Project

Department Fahrzeugtechnik und Flugzeugbau

Non Linear Aircraft Dynamics and PIO

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Abstract

The purpose of this report is to study the Pilot Induced Oscillations (PIO) phenomenon.
The method to show this phenomenon is to build the model of an airplane and its control system and then perform simulations on this plane with Matlab-Simulink software. The equations of flight dynamics used are the linear and the non-linear equations of dynamics given by **Mc Lean 1990**. The simulation is performed on a Douglas DC-8 with the help of derivatives data given by **Mc Ruer 1995.**

The model of the plane is built in different parts: flight dynamics model, pilot model, actuator model and control model. Then these parts are integrated in a bigger model representing the whole airplane with its controls. After checking that the aircraft is stable, a big gain representing a wrong reaction of the pilot is put inside this model. Then the plane starts to oscillate: the Pilot Induced Oscillations appear.

So, this report clearly shows that the PIO are caused by an overreaction of the pilot inside the loop of the control system.

DEPARTMENT OF AUTOMOTIVE AND AERONAUTICAL ENGINEERING

Nonlinear Aircraft Dynamics and PIO

Project

Background

Textbooks and lectures on aircraft dynamics investigate an aircraft primarily with a set of linear equations. Especially the approach to describe aircraft dynamics in state space form is particularly efficient. But not all effects of aircraft behavior can be studied with the simplified linear description. The equations themselves are nonlinear and so is the aerodynamic behavior of the aircraft. The flight control system is nonlinear: a) with respect to signal transmission from cockpit to control surface and b) considering the control surface actuation. Furthermore, deflection maximum travel and deflection rates are limited. The nonlinear phenomena can lead to dangerous characteristics like pilot in the loop oscillation (PIO) also know as aircraft pilot coupling (APC).

Task

Start from linear aircraft dynamics and work your way through the topics as outlined above. Consider these key words on the way: definition of nonlinear dynamics, linear and nonlinear longitudinal and lateral dynamics, flying qualities, MIL-Spec and MIL-Standard, nonlinear set of equations, actuator dynamics, describing function, closed loop control, pilot model, pilot in the loop control, PIO basics, longitudinal PIO, lateral PIO, PIO prediction.

Always show example calculations related to the topics with MATLAB/Simulink. Use stability and control derivatives as internationally available e.g. in textbooks.

All relevant sources of information should be considered ranging from text books, internet resources, papers, dissertations, ...

The report has to be written in English based on German or international standards on report writing.

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Glossary

Eigenvalue

"Complex numbers, λ that satisfy $A\,x = \lambda\,x$ where A is a $n*n$ matrix and x is some vector. In this case, x is called an eigenvector." (**LBM**)

Static error

Difference between the input and the output values when $t \to \infty$ and the temporal evolution of these signals is almost null.

PID Controller

"A PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly and rapidly, to keep the error minimal." (**Wikipedia 2009**)

Dirac impulse

"Function representing an infinitely sharp peak bounding unit area. This function $\delta(x)$ has the value zero everywhere except at x = 0 where its value is infinitely large in such a way that its total integral is 1." (**Wikipedia 2009**)

Step function

Discontinuous function whose value is zero before the step time and A after, where A is a constant different from zero.

1. Introduction

Aircrafts are never stable by themselves, they need to control. Otherwise they would fly in a constant turn. In order to make this aircraft fly in the direction we want, we have to make corrections at any time. There are two ways of making these corrections: the first one is a correction by the pilot, and the other is by the automatic flight control system (AFCS). This AFCS uses a feedback control which is much more accurate and faster, and which can eliminate some disturbances. **(Mc Lean 1990)**

But sometimes and under some conditions theses feedback control systems tend to oscillate. That is why the AFCS system is not really stable. If we use a high amplification, the response will be more accurate and faster but the system will be less stable. Then we have to find a compromise between stability and accuracy when we design a system like the AFCS.

As we want to study aircraft stability, we have to make a model of the aircraft. The first step for this modeling is to know which equations are used for the flight dynamics. Then we will integrate them in the aircraft model. Most of these equations are non-linear but we will see that we can get a linear model which is really close from the non-linear model but much easier and faster for the simulations.

One of these phenomena is the Pilot Induced Oscillation (PIO). We will explain this phenomenon more in detail in the PIO chapter. The main thing to know about this phenomenon is that it is an undesired movement of the aircraft which makes it oscillate and loose its stability. As the control system of the aircraft uses a closed loop, these oscillations are amplified then they become bigger and the aircraft stability is much worse.

