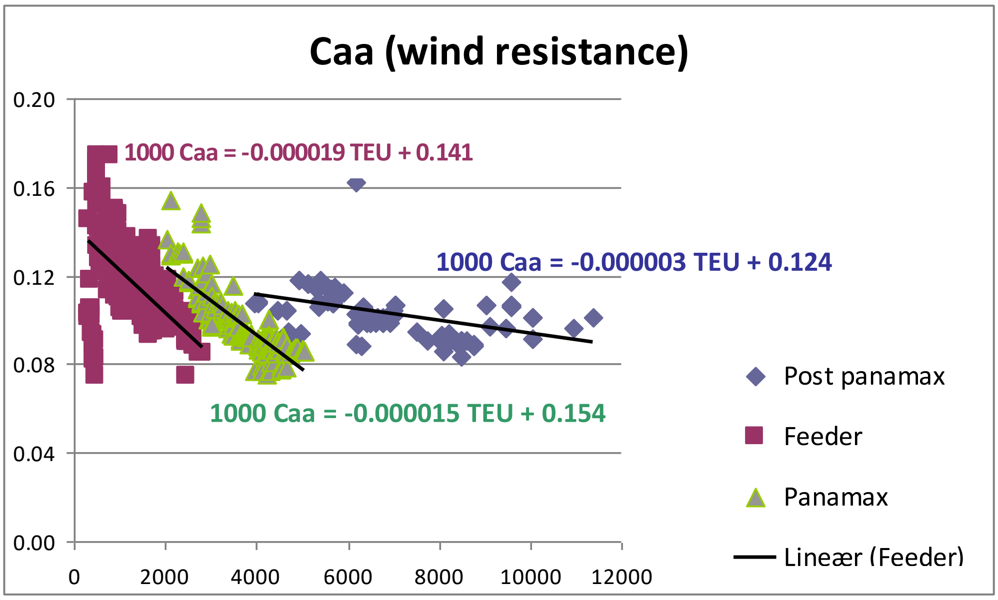


Fig. A4 Air resistance coefficient for container ships as function of container capacity (TEU)

In Fig. A4 are shown the calculated C_{AA} value for the different container ship sizes. However in order to obtain a continuous curve for all container ships a single curve has been deduced (Fig. A5) which is given by following expression:

$C_{AA} \cdot 1000 = 0.28 \cdot TEU^{-0.126}$ but newer less than 0.09

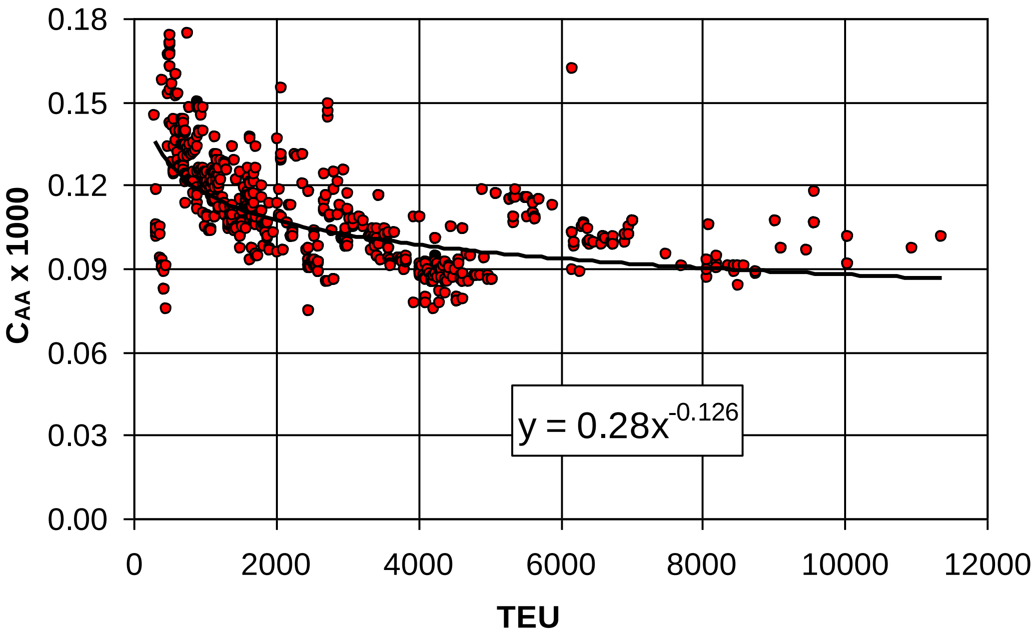


Fig. A5 Air resistance coefficient for container ships as function of container capacity (TEU)

Appendix B – Wetted surface

Tankers and bulk carriers

The equation used for calculation of the wetted surface in the present project is Mumfords formula according to [Harvald 1983, p. 131]:

$$S = 1.025 \cdot L_{pp} \cdot (C_B \cdot B + 1.7 \cdot T) = 1.025 \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{pp} \cdot T\right)$$

An analysis of wetted surface data of nearly 125 different newer ships (of different type as well as size) shows that the wetted surface according to the above mentioned version of Mumford´s formula can be up to 7 % too small or too high for container ships, bulk carriers and tankers and up to 15 % for Ro-Ro ships. Therefore it has been analysed if the formula can be adjusted slightly in order to increase the accuracy.

The wetted surface for tankers and bulk carriers (based on analysis of 35 vessels) can be calculated according to following formula (Fig. B1):

$$S = 0.99 \cdot \left(\frac{\nabla}{T} + 1.9 \cdot L_{wl} \cdot T\right)$$

The analysis shows (Fig. B1) that for 89 % of the ships the wetted surface is calculated with an uncertainty of less than 2 % when using the new proposal for the wetted surface. The uncertainty is less than 1 % for 49 % of the ships in the analysis which is a considerable improvement of the original Mumford formula.

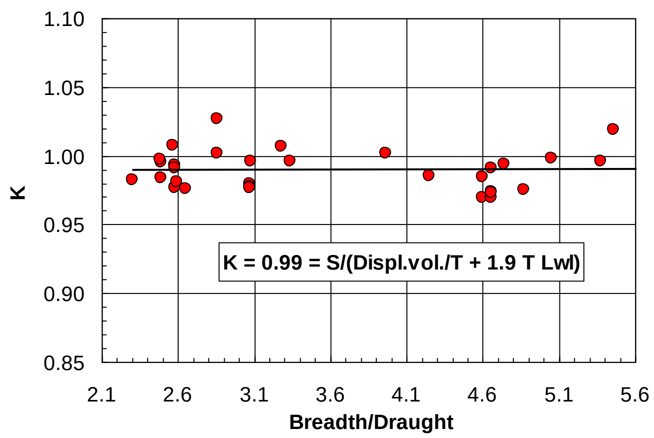


Fig. B1 Wetted surface coefficient for tankers and bulker carriers

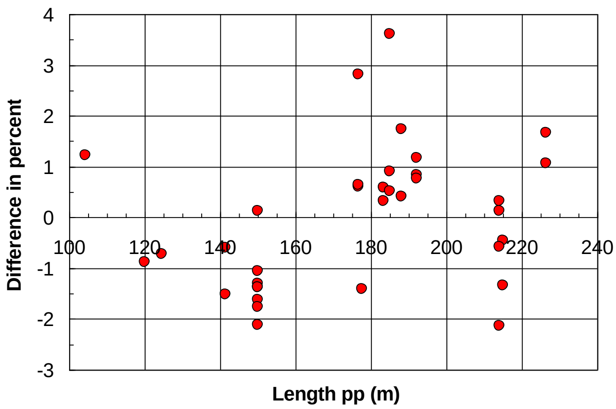


Fig. B2 Difference between actual and calculated wetted surface for tankers and bulk carriers

Container ships

The wetted surface for container ships (based on analysis of 38 vessels) can be calculated according to following formula (Fig. B3):

$$S = 0.995 \cdot \left(\frac{\nabla}{T} + 1.9 \cdot L_{wl} \cdot T\right)$$

The analysis shows (Fig. B4) that for more than 87 % of the ships the wetted surface is calculated with an uncertainty of less than 2 % when using the new proposal for the wetted surface. The uncertainty is less than 1 % for 47 % of the ships in the analysis which is a considerable improvement of the original Mumford formula.

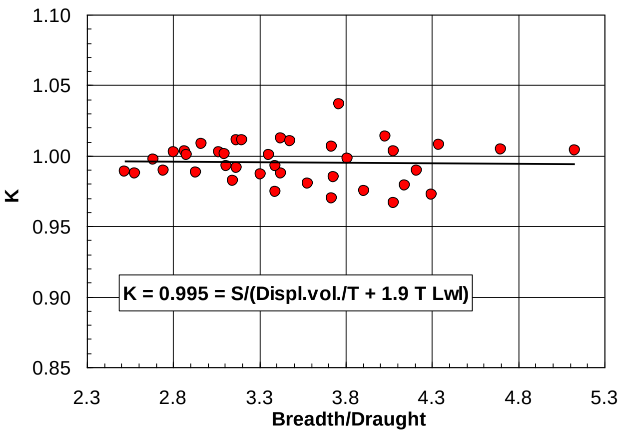


Fig. B3 Wetted surface coefficient for container ships

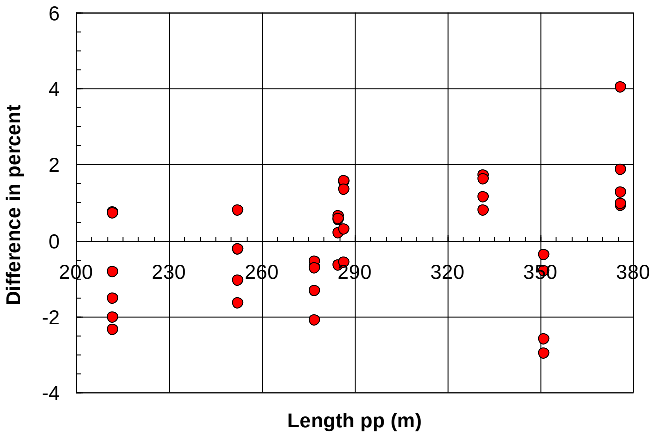


Fig. B4 Difference between actual and calculated wetted surface for container ships

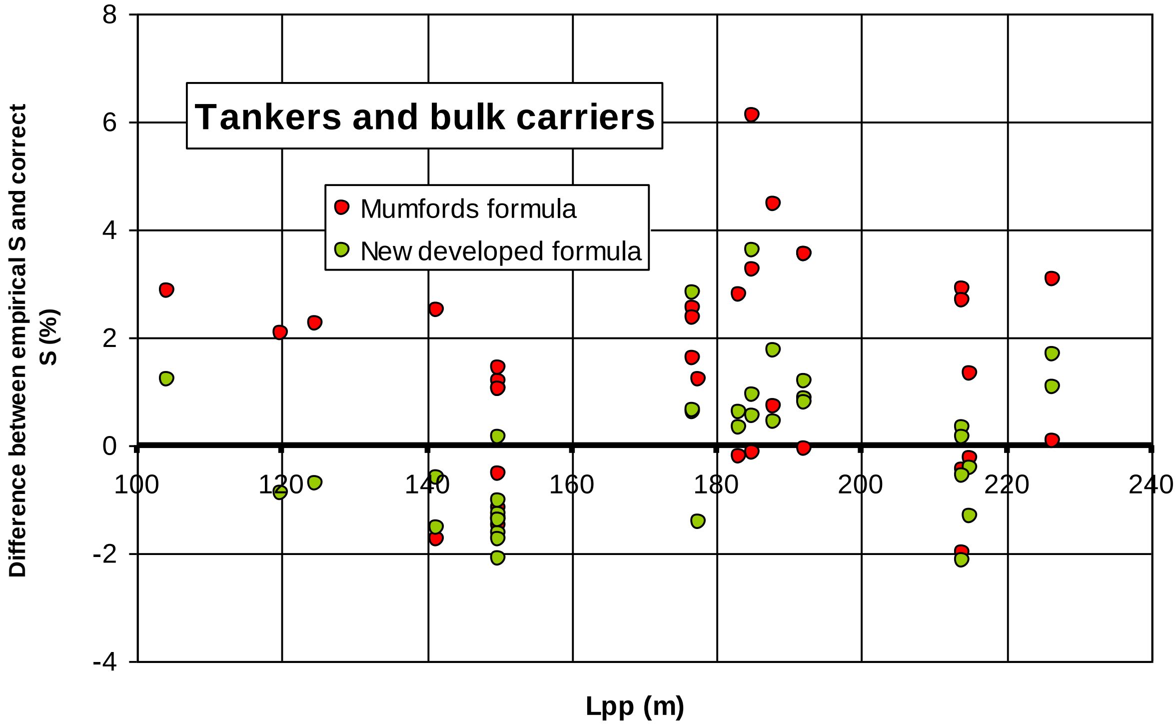


Fig. B5 Comparison between old and new formula for calculation of wetted surface, S, for tankers and bulk carriers

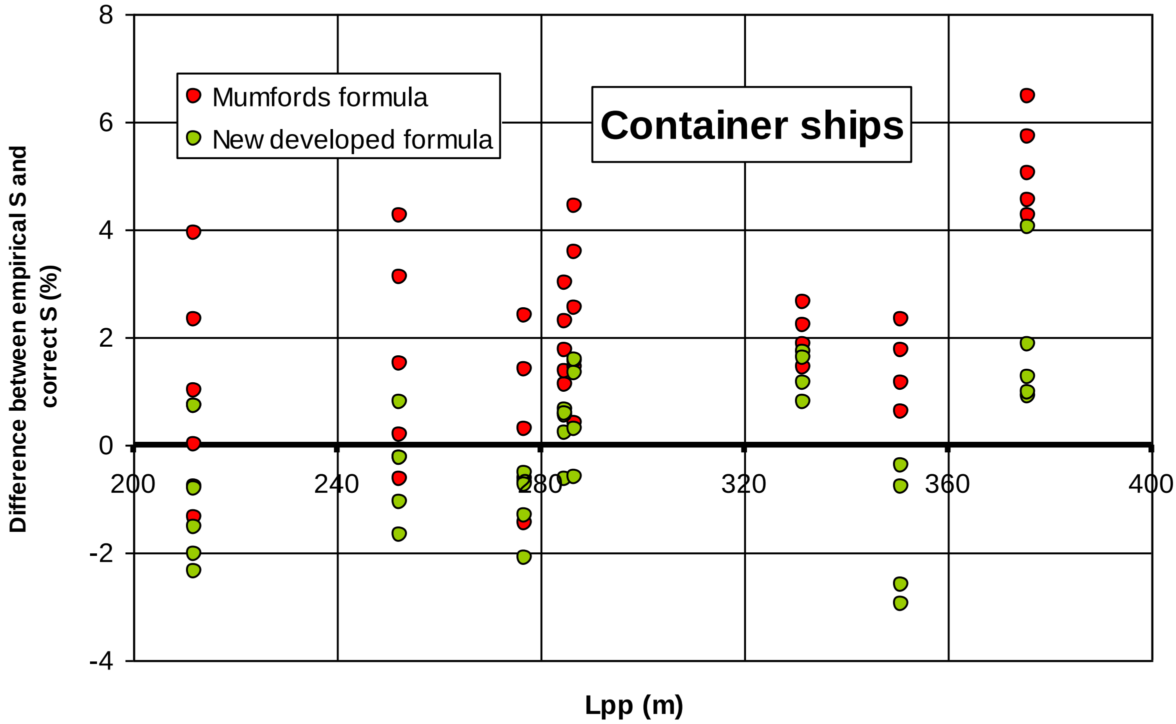


Fig. B6 Comparison between old and new formula for calculation of wetted surface, S, for container ships

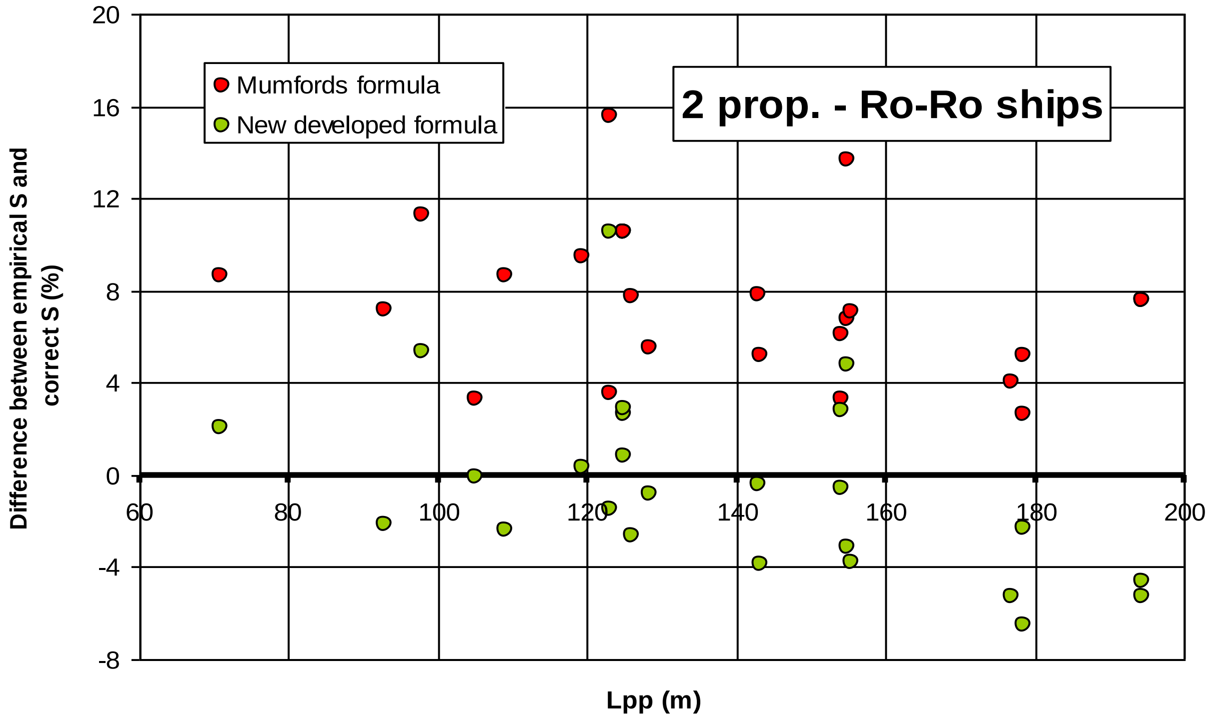


Fig. B7 Comparison between old and new formula for calculation of wetted surface, S, for conventional 2 propeller Ro-Ro ships

Wetted surface of Ro-Ro ships

An analysis of wetted surface data of 52 different Ro-Ro ships (of different type as well as size) shows that the wetted surface according to the above mentioned version of Mumford´s formula can be up to 15 % too small or too high (Fig. B6 and B7). Therefore it has been analysed if the formula can be adjusted to increase the accuracy.