In order to understand this PIO phenomenon we will first study the flight dynamics of the aircraft without trying to control it. Then we will see the simple models of aircraft control and pilot models without aircraft. The next step will be to integrate these basics linear models in an aircraft in order to control it. The last step will be to integrate a non linear model of actuator in the aircraft. Then we will have to put the right conditions in order to simulate the PIO, which means introducing high gains in the closed loop.

The literature we will use in this study consists in writings on aircraft dynamics and NASA reports about the PIO phenomenon, as written in the list of references. All simulations will be made on Matlab-Simulink software.

The aircraft of which we are going to study the flight dynamics is the Douglas DC-8. The derivative data of this plane are included at the end of this report.

The basic model of an aircraft is described by **Mc Lean 1990**

	Velocities	Applied forces and moments	Distances
Forward	U	X	x
Side	V	Y	y
Vertical	W	Z	z
Roll ϕ	P	L	
Pitch ϑ	Q	M	
Yaw β	R	N	
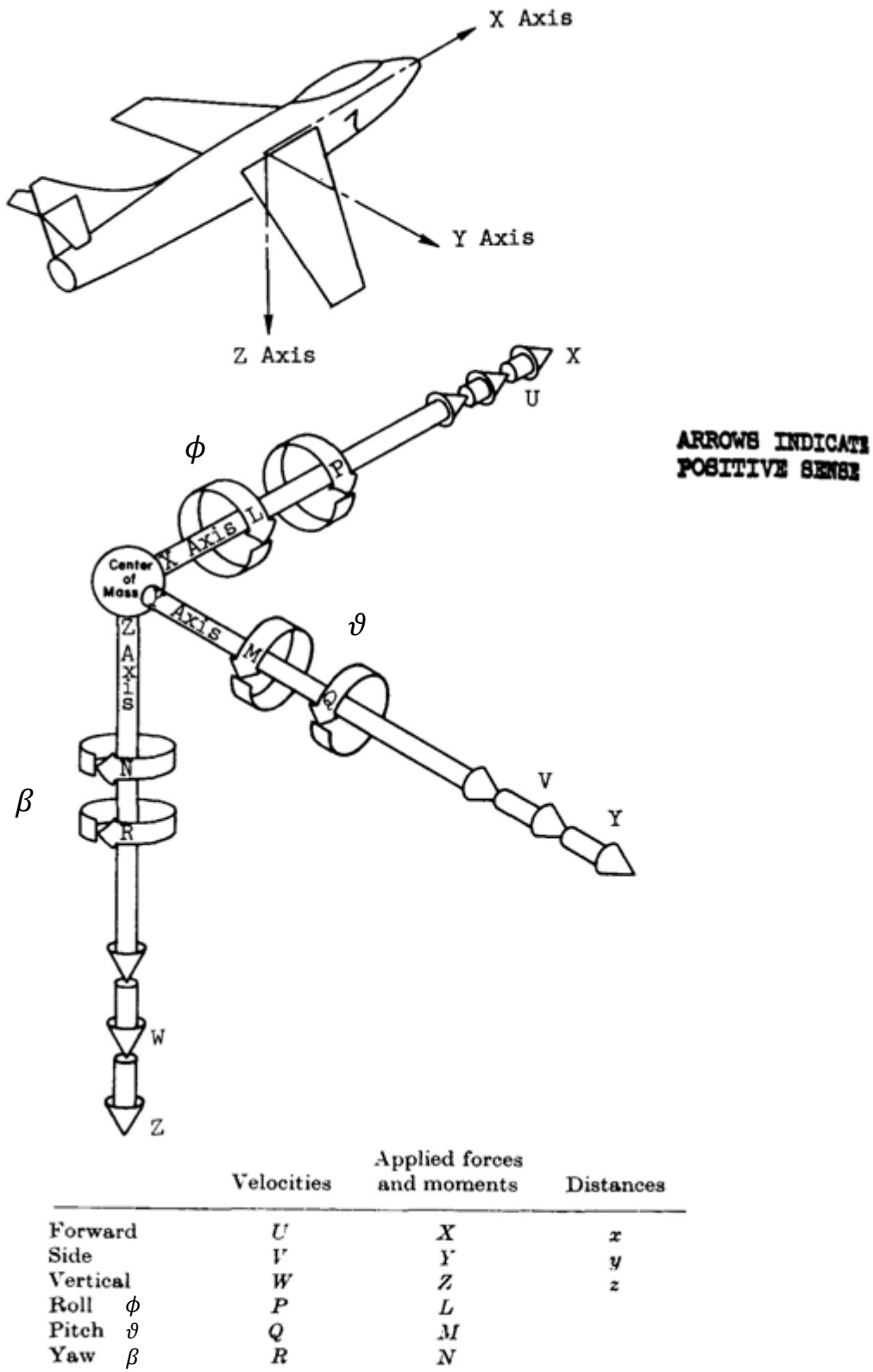
Fig 1.1   Conventions used for the aircraft axis and velocities **(Mc Lean 1990)**