Analysis of ship geometry data has shown that the wetted surface can be calculated according to following modified Mumford formulas:

$S = X \cdot \left(\frac{\nabla}{T} + 2.7 \cdot L_{wl} \cdot T\right)$ for single screw Ro-Ro ships

$S = X \cdot \left(\frac{\nabla}{T} + 1.3 \cdot L_{wl} \cdot T\right)$ for twin screw Ro-Ro ships

$S = X \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{wl} \cdot T\right)$ for twin-skeg Ro-Ro ships

The X- value for the three different ships types are show in Fig. B8

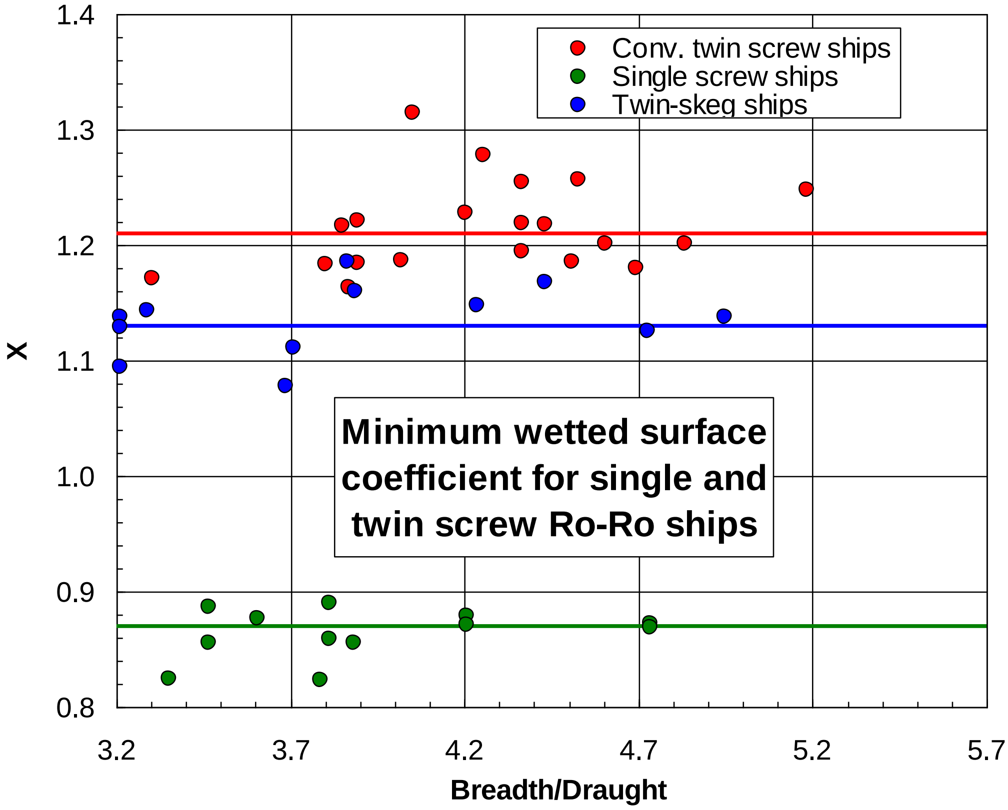


Fig. B8 Constant X is the modified Mumford formula

Using the modified Mumford formulas increases the accuracy of calculation of the wetted surface. However a further analysis reveals that the block coefficient also has an influence on the wetted surface, which can be seen by comparing the actual wetted surface with the wetted surface calculated according to the revised Mumford formula.

The results of this comparison are shown on Fig. B9 – B11. Based on the correction factors following equations for calculation of the wetted surface have been deducted:

Single screw Ro-Ro ships	$S = 0.87 \cdot \left(\frac{\nabla}{T} + 2.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin screw ship Ro-Ro ships with open shaft lines and twin rudders	$S = 1.21 \cdot \left(\frac{\nabla}{T} + 1.3 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.34 \cdot C_{BW})$
Twin-skeg Ro-Ro ships with two propellers and twin rudders	$S = 1.13 \cdot \left(\frac{\nabla}{T} + 1.7 \cdot L_{wl} \cdot T\right) \cdot (1.2 - 0.31 \cdot C_{BW})$

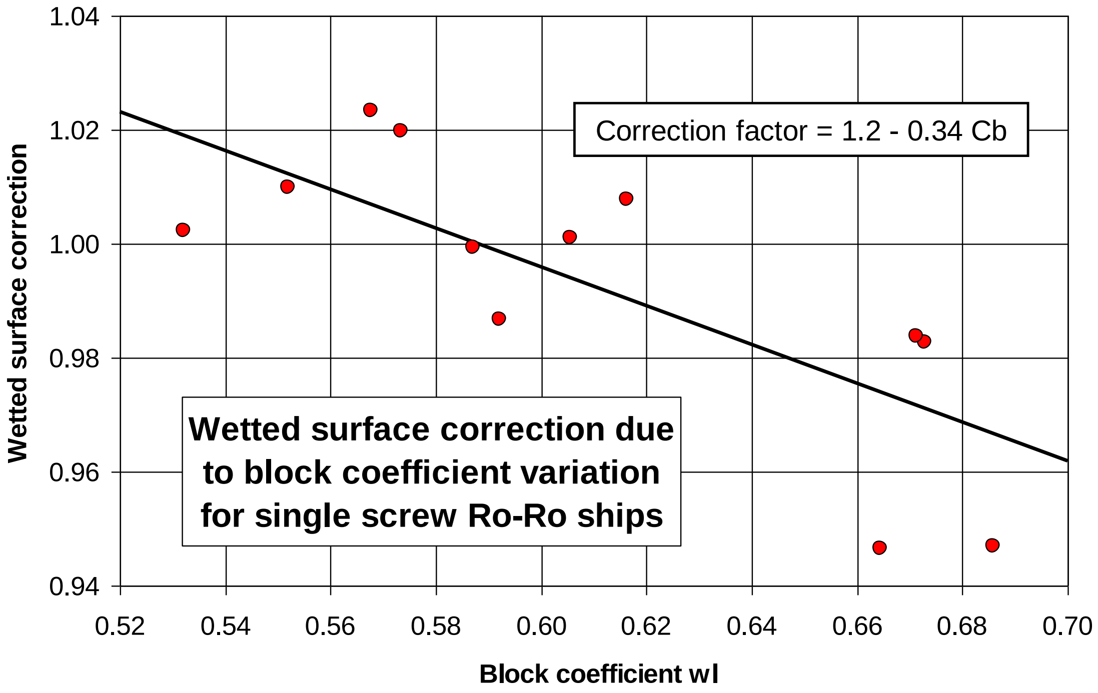


Fig. B9 Wetted surface correction for single screw Ro-Ro ships

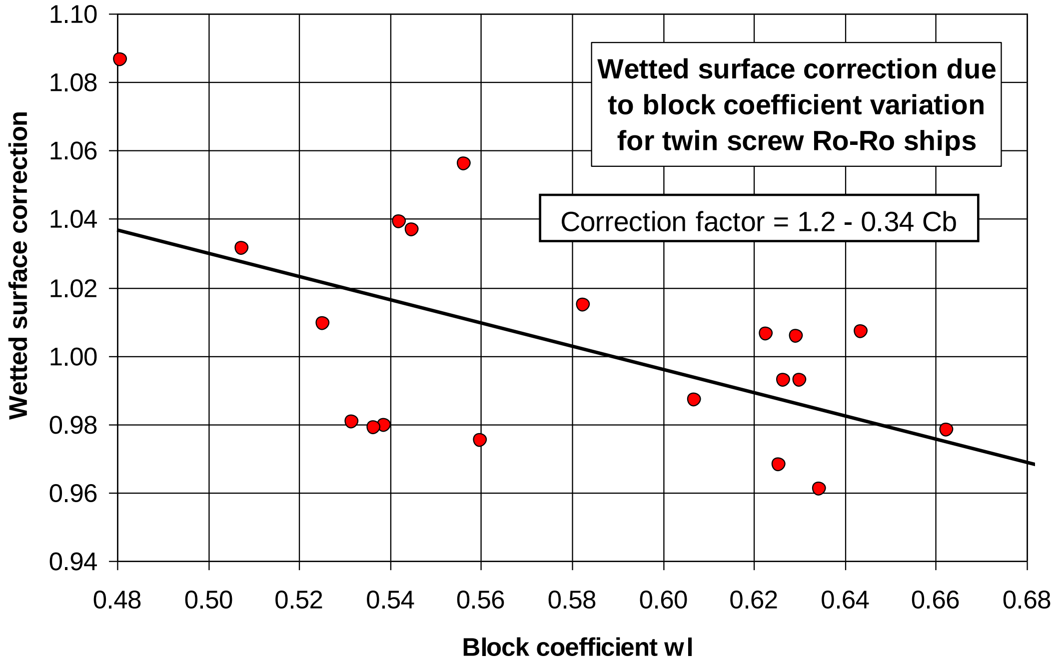


Fig. B10 Wetted surface correction for twin screw Ro-Ro ships

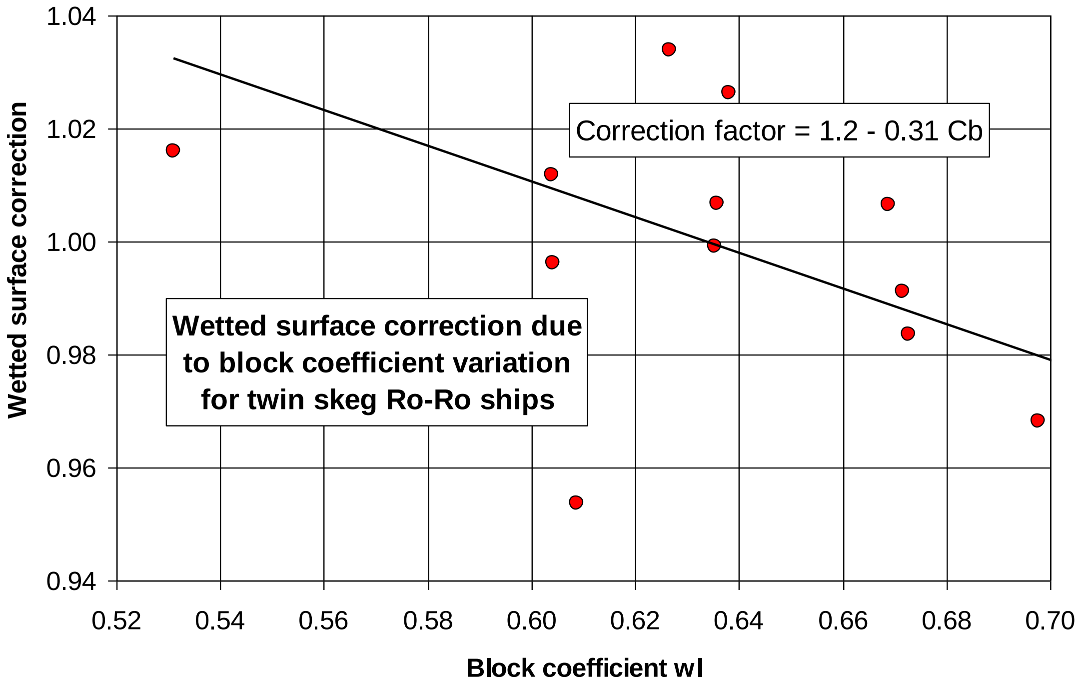


Fig. B11 Wetted surface correction for twin-skeg Ro-Ro ships

Comparisons of the wetted surface using the different formulas with the actual wetted surface are shown in Fig. B12 – B14. It is seen that the modified versions of Mumfords formula increases the accuracy considerable – with the smallest difference using the formula with block coefficient correction. It is seen that the difference is less than 3 % for 86 % of the single screw ships and 69 % of the conventional twin screw ships. For the twin-skeg ships the accuracy is even better as the difference is below 2 % for 79 % of these ships.

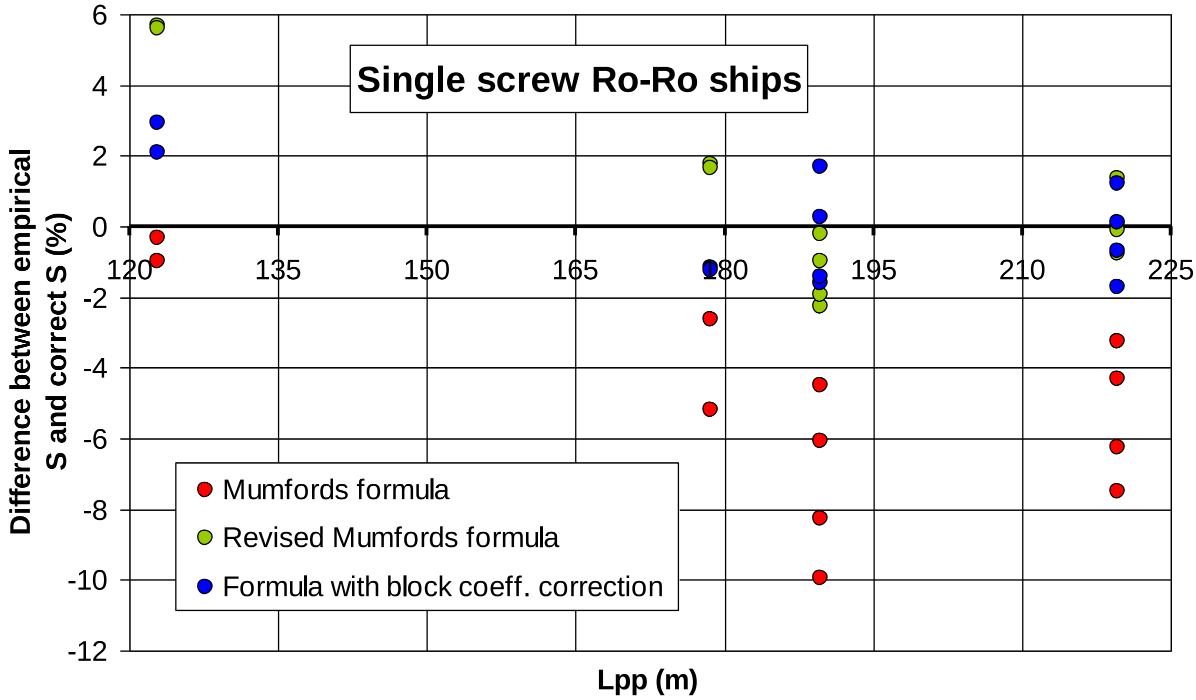


Fig. B12 Difference between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for single screw Ro-Ro ships

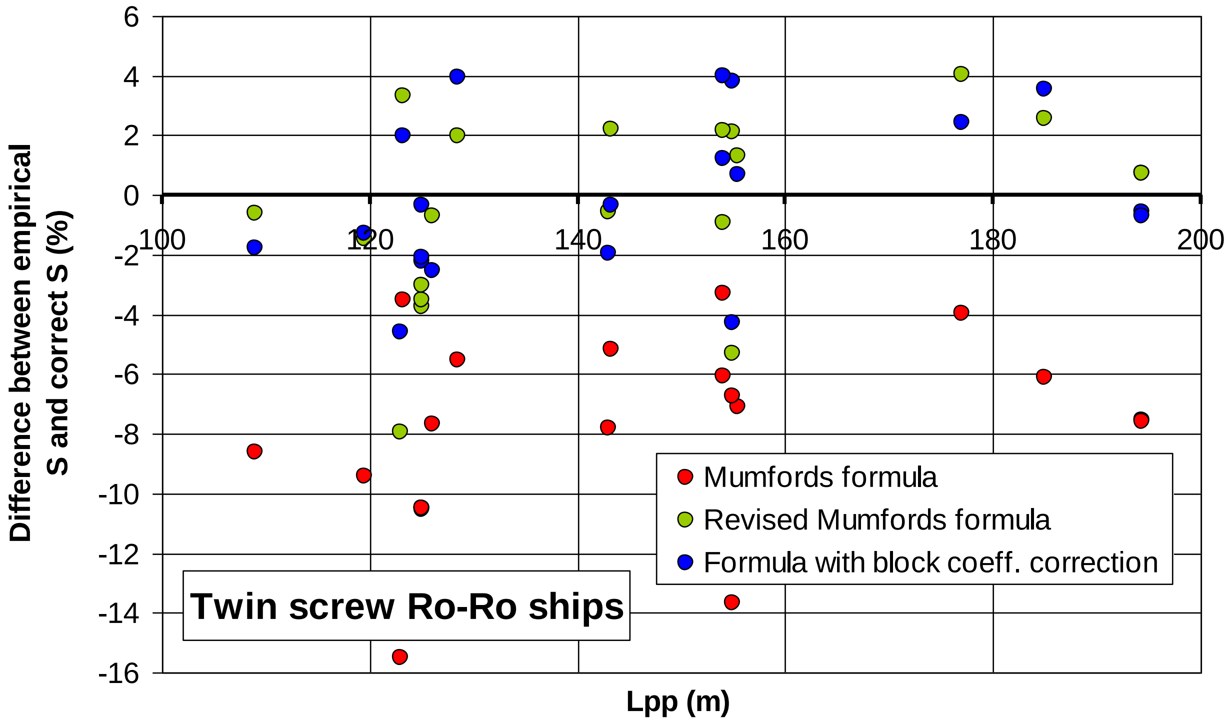


Fig. B13 Difference between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for conventional twin screw Ro-Ro ships

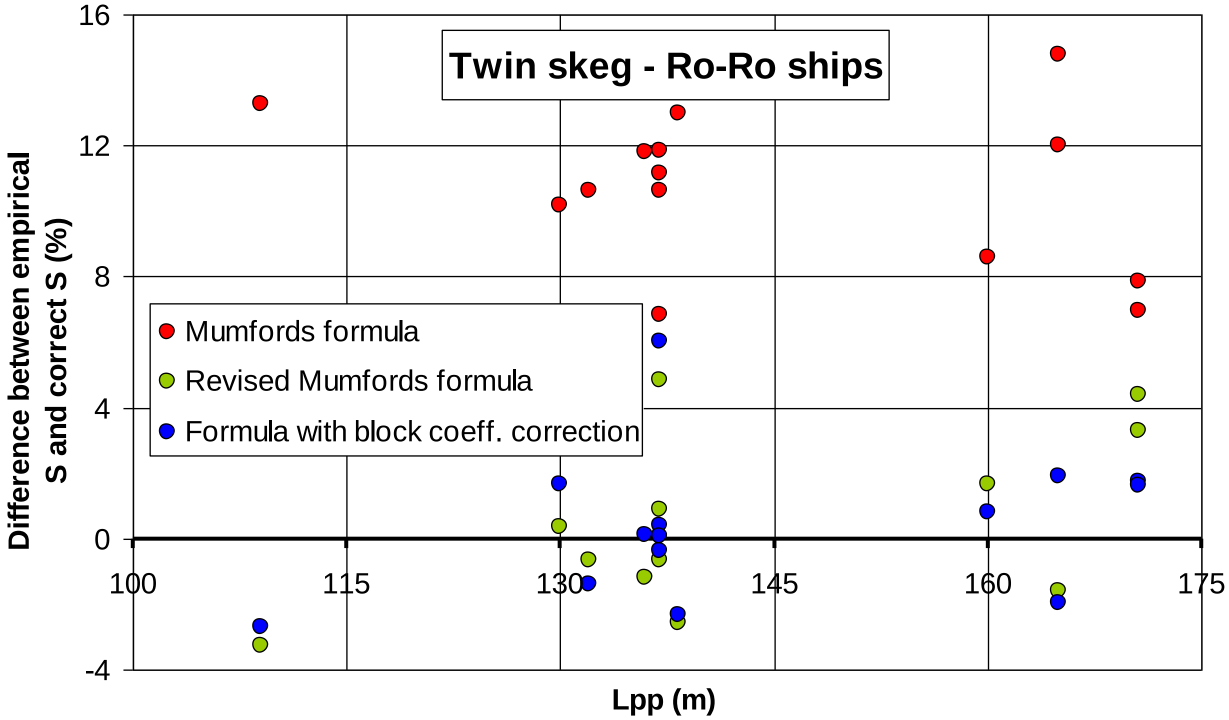


Fig. B14 Difference between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for twin-skeg Ro-Ro ships

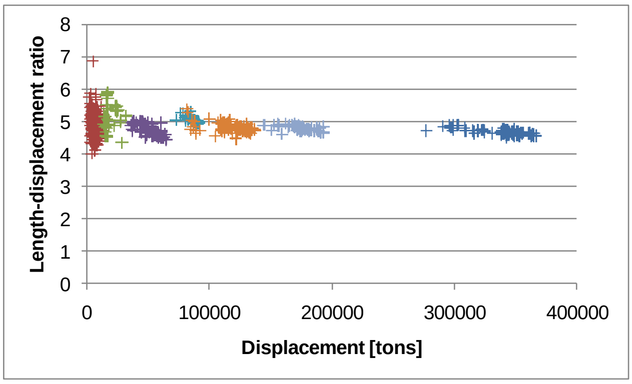
Table B1 Average difference in % between the wetted surface according to different versions of Mumfords formula and the actual wetted surface for Ro-Ro ships

Ship type	Original Mumford formula	Modified Mumford formula	Modified Mumford formula with block coefficient correction
Single screw ship	4.94	1.86	1.34
Conventional twin screw ship	5.80	2.80	2.53
Twin-skeg ship	10.68	2.15	1.65

Appendix C - Comments on *M* and C_P

Assuming C_M constant equals 0.990 - 0.995, the prismatic coefficient, C_P, can approximately be set to C_B, which is nearly constant for each vessel size. From an overall perspective the prismatic coefficient will for most vessels be constant or slightly decrease for decreasing draft.

The length-displacement ratio, *M*, varies dependent on the vessel size as shown in Fig. C1. For small and handysize vessels a large scatter is seen.