$$+\delta_A = \tfrac{1}{2}(\delta_{A_L} + \delta_{A_R})$$
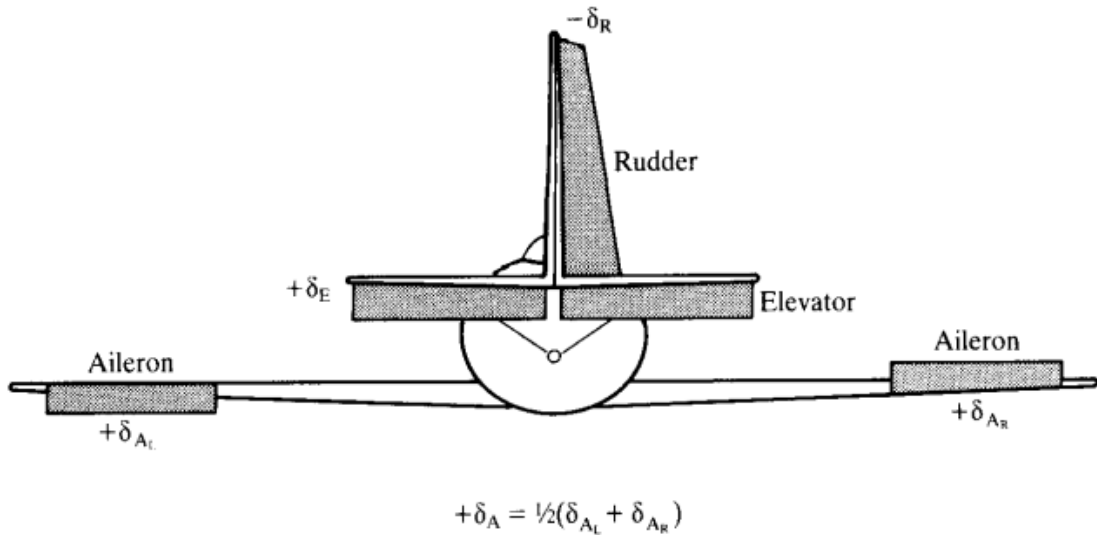
Fig 1.2   Control surfaces deflections conventions **(Mc Lean 1990)**
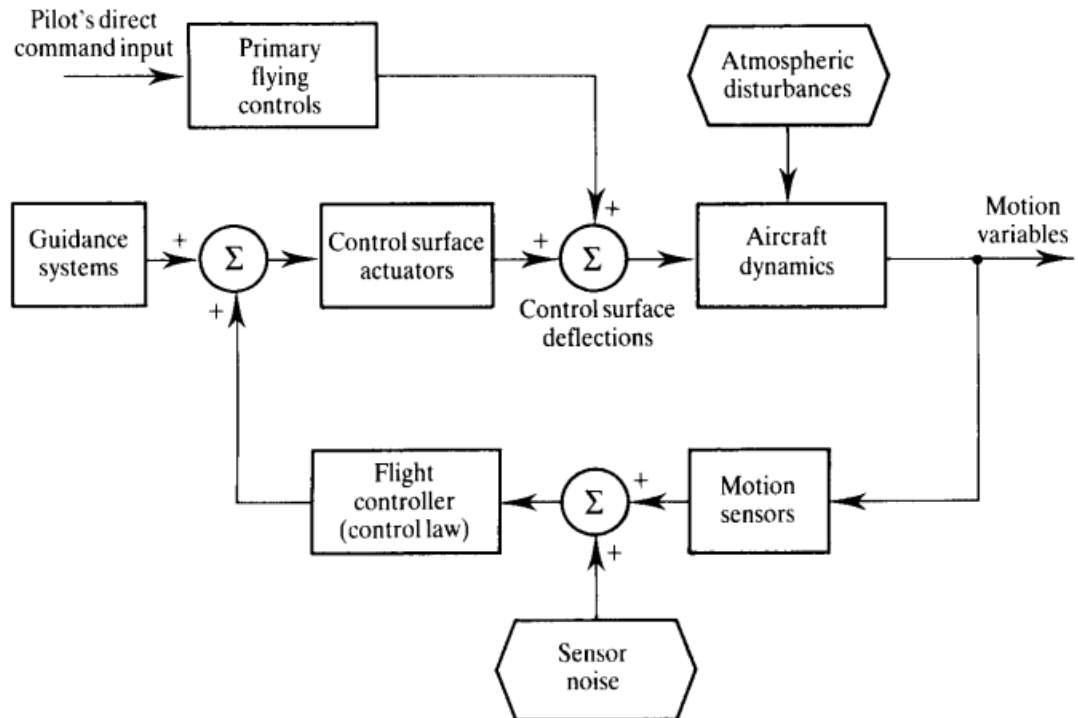
Fig 1.3   General structure of an AFCS **(Mc Lean 1990)**