	Mean	St. dev.
Small	4.88	0.34
Handysize	5.13	0.44
Handymax	4.66	0.12
Panamax	5.05	0.07
Aframax	4.78	0.10
Suezmax	4.77	0.06
VLCC	4.65	0.06

Figure C1. Length displacement ratio for tankers (standard vessels).

Appendix D – Bulbous Bow Resistance Correction for Tankers and Bulk Carriers

For the present project several model tests results for ships having bulbous bows have been analysed in order to find a suitable bulbous bow resistance correction. The total resistance coefficient of each individual ship has been calculated by Harvald´s method without any corrections for bulbous bow. Subtracting this value from the total resistance coefficient found by model tests gives the bulbous bow correction which is needed for updating of the resistance calculation method.

The results of this analysis for 277 model test values for ships with a bulbous bow are shown in figure D1. The figure shows positive influence of the bow for increasing Froude number.

The model tests:
Sixteen different vessels some of them in different loading condition giving 27 test vessels in total. All vessels are tested at various speeds giving 277 results in total.

The vessels: 6 bulk carriers and 9 tankers, 1 small, 3 handysize, 4 handymax, 6 Panamax and 1 Aframax.

For tankers and bulk carriers the correction can be approximated by a linear function, see Fig. D1:

$$\Delta C_{R,bulb} = \max(-0.4; -0.1 - 1.6 \cdot Fn)$$

Standard deviation: $0.15 \cdot 10^{-3}$

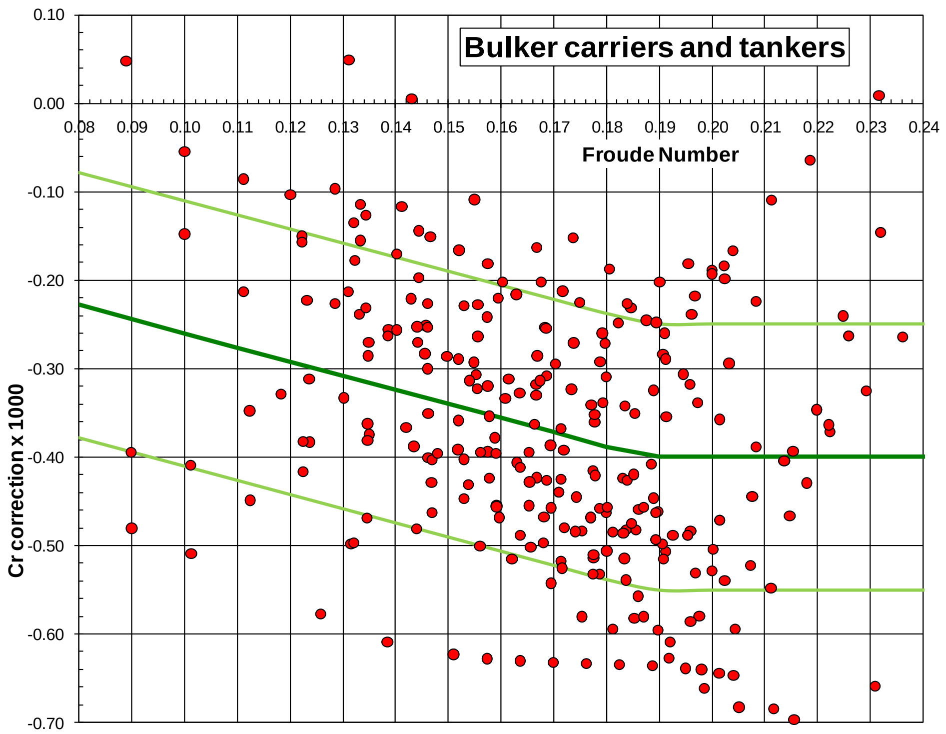


Fig. D1 Bulbous Cr correction from model tests

Appendix E – Bulb Bow Resistance Correction for Container Vessels and other Ships with low Block Coefficient

For the present project several model tests results for ships having a bulbous bow have been analysed in order to find a suitable bulbous bow resistance correction for ships having a block coefficient in the range from 0.5 to 0.7, i.e. the range for container ships, general cargo ships and Ro-Ro ships.

The total resistance coefficient of each individual ship has been calculated by Harvald´s method without any corrections for bulbous bow. Subtracting this value from the total resistance coefficient found by model tests (with the influence of the bulbous bow) gives the bulbous bow correction/influence which is needed for updating of the resistance calculation method.

After several investigations it was decided to calculate the correction due to the bulbous influence in per cent of the residual resistance as shown in Fig. E1, showing the results for 229 model test values for 21 different vessels (13 Ro-Ro ships and 8 cargo ships). By using different approaches following bulbous bow correction has obtained:

$$\Delta C_{R,bulb} = (250 \cdot Fn - 90) \cdot \frac{C_{R\ \text{NO bulbous bow}}}{100}$$

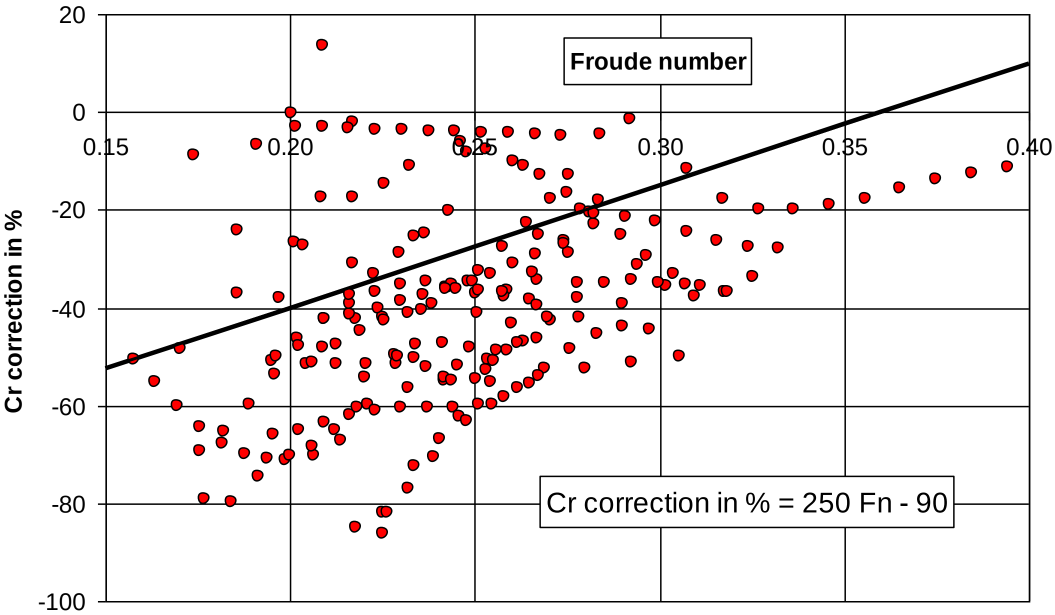


Fig. E1 Residual resistance coefficient correction due to the influence of a bulbous bow found by model tests

The percentage correction could be determined by making a regression analysis of the results in Fig. E1. This was tried, but resulted in *Ct* values which were generally too optimistic compared with the model test results. The bulbous bow correction was therefore slightly modified until the results in Fig. E2 were obtained.

The validity of the proposed bulbous bow residual resistance correction has been tested by applying the new bulbous bow correction on the ships which have been model tested. The ratio between the total resistance coefficient based on the new proposal and the total resistance coefficient found by model tests are shown in Fig. E2. It is seen that the revised Harvald method predicts approximately 0 - 10 % higher resistance coefficients than the model tests, which shows that the proposed *Cr* correction is slightly pessimistic compared with the actual model test values.

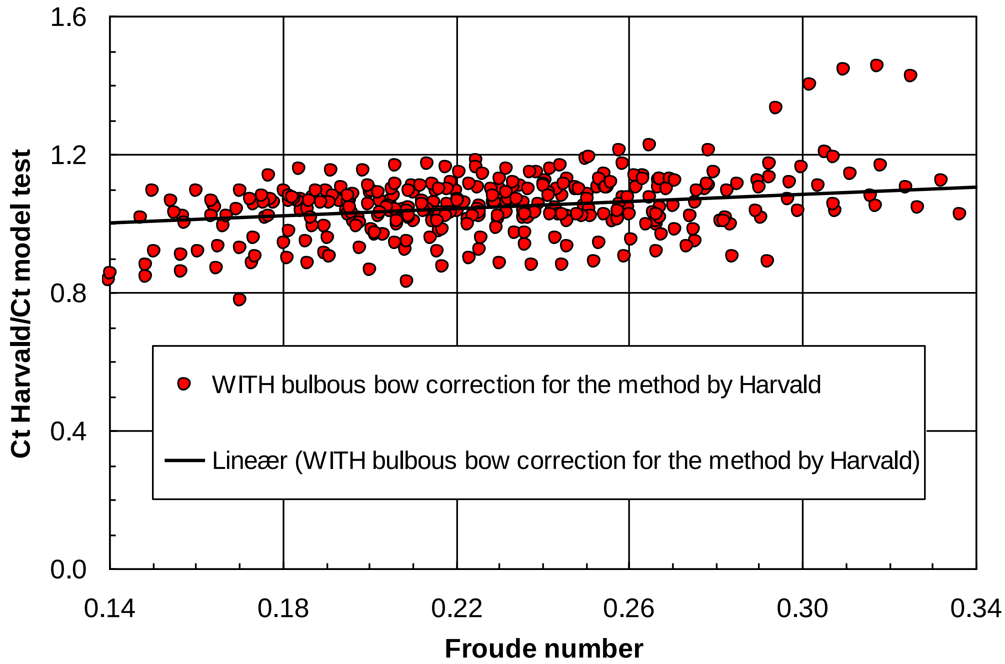


Fig. E2 Ratio between total resistance coefficients found by the revised method by Harvald and resistance coefficient found by model tests.

The total resistance coefficients with no bulbous bow correction have also been compared with the model test values and the results of this comparison is shown in Fig. E3. It is seen that the total resistance coefficient with no correction is approximately 15 - 21 % higher than the model test values. Together with the results in Fig. E2, this shows that the bulbous bow in average reduces the resistance with approximately 12 %, which is in line with tests with 3 ship models which have been tested without and with a bulbous bow. The results of these tests are shown in Fig. E4, which shows a reduction of the resistance of 10 – 20 % due to the influence of a bulbous bow.

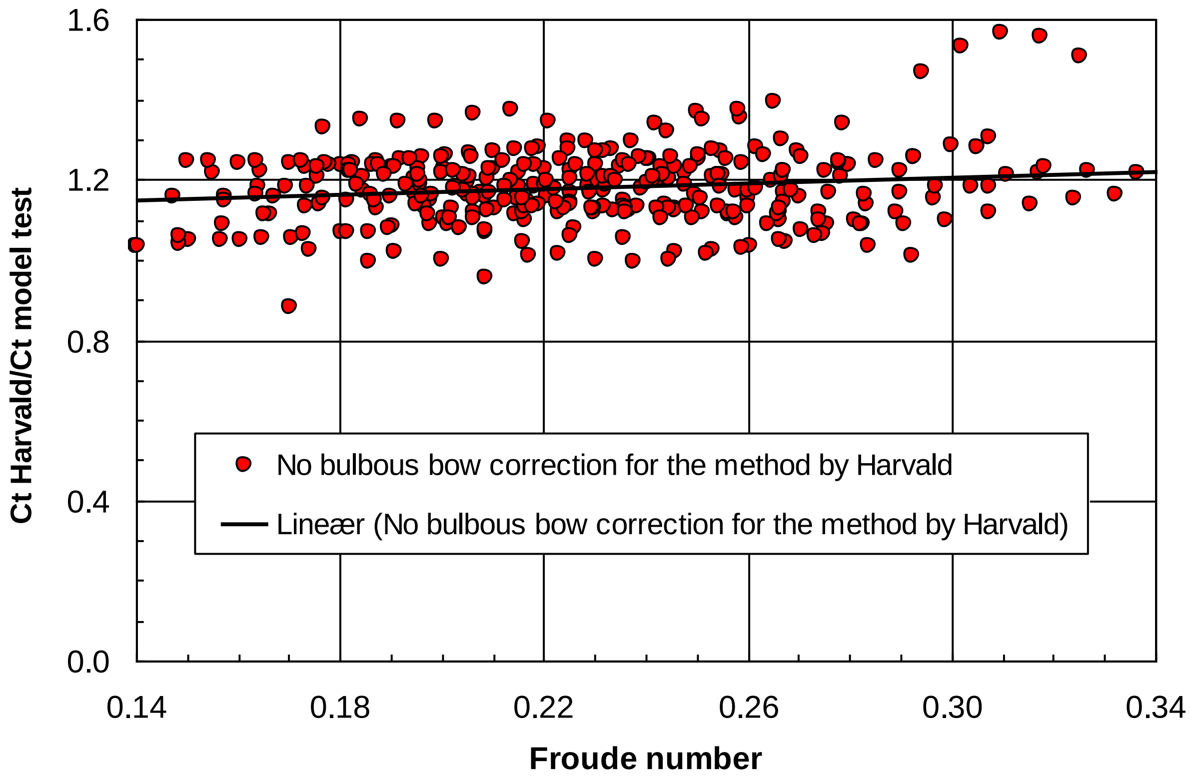


Fig. E3 Ratio between total resistance coefficients found using Harvald´s method without bulbous bow correction and total resistance coefficient found by model tests.

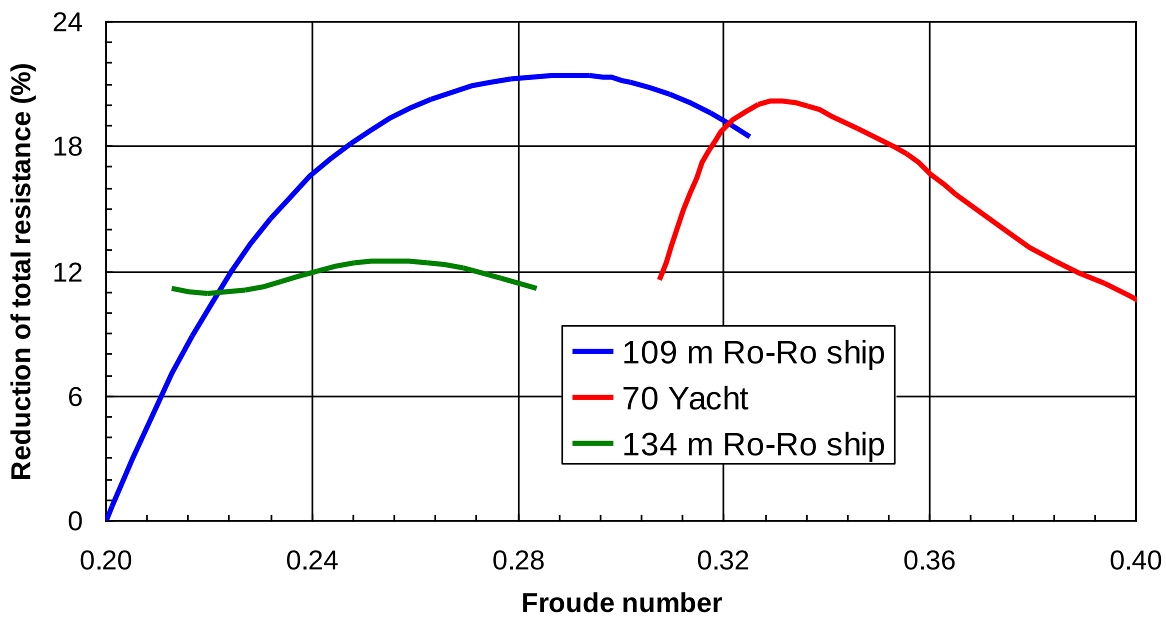


Fig. E4 Reduction of total resistance due to the influence of a bulbous bow. Found by model tests for three ships which were tested with and without a bulbous bow.

Appendix F - Propeller diameter

The propeller diameter shall be as large as possible to obtain the highest efficiency. But in order to avoid cavitation and air suction, the diameter is restricted by the draught. In this appendix expressions for the propeller diameter as function of the maximum draught are given and documented by relevant statistical data, Significant Ships (1990 – 2010).

Bulk carriers and tankers (Fig. F1 and F2)

$$D_{prop} = 0.395 \cdot \text{max. draught} + 1.30$$

It is seen that the diameter to draught ratio decreases with increasing draught from 0.6 to 0.4

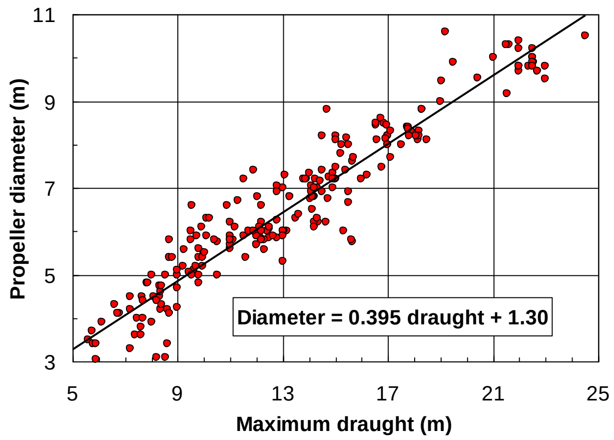


Fig. F1 Propeller diameter for tankers and bulk carriers

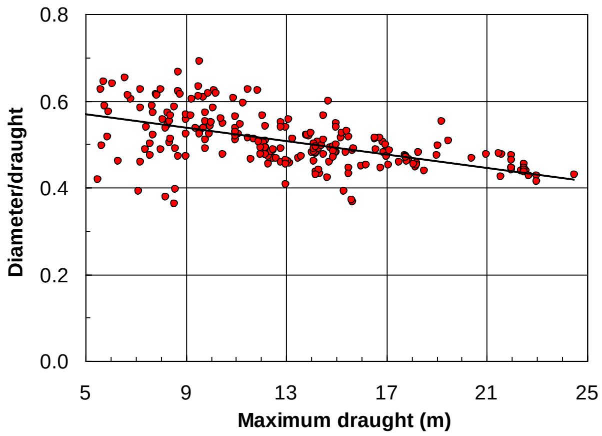


Fig. F2 Propeller diameter to draught ratio for tankers and bulk carriers

Container ships (Fig. F3 and F4)

$$D_{prop} = 0.623 \cdot \text{max. draught} - 0.16$$

It is seen that the diameter to draught ratio is in average nearly constant around 0.6 however with some variation from 0.5 to 0.7

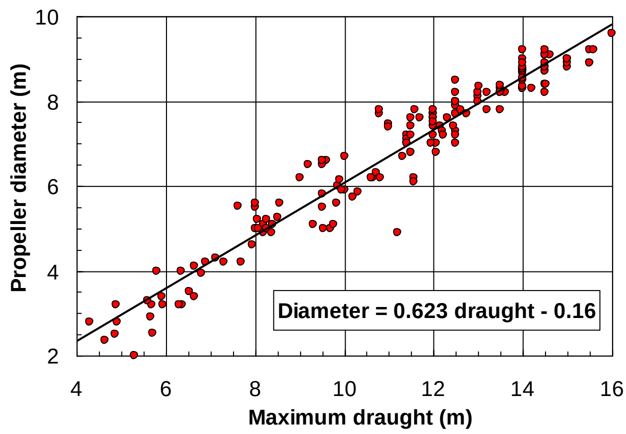


Fig. G3 Propeller diameter for container ships

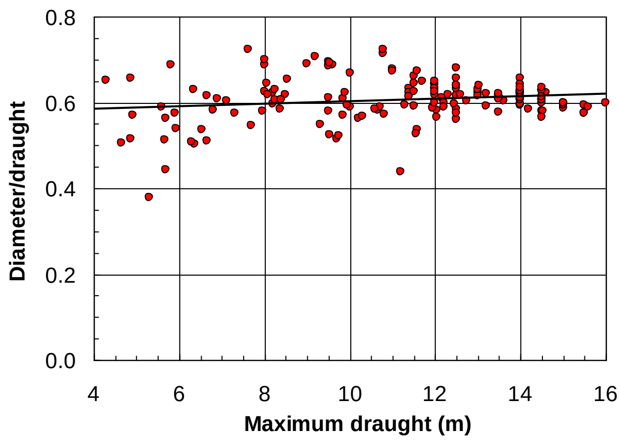


Fig. G4 Propeller diameter to draught ratio for container ships

Twin screw Ro-Ro ships (Fig. F5 and F6)

$$D_{prop} = 0.713 \cdot \text{max. draught} - 0.08$$

It is seen that the diameter to draught ratio is in average nearly constant around 0.7 however with quite large variations from 0.4 to 0.95.

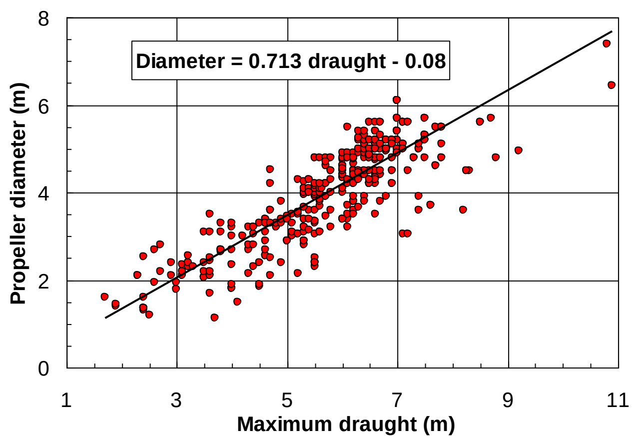


Fig. F5 Propeller diameter for twin screw Ro-Ro ships

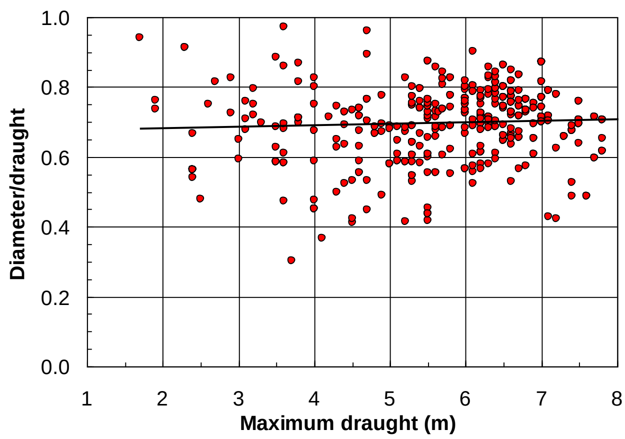


Fig. F6 Propeller diameter to draught ratio for twin screw Ro-Ro ships

Appendix G – Wake fraction and thrust deduction fraction for tankers and bulk carriers

Wake fraction

For 26 single screw tankers and bulk carriers, the wake fraction has been calculated using Harvald´s formulas. The calculated wake fraction is the trial wake fraction (i.e. clean hull conditions) which has been compared with the values found from model tests for a sample of full load and ballast conditions. In fig. G1 is shown a comparison between the calculated and the measured wake fraction from model tests. For 38 % of the values the difference between the measured and calculated value is less than 10 % and for 73 % less than 25 %. The calculated wake fraction seems to be slightly higher than the measured values obtained from model tests.

Thrust deduction fraction

For the same 26 single screw tankers and bulk carriers the thrust deduction fraction has also been calculated using Harvald´s formulas. The calculated thrust deduction fraction has been compared with the values found from the model tests. In Fig. G2 is shown a comparison between the calculated and the measured thrust deduction. For 38 % of the values the difference between the measured and calculated value is less than 10 % and for 65 % less than 25 % In general the calculated thrust deduction fraction seems to be higher than the measured values obtained from model tests.

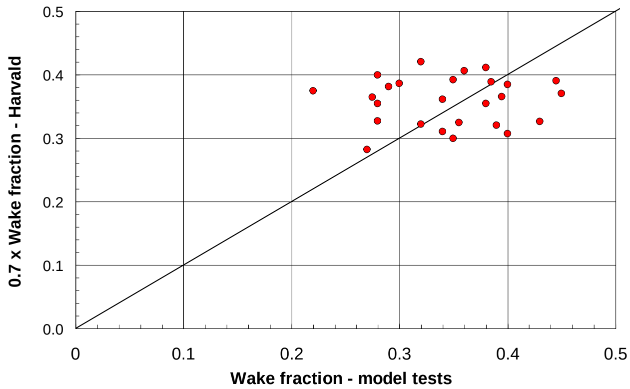


Fig. G1 Comparison of measured and calculated wake fraction.

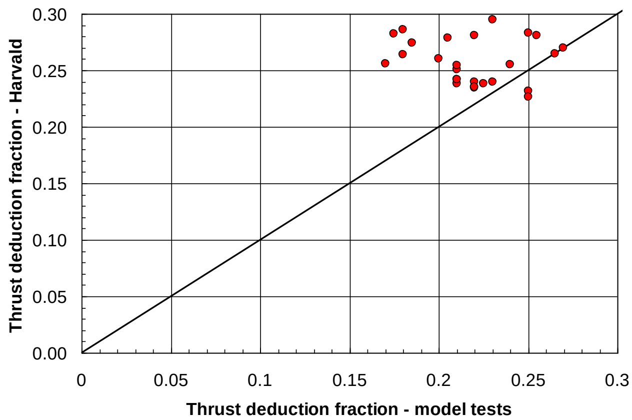


Fig. G2 Comparison of measured and calculated thrust deduction fraction.

Hull efficiency

The resulting hull efficiency has also been analyzed (Fig. G3). A relatively good agreement between the calculated efficiency and the measured hull efficiency is seen. For 62 % of the values the difference between the measured and calculated value is less than 10 % and for more than 90 % the difference is less than 15 % The value obtained from model tests is in average 3 % higher than the hull efficiency obtained by using Harvald´s method, which means that Harvald´s method is slightly pessimistic.

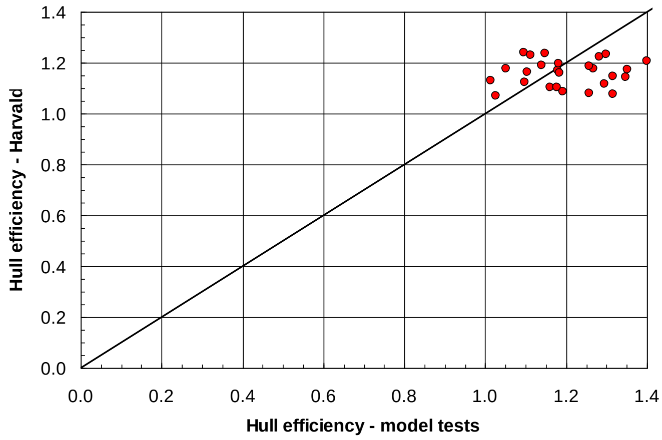


Fig. G3 Comparison of measured and calculated hull efficiency.

Correction of wake fraction and thrust deduction fraction

I order to obtain more correct values of w and t (which corresponds better with the model test values), the difference between the values obtained by model tests and calculated by Harvald´s formulas has been plotted as function of the length displacement ratio M (Fig. G4 and G5). It is seen that the difference depends on the length displacement ratio such that the difference is highest for the lowest length displacement ratios.

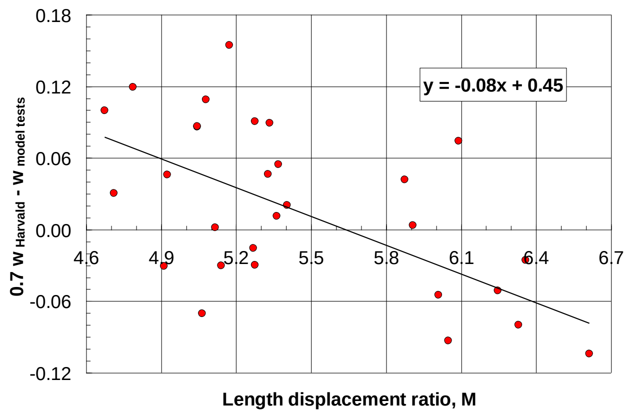


Fig. G4 Difference between calculated (Harvald) and measured (model tests) wake fraction

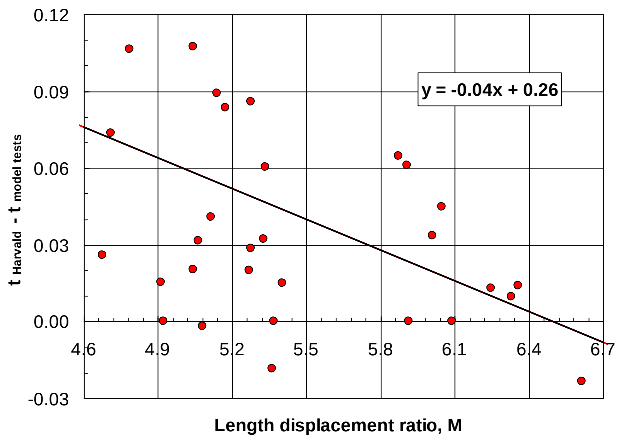


Fig. G5 Difference between calculated (Harvald) and measured (model tests) thrust deduction fraction

Based on the regression analysis in Fig. G4 and G5, following corrected formulas for calculation of the wake fraction and the thrust deduction fraction for tankers and bulk carriers have been derived:

$$w_{Corrected} = 0.7 \cdot w_{Harvald} - 0.45 + 0.08 \cdot M$$

$$t_{Corrected} = t_{Harvald} - 0.26 + 0.04 \cdot M$$

The updated values of w and t and the hull efficiency according to the new formulas are shown in Fig. G6 - G8. The mean value of hull efficiencies from model tests is identical with the mean value of the corresponding hull efficiencies calculated by using the corrected w and t formulas.

For 42 % of the tests, the difference between the measured and calculated wake fraction is less than 10 % and for 88 % less than 25 %. For 38 % of the test results, the difference between the measured and calculated thrust deduction fraction is less than 10 % and for 96 % less than 25 %. For 73 % of the test results the difference between the measured and calculated hull efficiency is less than 10 % and for 96 % the difference is less than 15 %.

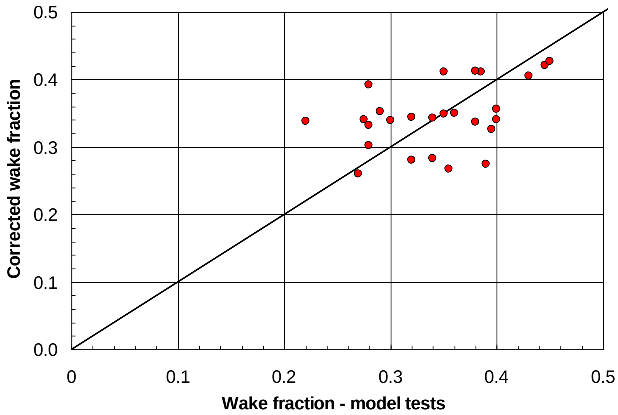


Fig. G6 Comparison of measured and calculated wake fraction.

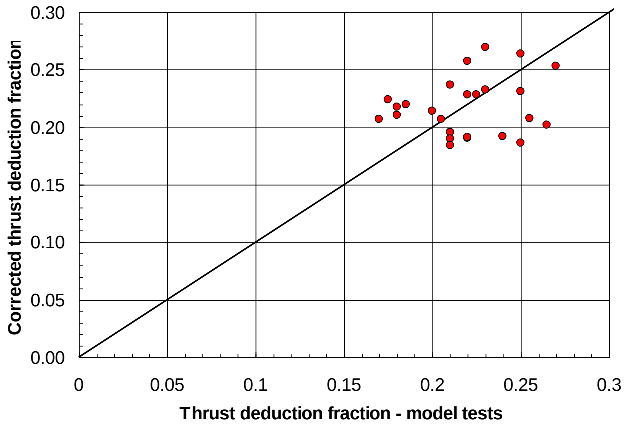


Fig. G7 Comparison of measured and calculated thrust deduction fraction.

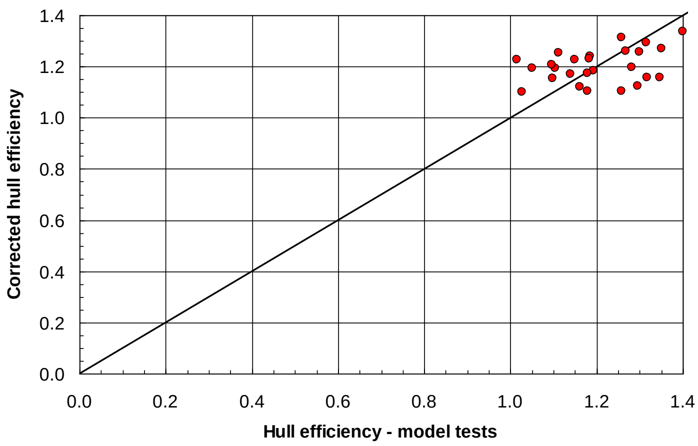


Fig. G8 Comparison of measured and calculated hull efficiency by revised w and t formulas.

Appendix H – Wake fraction and thrust deduction fraction for twin-screw ships including twin-skeg ships

For conventional twin screw ships the wake fraction and thrust deduction fraction are calculated according to formulas based on Harvald [Harvald 1983, Figure 6.5.8]:

$$w = 1.133 \cdot C_B^2 - 0.797 \cdot C_B + 0.215$$

$$t = 0.0665 + 0.62833 \cdot w$$

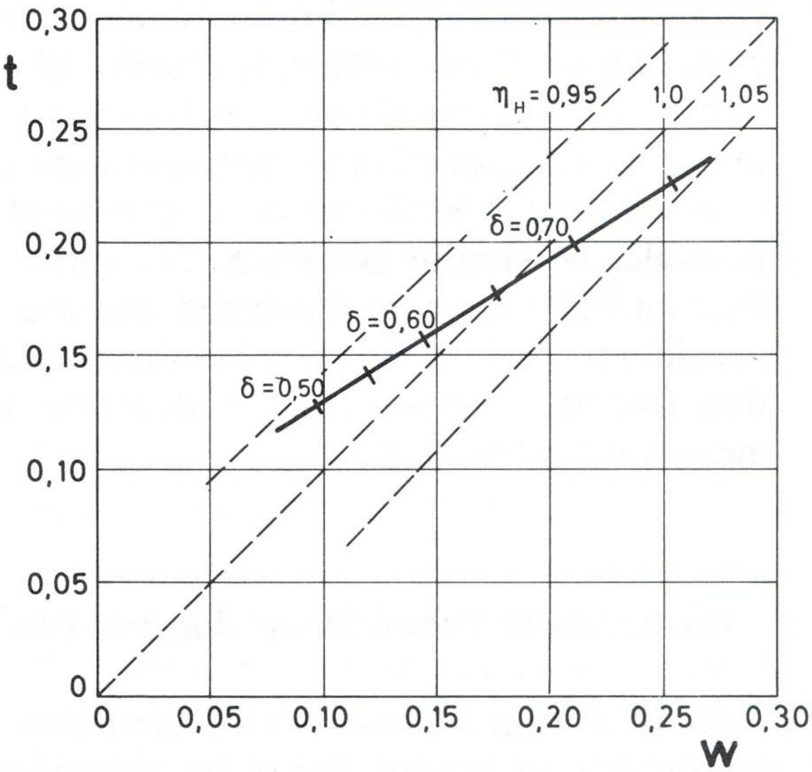


Figure 6.5.8. Relationship among the thrust deduction fraction, the wake fraction, and the hull efficiency for twin-screw ships having normal form and $D/L = 0.03$.

For twin-skeg ships the wake fraction will be higher due to the skeg in front of each propeller. Based on analysis of 15 model test results with twin-skeg Ro-Ro ships and twin skeg container ships (Fig. H1 and H2) following equations have been established for calculation of the wake fraction and the thrust deduction fraction of twin-skeg vessels as function of the water line block coefficient C_B for the wake fraction:

$$w = 0.7 \cdot C_B - 0.2$$

$$t = 0.19$$

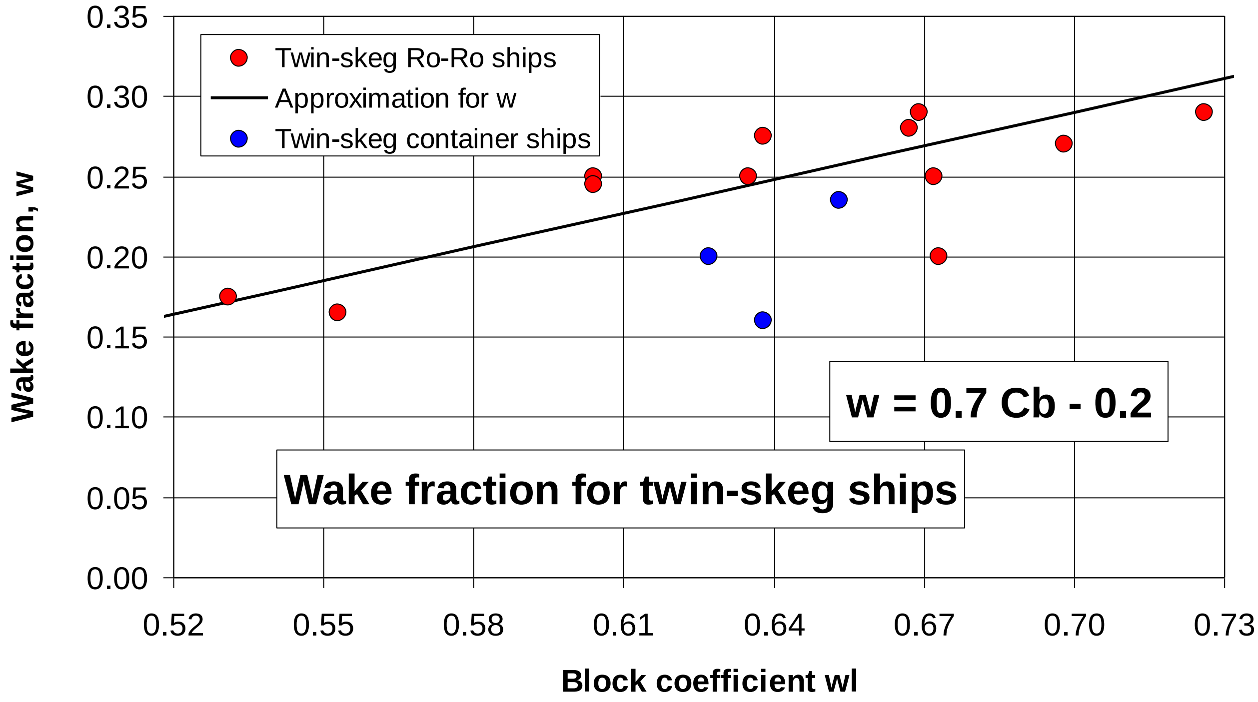


Fig. H1 Wake fraction, w, found by model tests for twin-skeg ships

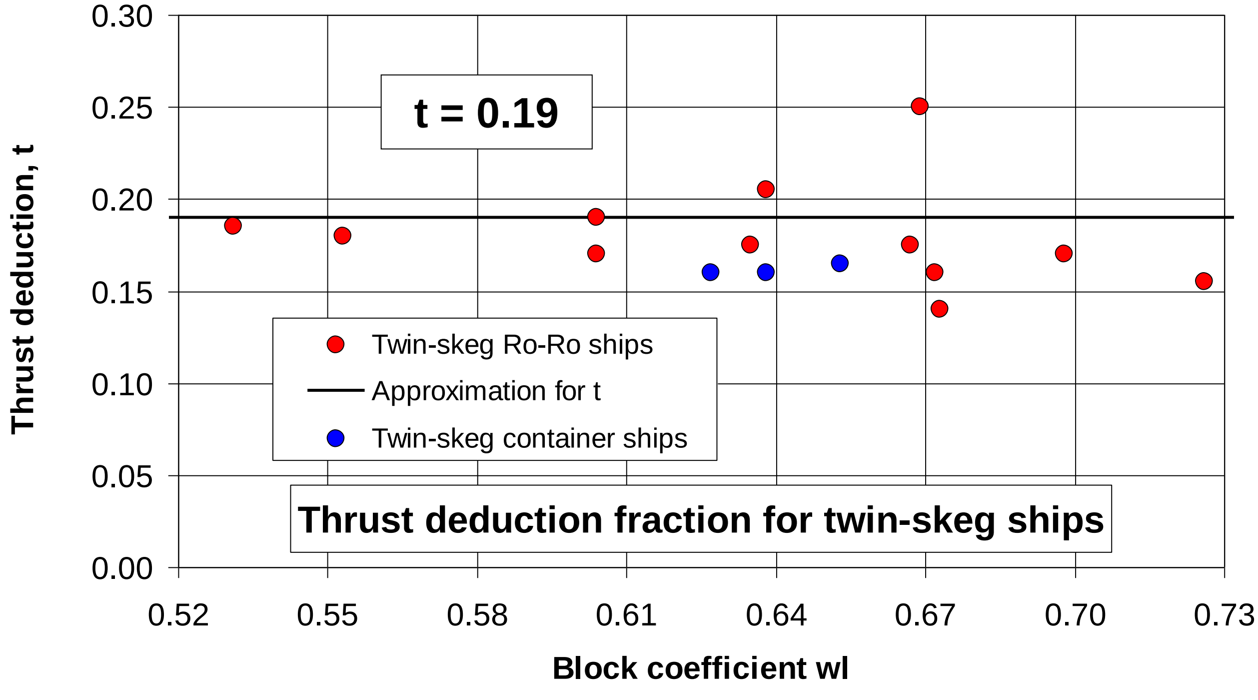


Fig. H2 Thrust deduction fraction, t, found by model tests for twin-skeg ships

Hull efficiency

The resulting hull efficiency has also been analyzed (Fig. H3). A relatively good agreement between the calculated efficiency and the measured hull efficiency is seen with a maximum deviation of approximately plus/minus 5 %