2. Linear and non linear flight dynamics

The first thing we need to do, in order to build the model of the aircraft, is to put the set of equation representing the flight dynamics inside a block, so we can simulate the flight of the aircraft.

The first step will be to use the linear equations which are easy to put inside the model, then we will use the non linear equations, which should be closer from the reality, but much harder and slower for the simulations.

When we use the linear dynamics, we have to separate two cases of flight: the longitudinal flight, which means that the aircraft flights straight, and can turn a few around the Y axis, so it changes its pitch angle. The other case of flight is the lateral flight which means that the aircraft can turn around the X or Z axis, but not on the Y axis, so it can change its roll or its yaw angle.

2.1. Longitudinal aircraft dynamics (linear)

2.1.1. Equations

The dynamic of the aircraft can be represented by this state space equation:

$\dot{x} = A \cdot x + B \cdot u$

Where x is called the state vector and u the control vector.

We have also: $x = \begin{bmatrix} u \\ w \\ q \\ \vartheta \end{bmatrix}$ and $u = \begin{bmatrix} \delta_E \\ \delta_F \\ u_g \\ w_g \\ q_g \end{bmatrix}$

u (of x vector) is the plane velocity on X axis, w on Z axis, q is the pitch velocity and ϑ the pitch angle.

δ_E is the command on the elevator, δ_F on the flaps and u_g, w_g, q_g are about the wind velocity.

The vector A is the state matrix and B is the control matrix. Both of these vectors are filled with derivatives.

$$A = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & U_0 & 0 \\ \widetilde{M}_u & \widetilde{M}_w & \widetilde{M}_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} X_{\delta_E} & X_{\delta_F} & -X_u & -X_w & 0 \\ Z_{\delta_E} & Z_{\delta_F} & -Z_u & -Z_w & -U_0 \\ \widetilde{M}_{\delta_E} & \widetilde{M}_{\delta_F} & -\widetilde{M}_u & -\widetilde{M}_w & \widetilde{M}_q \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

With:

$\widetilde{M}_u = M_u + M_{\dot{w}} \cdot Z_u$ $\qquad$ $\widetilde{M}_w = M_w + M_{\dot{w}} \cdot Z_w$

$\widetilde{M}_q = M_q + U_0 \cdot M_{\dot{w}}$ $\qquad$ $\widetilde{M}_{\delta_E} = M_{\delta_E} + M_{\dot{w}} \cdot Z_{\delta_E}$

2.1.2. Application

We use these equations to simulate the longitudinal dynamic of a DC8.
We do not use the second column of the B matrix (flaps command), so the control matrix and control vector are now:

$$B = \begin{bmatrix} X_{\delta_E} & -X_u & -X_w & 0 \\ Z_{\delta_E} & -Z_u & -Z_w & -U_0 \\ \tilde{M}_{\delta_E} & -\tilde{M}_u & -\tilde{M}_w & -\tilde{M}_q \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \text{and} \quad u = \begin{bmatrix} \delta_E \\ u_g \\ w_g \\ q_g \end{bmatrix}$$