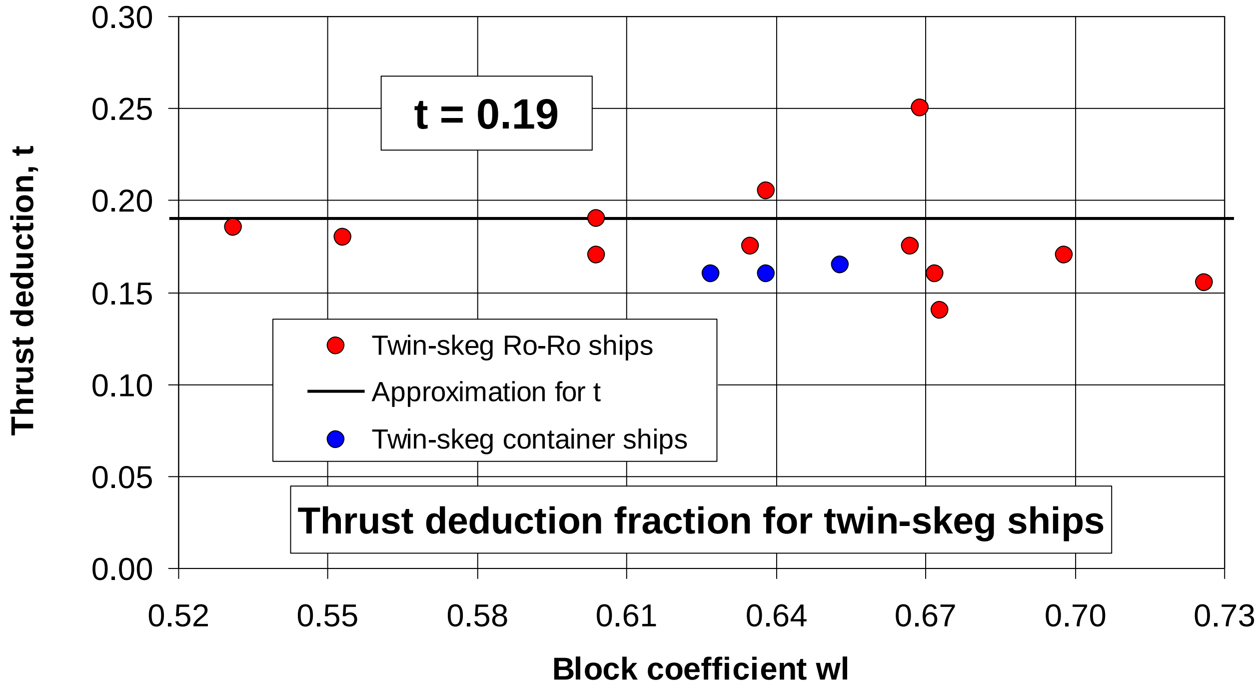
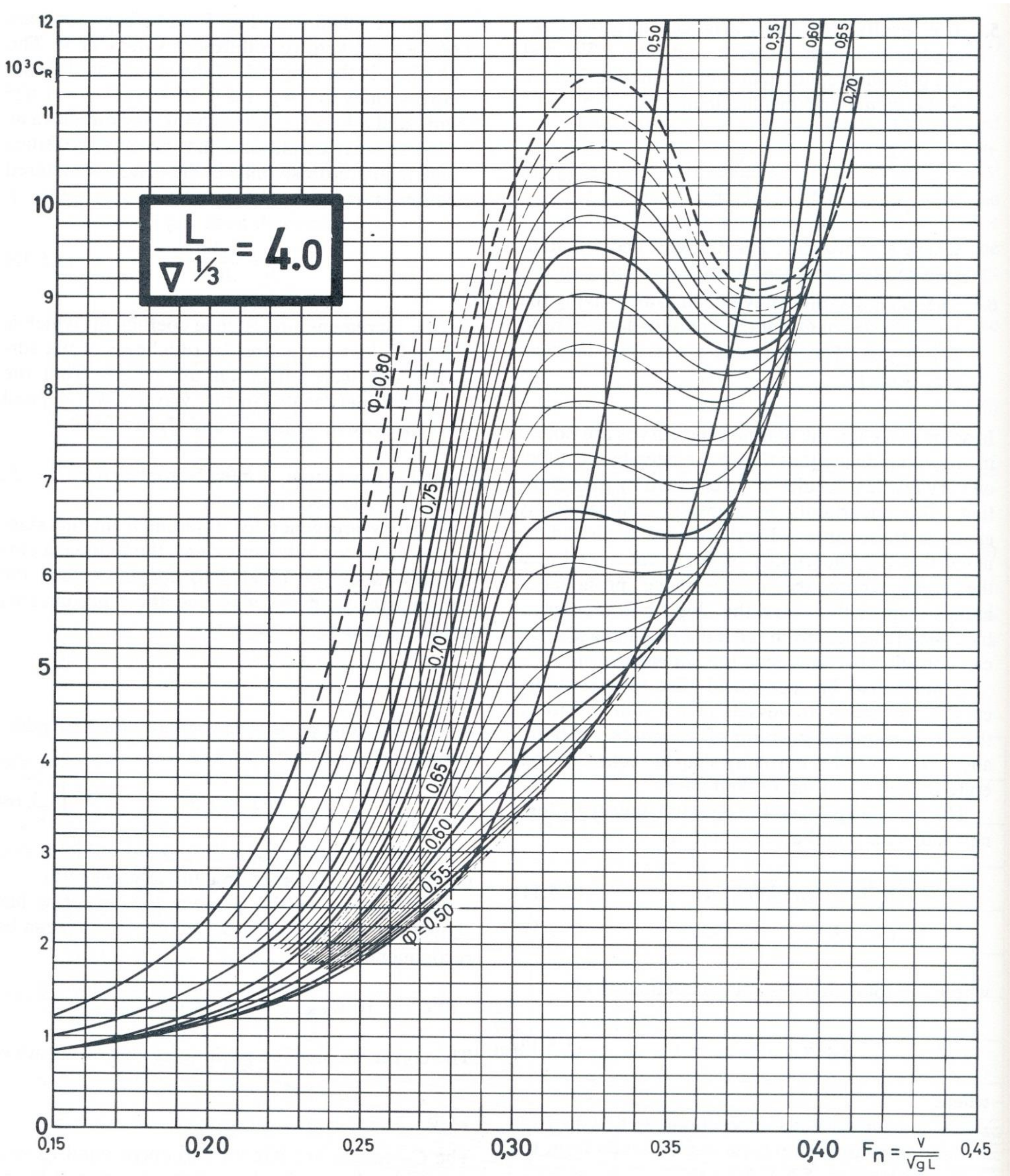
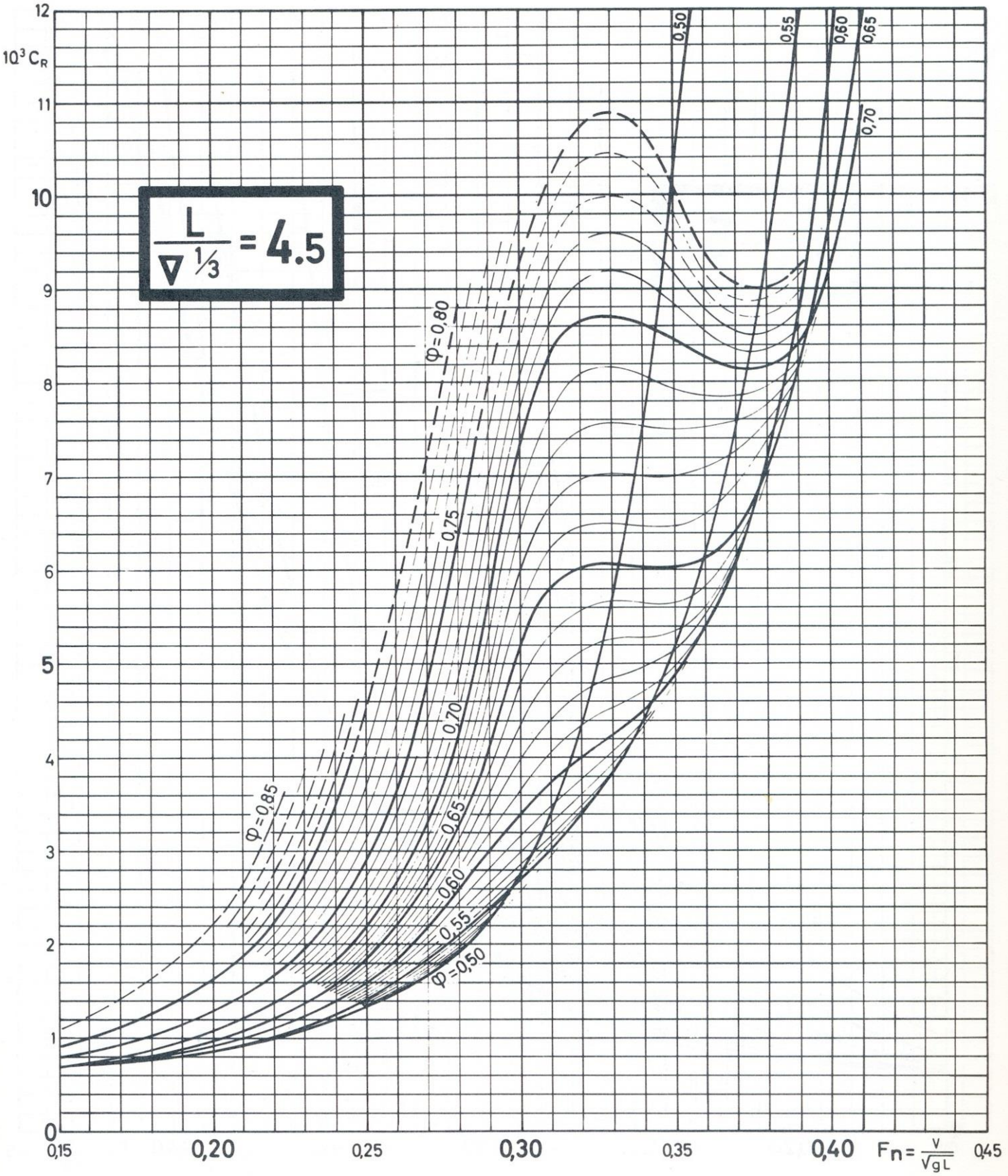


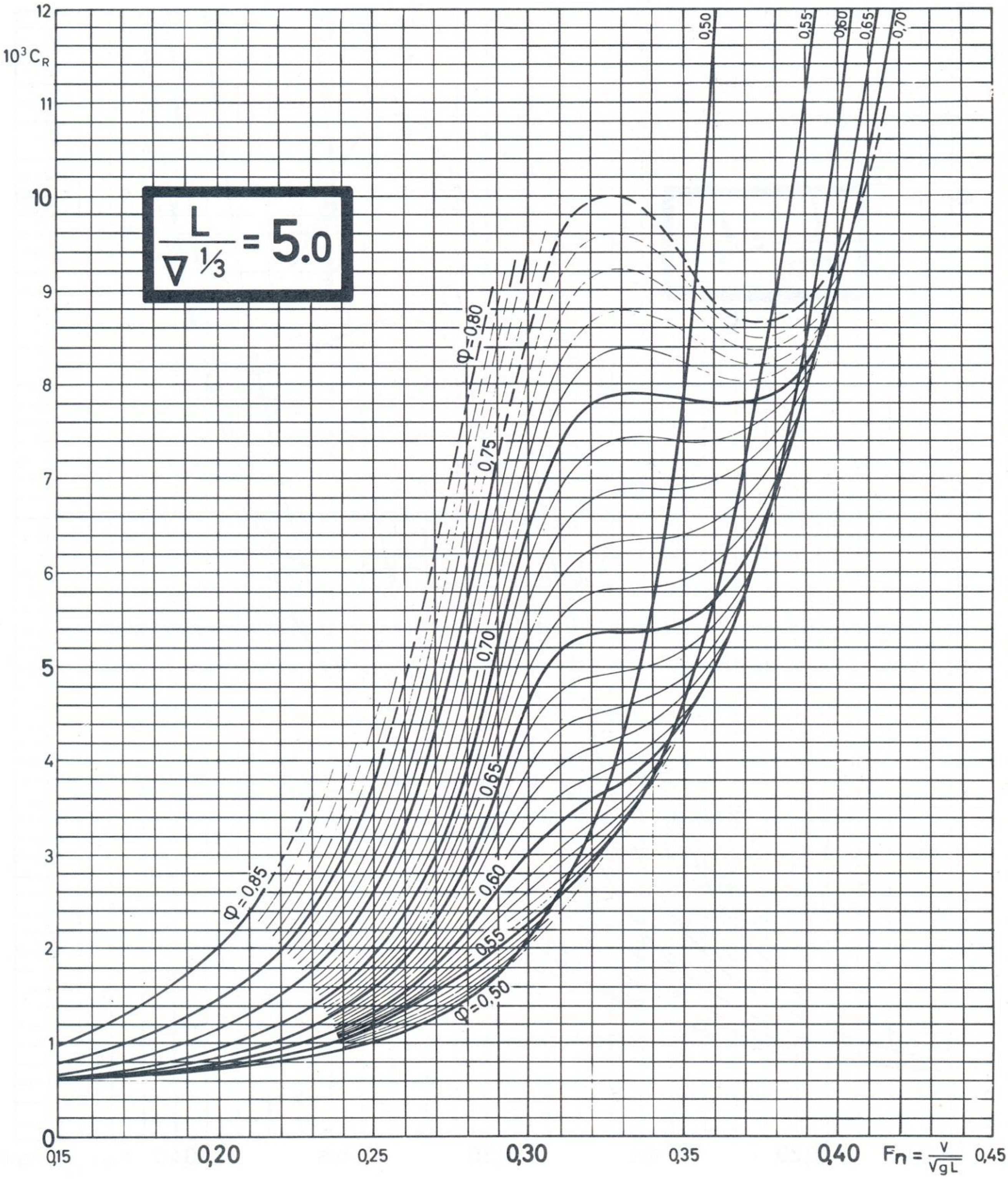
Fig. H3 Hull efficiency found by model tests for twin-skeg ships compared with the values calculated by developed empirical formulas for wake fraction and thrust deduction fraction

Appendix I – Cr diagrams according to Harvald

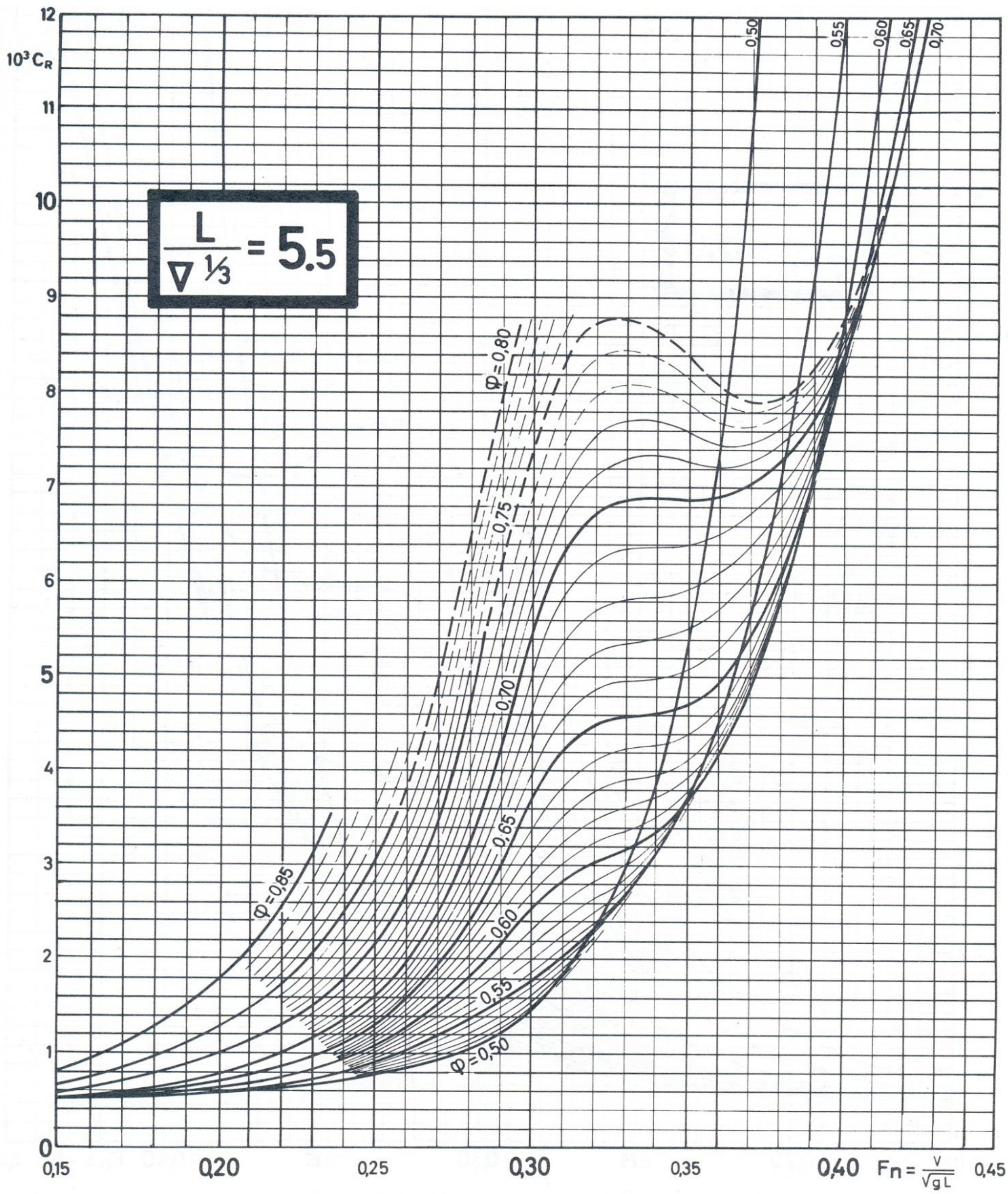




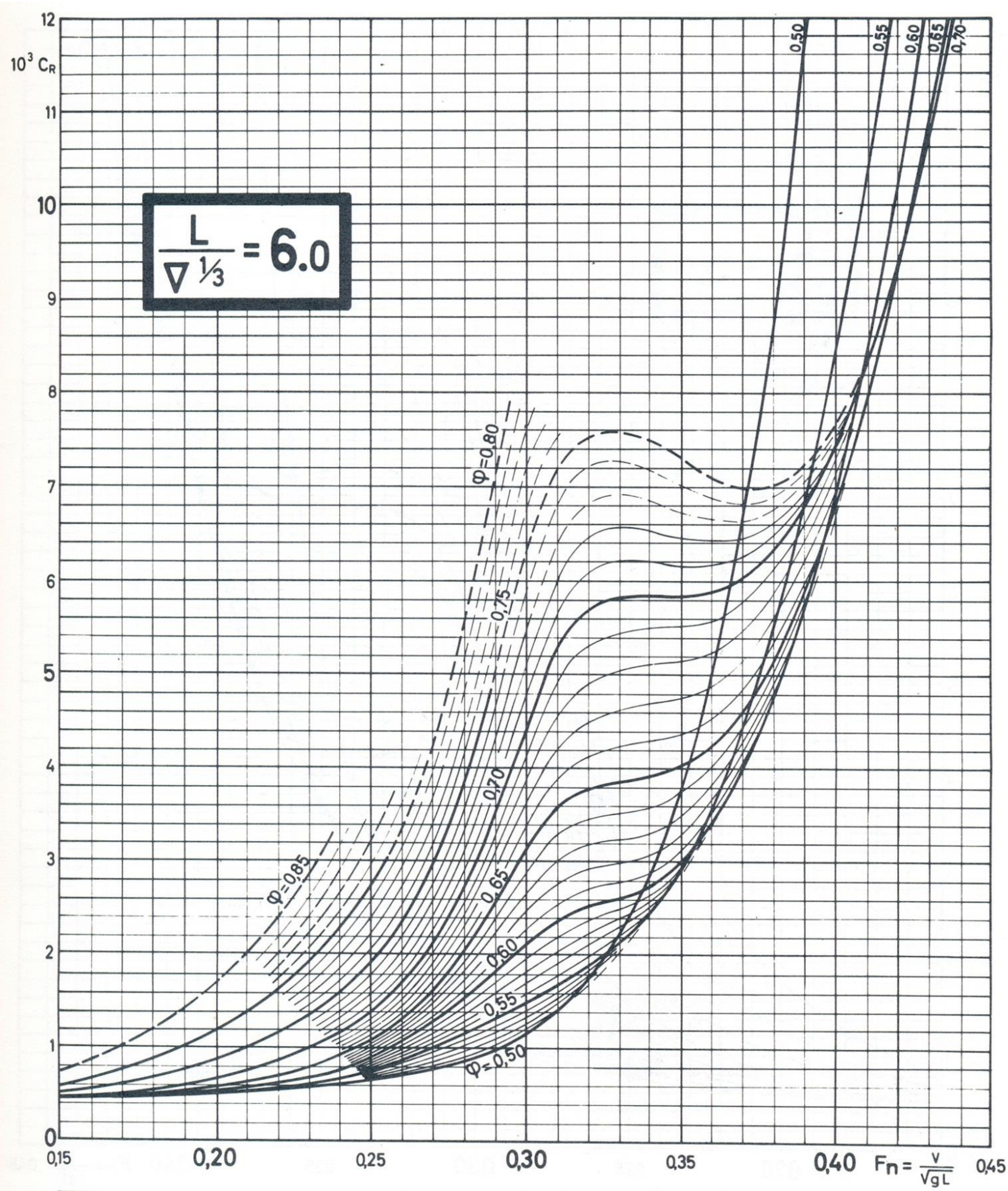
$10^3 C_R$
$\frac{L}{\nabla^{1/3}} = 4.5$
$\varphi = 0,85$
$\varphi = 0,80$
0,75
0,70
0,65
0,60
0,55
$\varphi = 0,50$
0,50
0,55
0,60
0,65
0,70
$F_n = \frac{V}{\sqrt{gL}}$
0,15
0,20
0,25
0,30
0,35
0,40
0,45
0
1
2
3
4
5
6
7
8
9
10
11
12

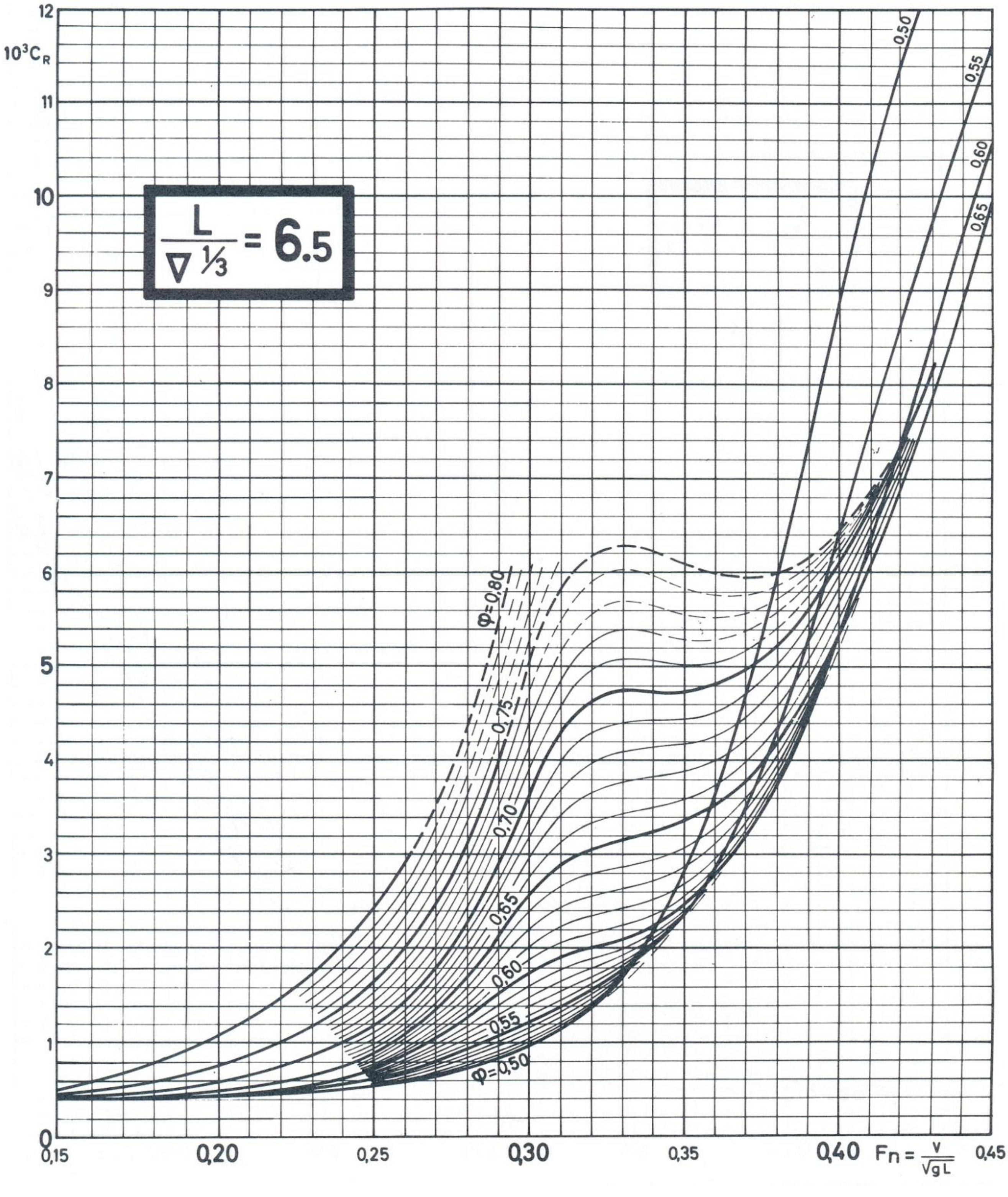


12
$10^3 C_R$
11
10
9
8
7
6
5
4
3
2
1
0
$\frac{L}{\nabla^{1/3}} = 5.0$
φ = 0,85
φ = 0,80
0,75
0,70
0,65
0,60
0,55
φ = 0,50
0,50
0,55
0,60
0,65
0,70
0,15
0,20
0,25
0,30
0,35
0,40
$F_n = \frac{v}{\sqrt{gL}}$
0,45

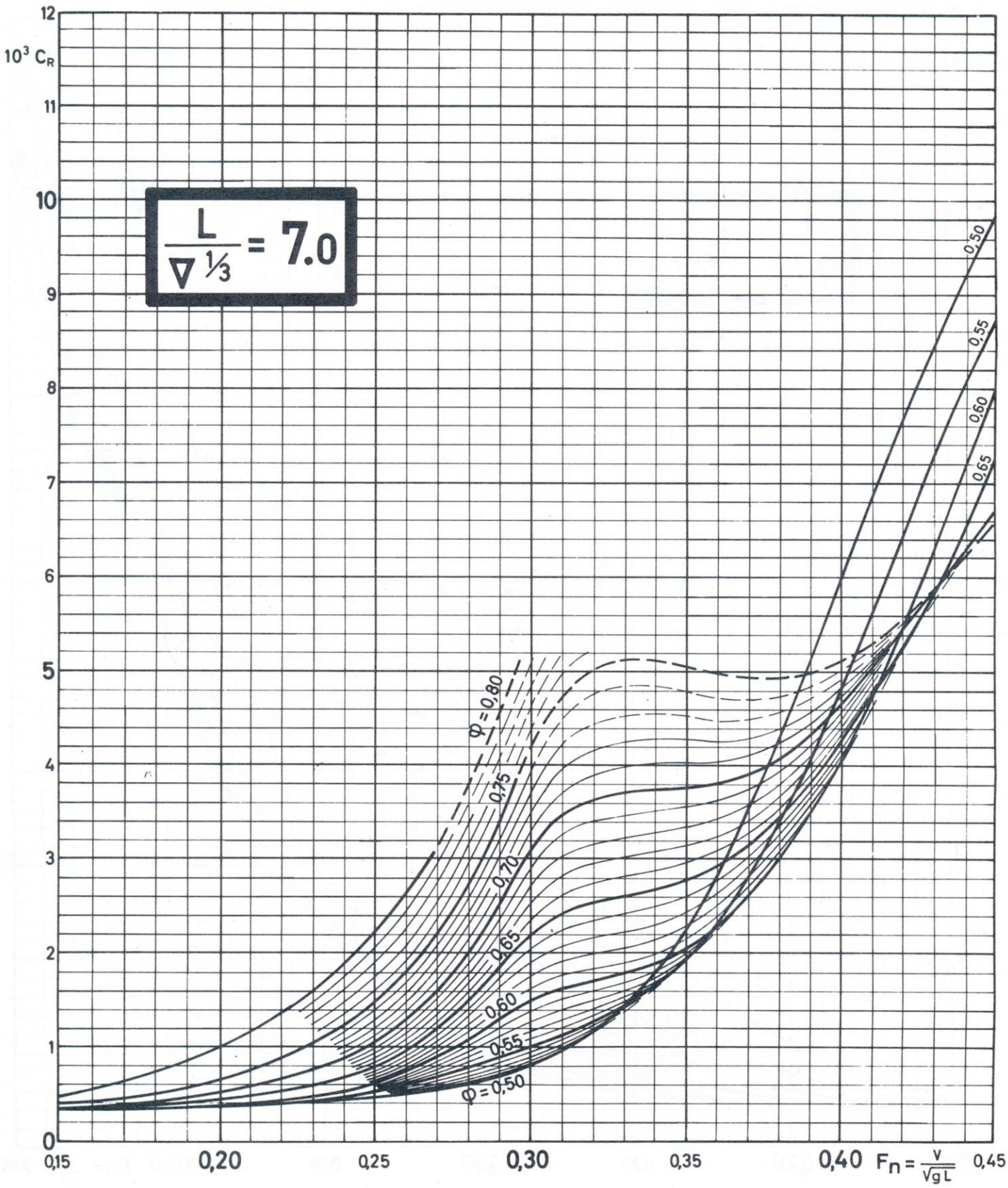


$10^3 C_R$
$\frac{L}{\nabla^{1/3}} = 5.5$
φ = 0,85
φ = 0,80
0,75
0,70
0,65
0,60
0,55
φ = 0,50
0,50
0,55
0,60
0,65
0,70
0
1
2
3
4
5
6
7
8
9
10
11
12
0,15
0,20
0,25
0,30
0,35
0,40
$Fn = \frac{V}{\sqrt{gL}}$
0,45

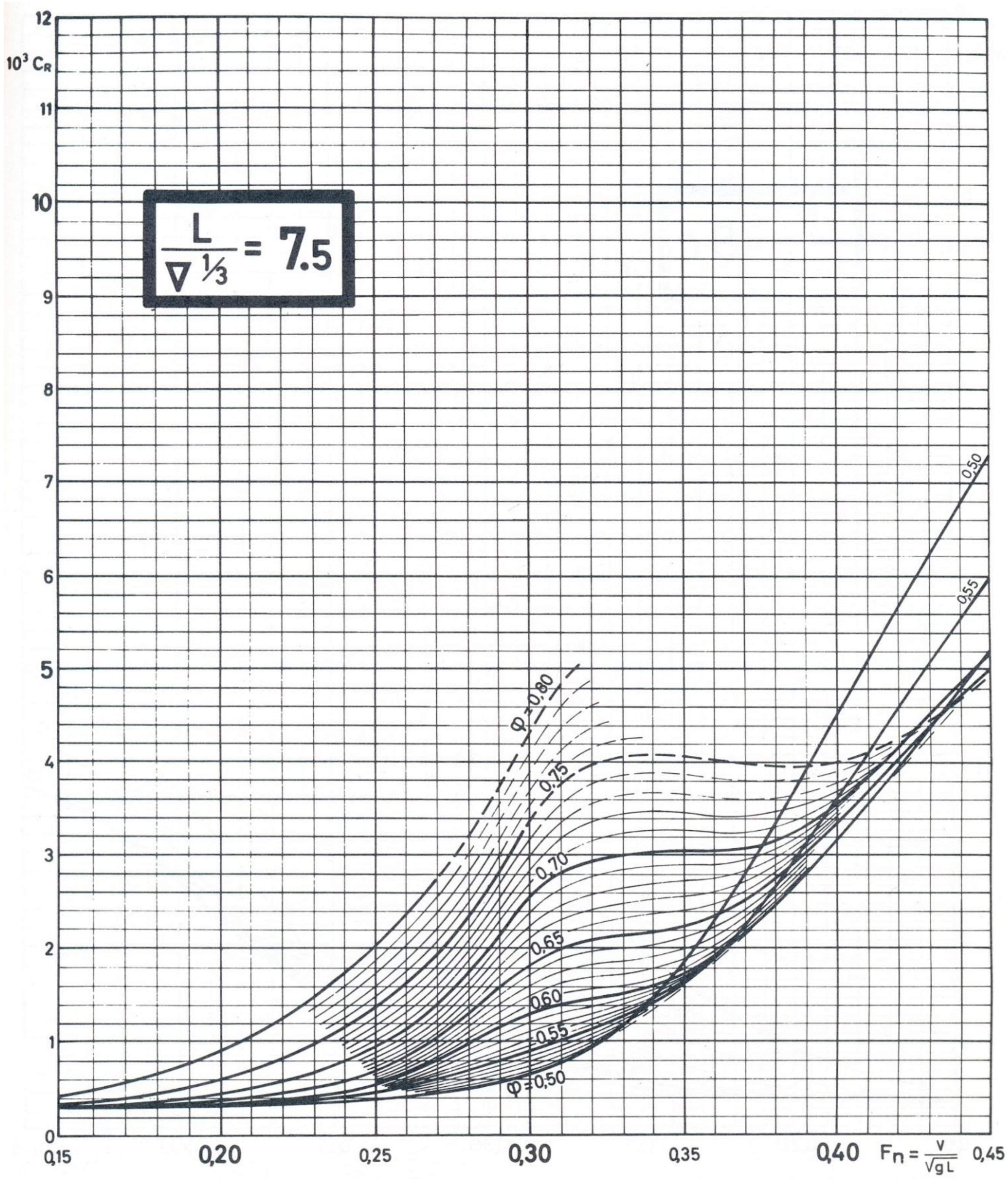

$10^3 C_R$
$\frac{L}{\nabla^{1/3}} = 6.0$
φ=0,85
φ=0,80
0,75
0,70
0,65
0,60
0,55
φ=0,50
0,50
0,55
0,60
0,65
0,70
0,15
0,20
0,25
0,30
0,35
0,40
$F_n = \frac{v}{\sqrt{gL}}$
0,45

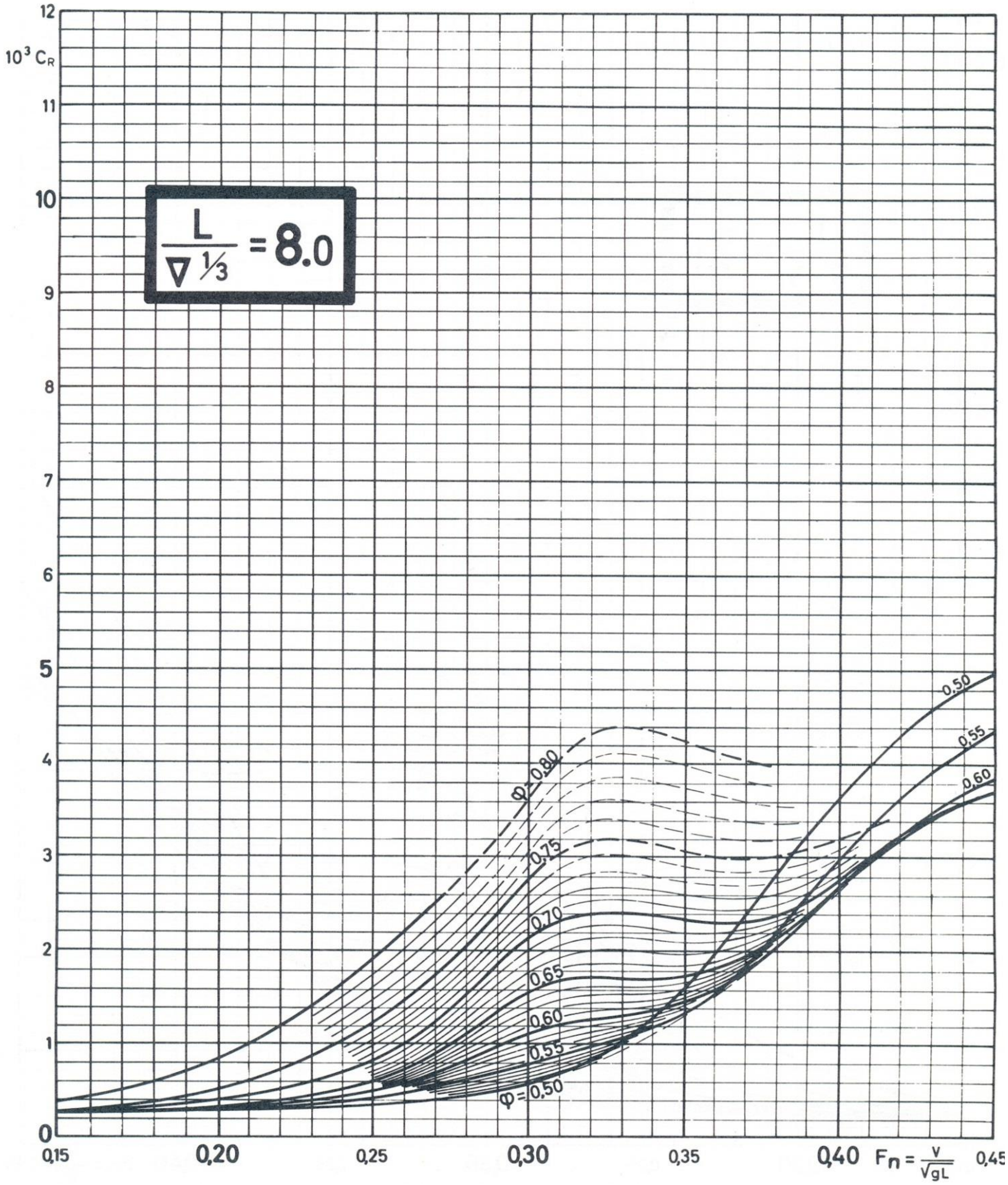


$10^3 C_R$
$\frac{L}{\nabla^{1/3}} = 6.5$
12
11
10
9
8
7
6
5
4
3
2
1
0
0,15
0,20
0,25
0,30
0,35
0,40
0,45
$Fn = \frac{v}{\sqrt{gL}}$
φ=0,80
0,75
0,70
0,65
0,60
0,55
φ=0,50
0,50
0,55
0,60
0,65

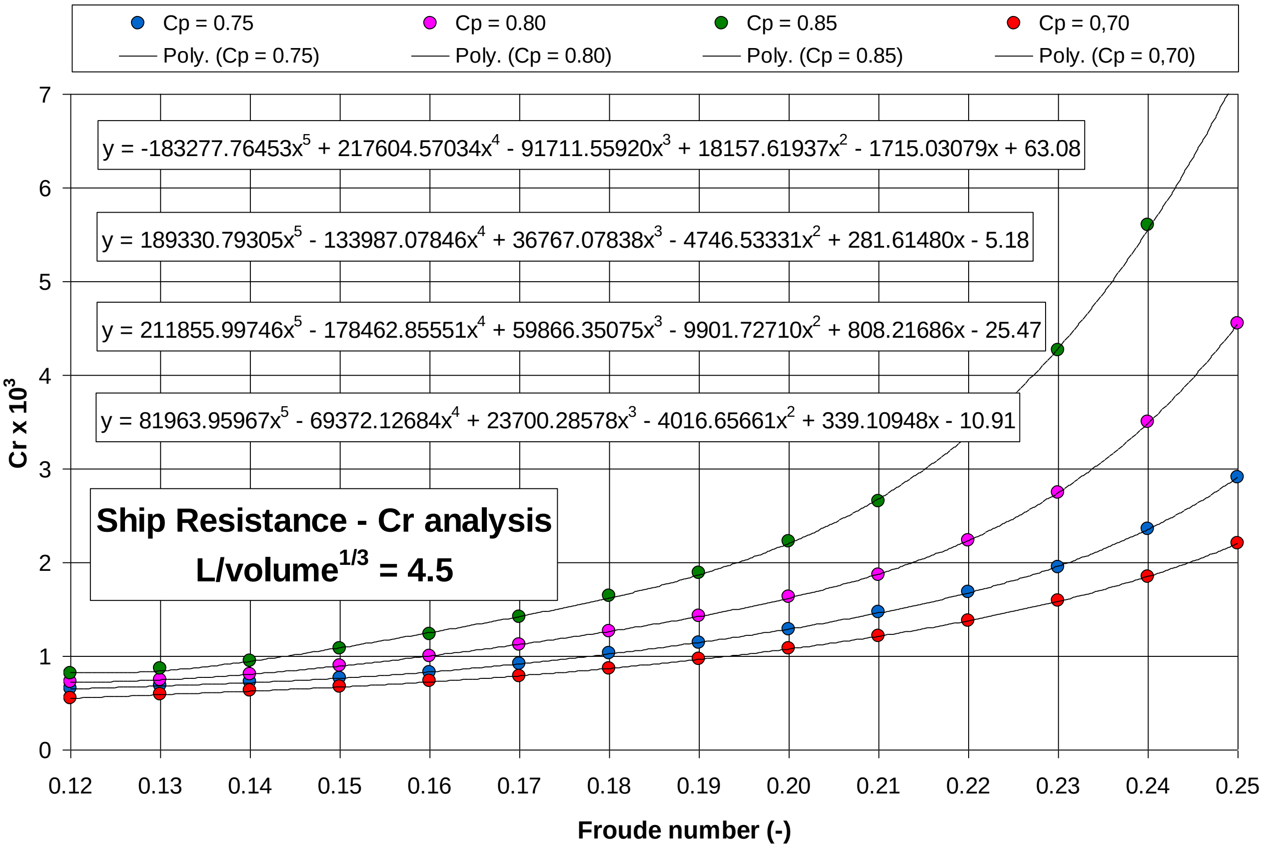


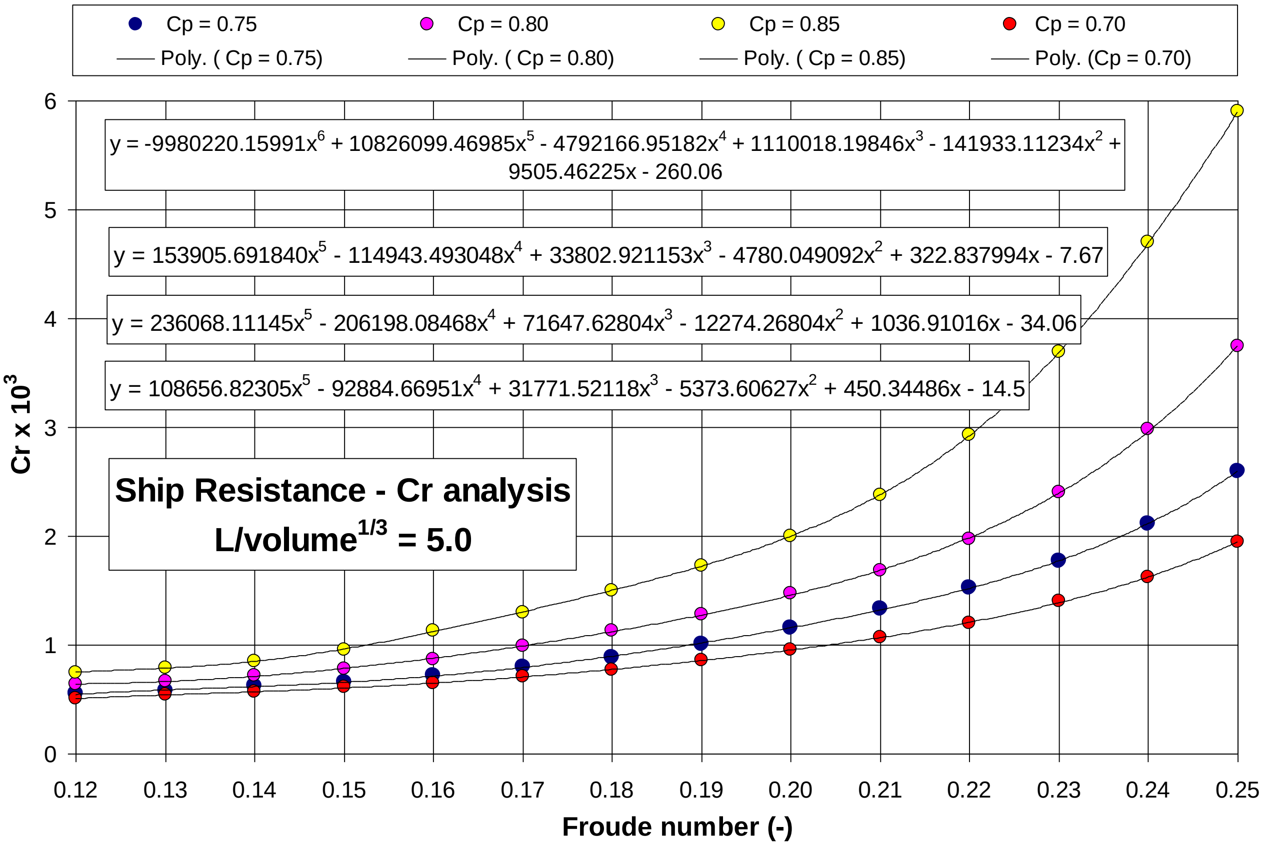
$10^3 C_R$
$\frac{L}{\nabla^{1/3}} = 7.0$
12
11
10
9
8
7
6
5
4
3
2
1
0
0,15
0,20
0,25
0,30
0,35
0,40
0,45
$F_n = \frac{v}{\sqrt{gL}}$
φ = 0,80
0,75
0,70
0,65
0,60
0,55
φ = 0,50
0,50
0,55
0,60
0,65

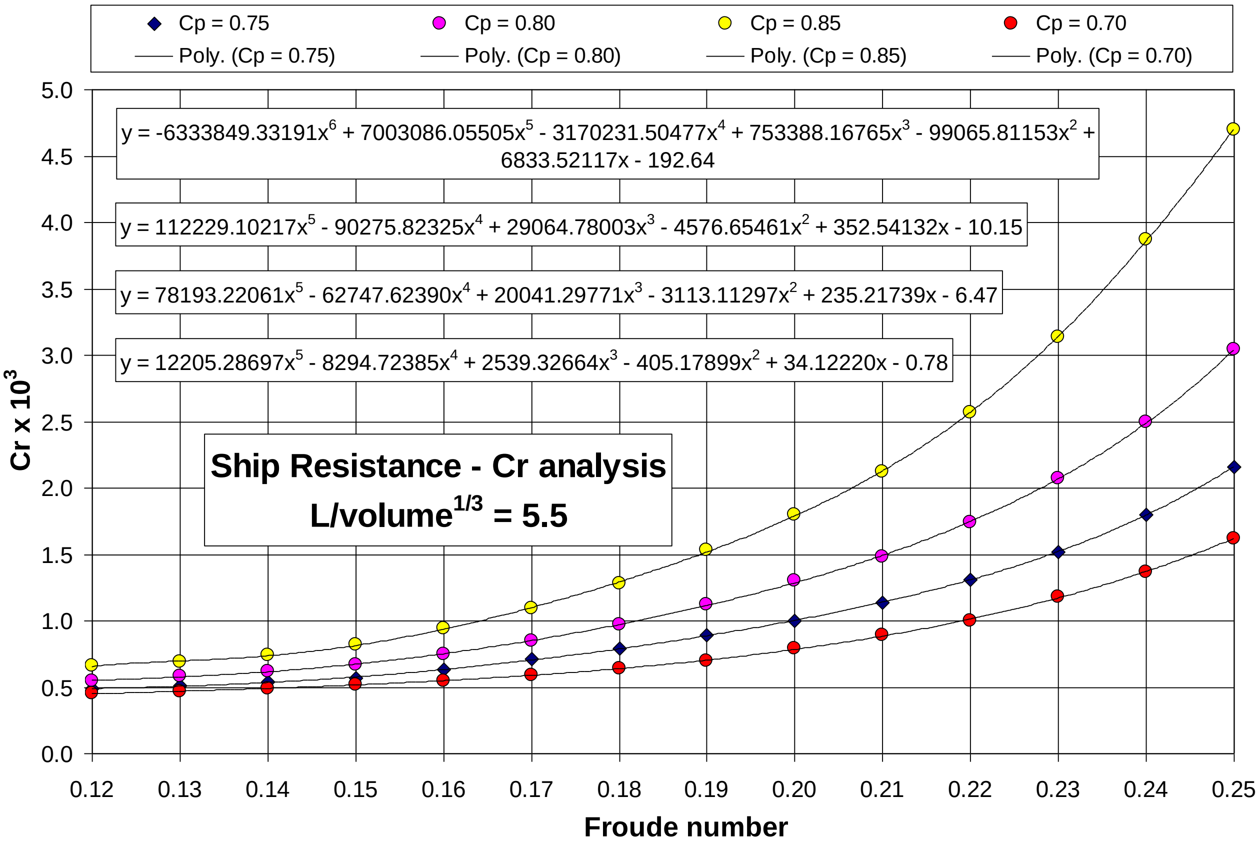

12
$10^3 C_R$
11
10
9
8
7
6
5
4
3
2
1
0
$\frac{L}{\nabla^{1/3}} = 7.5$
φ = 0,80
0,75
0,70
0,65
0,60
0,55
φ = 0,50
0,50
0,55
0,15
0,20
0,25
0,30
0,35
0,40
$F_n = \frac{V}{\sqrt{gL}}$
0,45

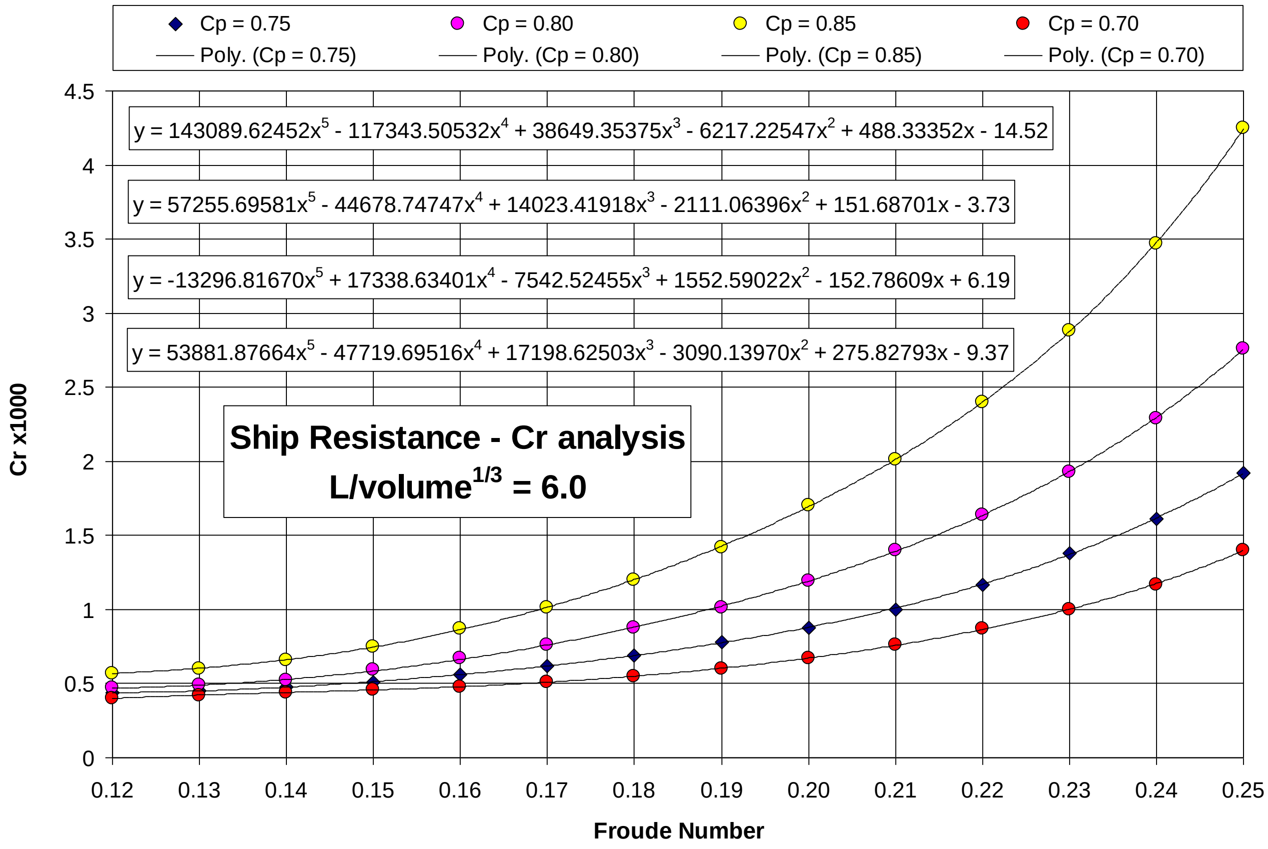

12
$10^3 C_R$
11
10
$\frac{L}{\nabla^{1/3}} = 8.0$
9
8
7
6
5
4
3
2
1
0
0,15
0,20
0,25
0,30
0,35
0,40
$F_n = \frac{v}{\sqrt{gL}}$
0,45
φ = 0,80
0,75
0,70
0,65
0,60
0,55
φ = 0,50
0,50
0,55
0,60

Appendix J – Cr equations found from regression analysis of Cr curves from “Ship Resistance” for bulky ships, i.e. prismatic coefficient larger than 0.70

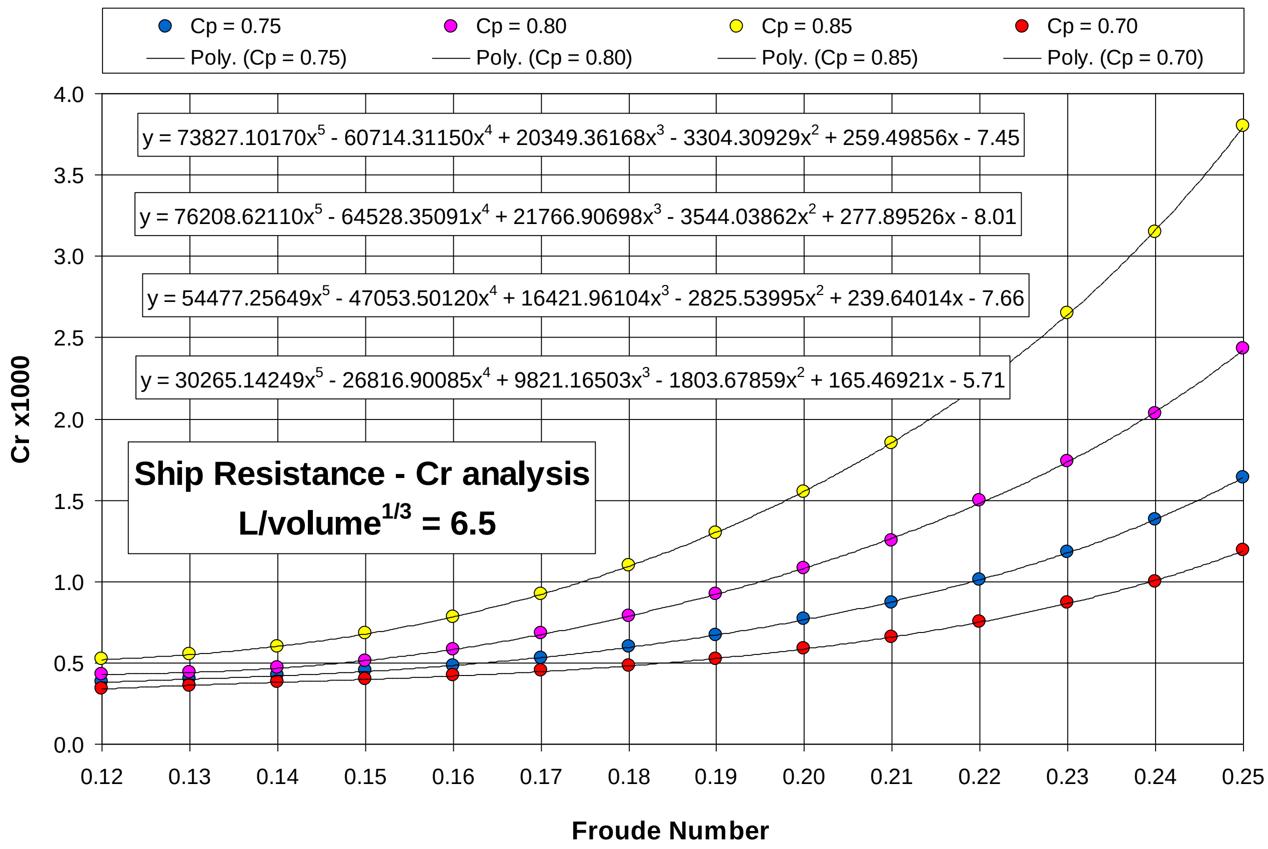



Cp = 0.75
Cp = 0.80
Cp = 0.85
Cp = 0.70
Poly. (Cp = 0.75)
Poly. (Cp = 0.80)
Poly. (Cp = 0.85)
Poly. (Cp = 0.70)
y = -9980220.15991x6 + 10826099.46985x5 - 4792166.95182x4 + 1110018.19846x3 - 141933.11234x2 + 9505.46225x - 260.06
y = 153905.691840x5 - 114943.493048x4 + 33802.921153x3 - 4780.049092x2 + 322.837994x - 7.67
y = 236068.11145x5 - 206198.08468x4 + 71647.62804x3 - 12274.26804x2 + 1036.91016x - 34.06
y = 108656.82305x5 - 92884.66951x4 + 31771.52118x3 - 5373.60627x2 + 450.34486x - 14.5
Ship Resistance - Cr analysis
L/volume1/3 = 5.0
Cr x 103
Froude number (-)

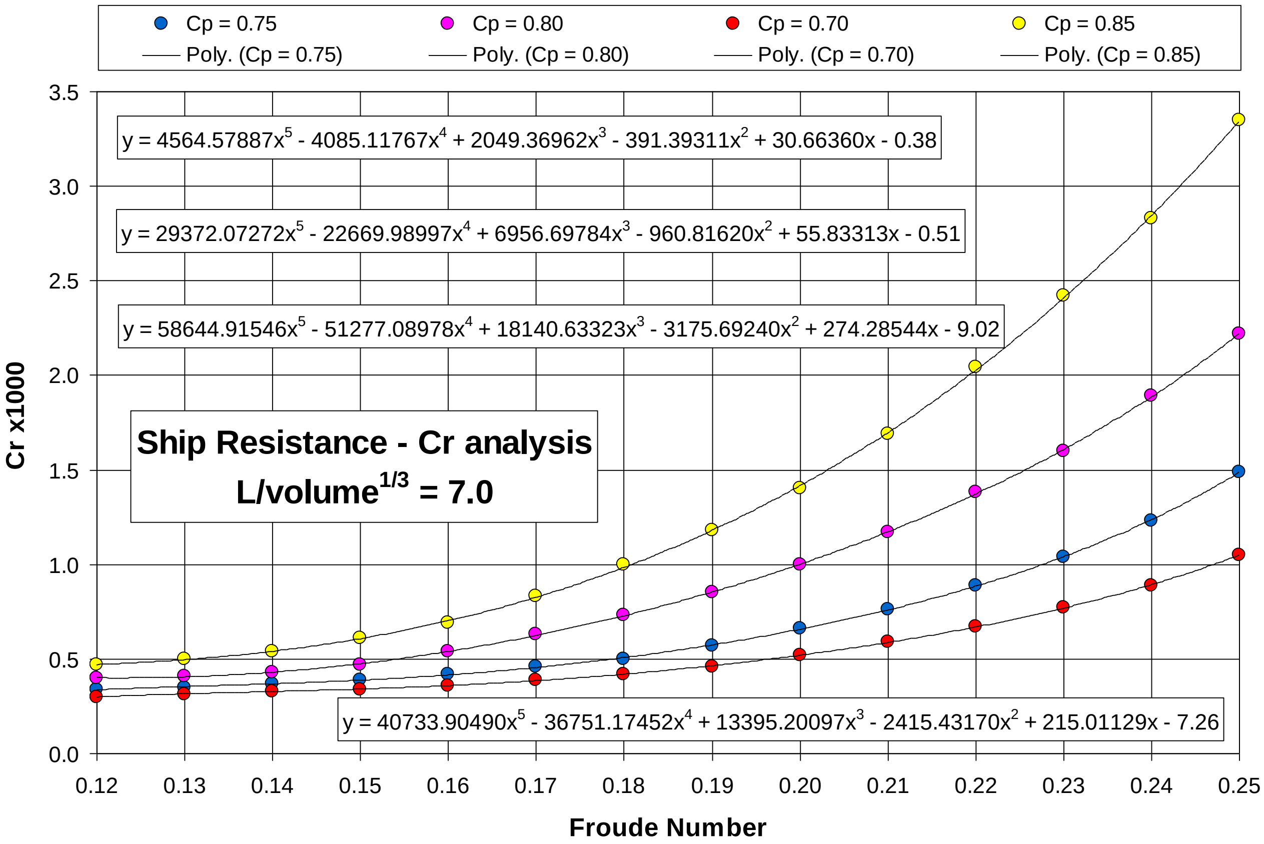

Cp = 0.75
Cp = 0.80
Cp = 0.85
Cp = 0.70
Poly. (Cp = 0.75)
Poly. (Cp = 0.80)
Poly. (Cp = 0.85)
Poly. (Cp = 0.70)
y = -6333849.33191x6 + 7003086.05505x5 - 3170231.50477x4 + 753388.16765x3 - 99065.81153x2 + 6833.52117x - 192.64
y = 112229.10217x5 - 90275.82325x4 + 29064.78003x3 - 4576.65461x2 + 352.54132x - 10.15
y = 78193.22061x5 - 62747.62390x4 + 20041.29771x3 - 3113.11297x2 + 235.21739x - 6.47
y = 12205.28697x5 - 8294.72385x4 + 2539.32664x3 - 405.17899x2 + 34.12220x - 0.78
Ship Resistance - Cr analysis
L/volume1/3 = 5.5
Cr x 103
Froude number



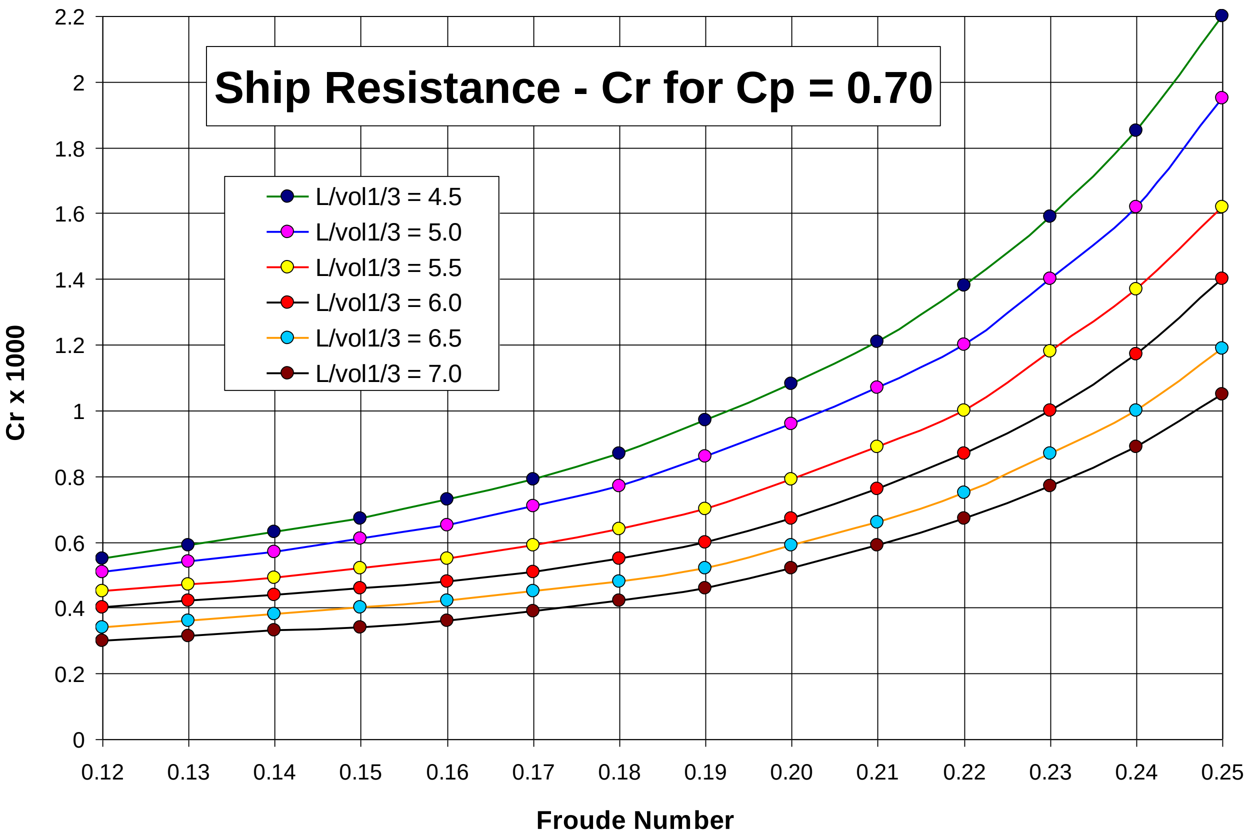
Cp = 0.75
Poly. (Cp = 0.75)
Cp = 0.80
Poly. (Cp = 0.80)
Cp = 0.85
Poly. (Cp = 0.85)
Cp = 0.70
Poly. (Cp = 0.70)
y = 143089.62452x5 - 117343.50532x4 + 38649.35375x3 - 6217.22547x2 + 488.33352x - 14.52
y = 57255.69581x5 - 44678.74747x4 + 14023.41918x3 - 2111.06396x2 + 151.68701x - 3.73
y = -13296.81670x5 + 17338.63401x4 - 7542.52455x3 + 1552.59022x2 - 152.78609x + 6.19
y = 53881.87664x5 - 47719.69516x4 + 17198.62503x3 - 3090.13970x2 + 275.82793x - 9.37
Ship Resistance - Cr analysis
L/volume1/3 = 6.0
Cr x1000
Froude Number



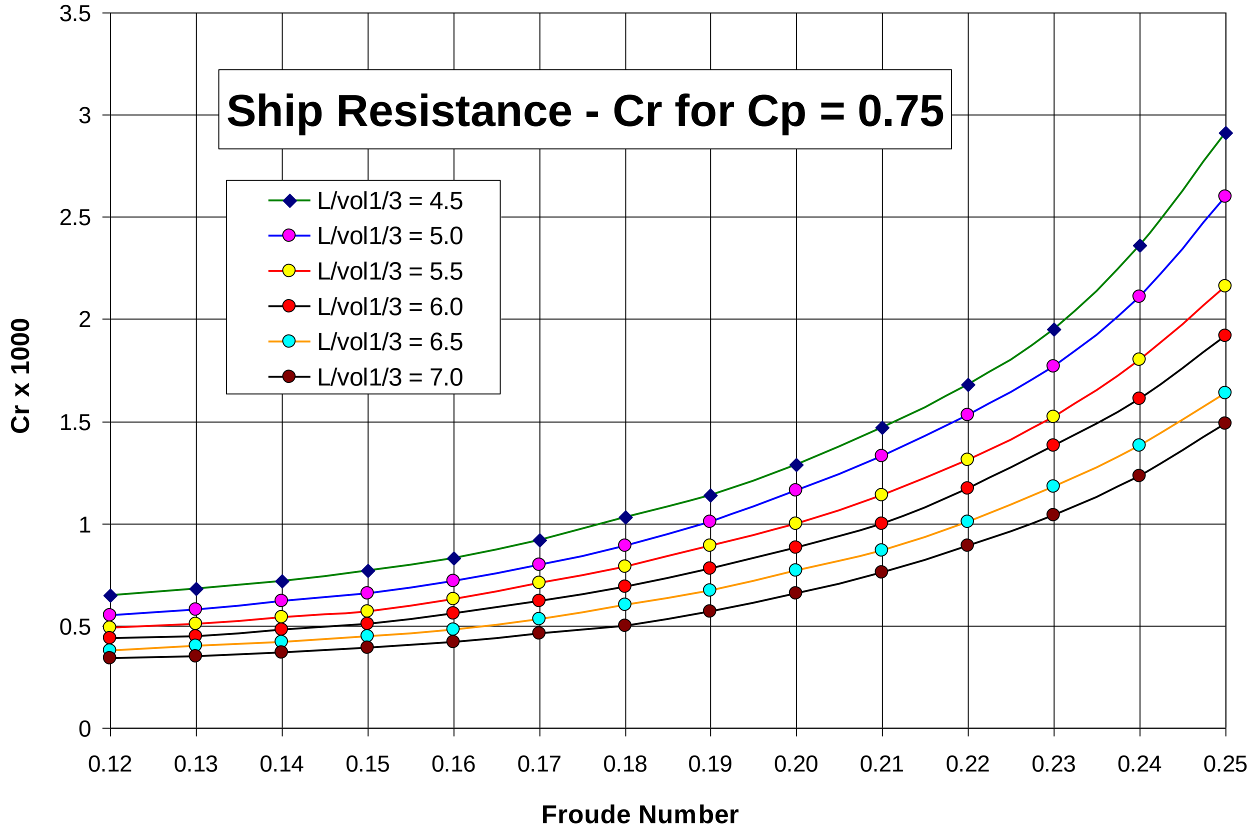
Cp = 0.75
Poly. (Cp = 0.75)
Cp = 0.80
Poly. (Cp = 0.80)
Cp = 0.85
Poly. (Cp = 0.85)
Cp = 0.70
Poly. (Cp = 0.70)
y = 73827.10170x5 - 60714.31150x4 + 20349.36168x3 - 3304.30929x2 + 259.49856x - 7.45
y = 76208.62110x5 - 64528.35091x4 + 21766.90698x3 - 3544.03862x2 + 277.89526x - 8.01
y = 54477.25649x5 - 47053.50120x4 + 16421.96104x3 - 2825.53995x2 + 239.64014x - 7.66
y = 30265.14249x5 - 26816.90085x4 + 9821.16503x3 - 1803.67859x2 + 165.46921x - 5.71
Ship Resistance - Cr analysis
L/volume1/3 = 6.5
Cr x1000
Froude Number



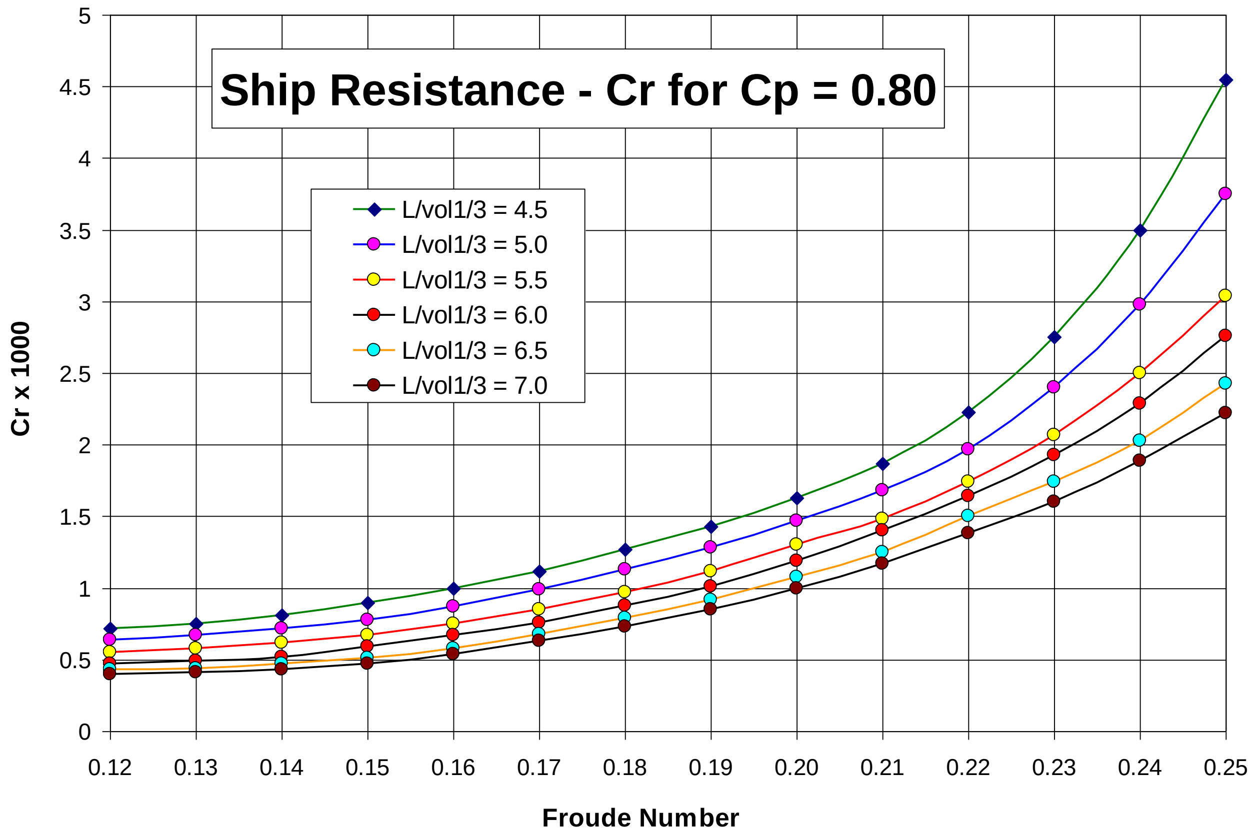
Cp = 0.75
Poly. (Cp = 0.75)
Cp = 0.80
Poly. (Cp = 0.80)
Cp = 0.70
Poly. (Cp = 0.70)
Cp = 0.85
Poly. (Cp = 0.85)
y = 4564.57887x5 - 4085.11767x4 + 2049.36962x3 - 391.39311x2 + 30.66360x - 0.38
y = 29372.07272x5 - 22669.98997x4 + 6956.69784x3 - 960.81620x2 + 55.83313x - 0.51
y = 58644.91546x5 - 51277.08978x4 + 18140.63323x3 - 3175.69240x2 + 274.28544x - 9.02
Ship Resistance - Cr analysis
L/volume1/3 = 7.0
y = 40733.90490x5 - 36751.17452x4 + 13395.20097x3 - 2415.43170x2 + 215.01129x - 7.26
Cr x1000
Froude Number
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
0.24
0.25



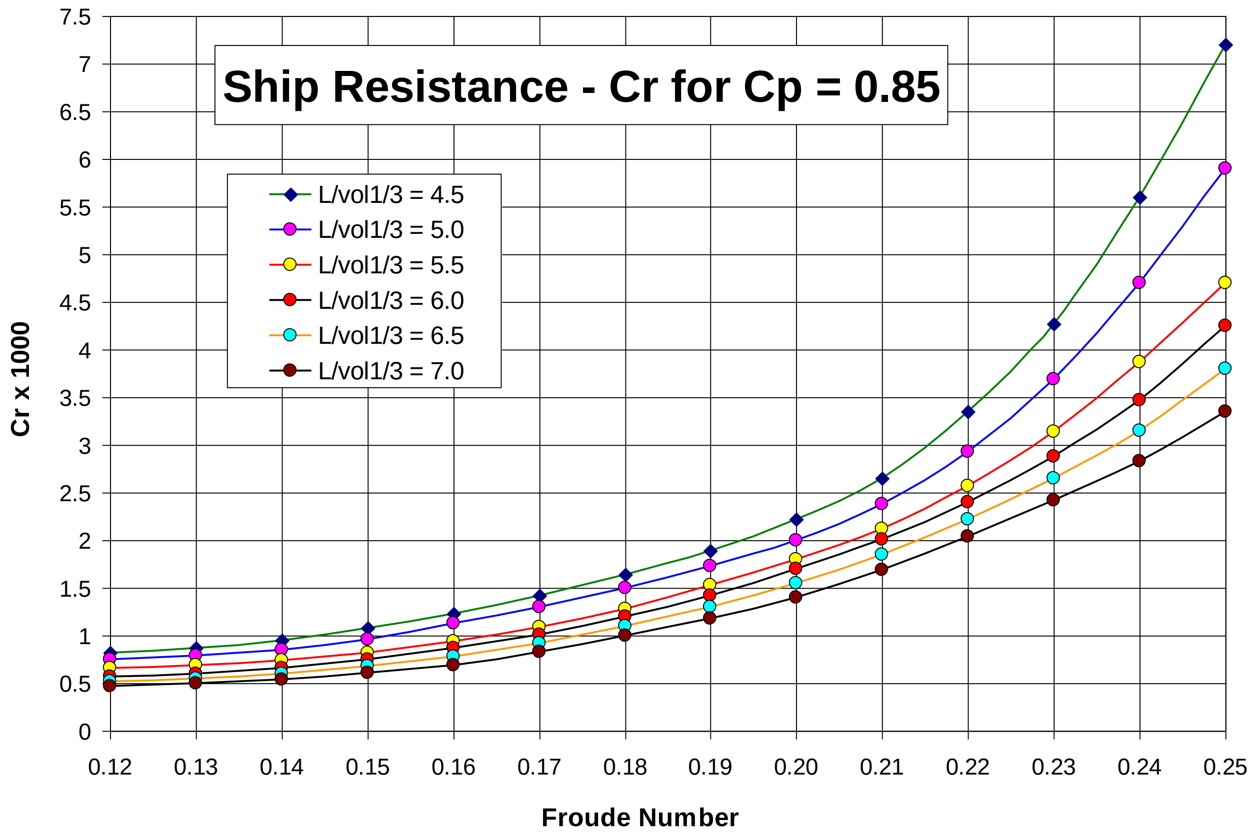
Ship Resistance - Cr for Cp = 0.70
L/vol1/3 = 4.5
L/vol1/3 = 5.0
L/vol1/3 = 5.5
L/vol1/3 = 6.0
L/vol1/3 = 6.5
L/vol1/3 = 7.0
Cr x 1000
Froude Number
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
0.24
0.25



Ship Resistance - Cr for Cp = 0.75
L/vol1/3 = 4.5
L/vol1/3 = 5.0
L/vol1/3 = 5.5
L/vol1/3 = 6.0
L/vol1/3 = 6.5
L/vol1/3 = 7.0
Cr x 1000
3.5
3
2.5
2
1.5
1
0.5
0
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
0.24
0.25
Froude Number



Ship Resistance - Cr for Cp = 0.80
L/vol1/3 = 4.5
L/vol1/3 = 5.0
L/vol1/3 = 5.5
L/vol1/3 = 6.0
L/vol1/3 = 6.5
L/vol1/3 = 7.0
Cr x 1000
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
0.24
0.25
Froude Number



Ship Resistance - Cr for Cp = 0.85
L/vol1/3 = 4.5
L/vol1/3 = 5.0
L/vol1/3 = 5.5
L/vol1/3 = 6.0
L/vol1/3 = 6.5
L/vol1/3 = 7.0
Cr x 1000
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
0.21
0.22
0.23
0.24
0.25
Froude Number