The output equation of the simulation is: $y = C \cdot x + D \cdot u$

As we only want to know the evolution of the state vector, we use the following parameters for C and D:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
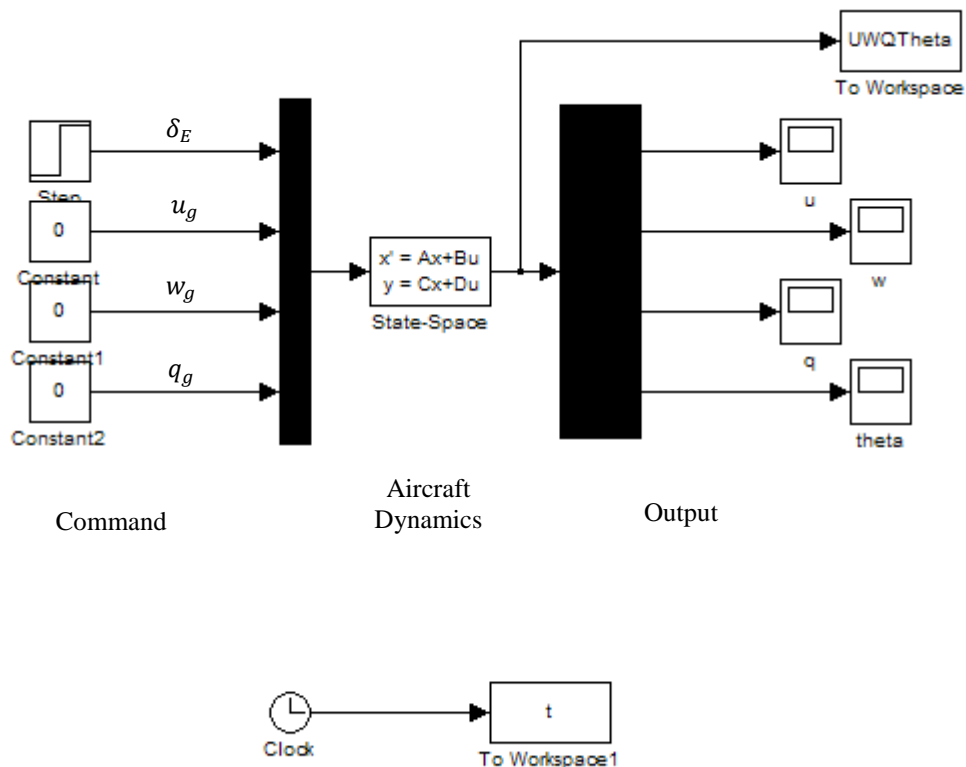
Fig 2.1   Simulink model used for the simulation

As we can see on the model, the command is only an action on the elevator (step).
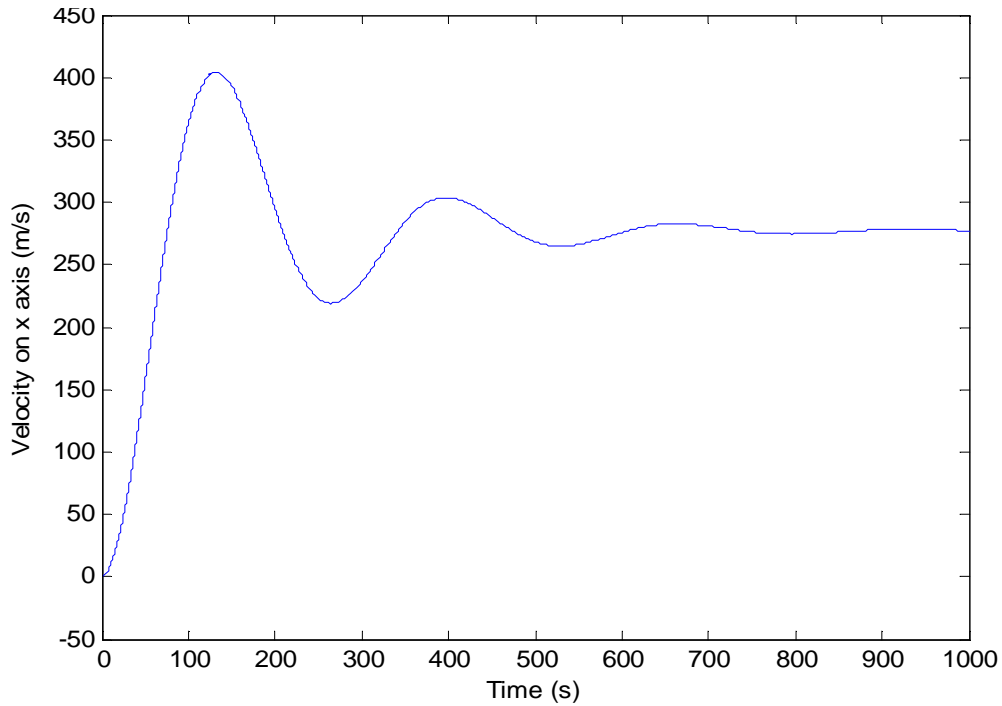
Fig 2.2   Velocity on x axis

As we can see on this graph, the velocity on x-Axis is null at the beginning, and then it increases and reaches its final value after about 150 seconds. There are some few but quite high oscillations of the speed before it reaches the final value.
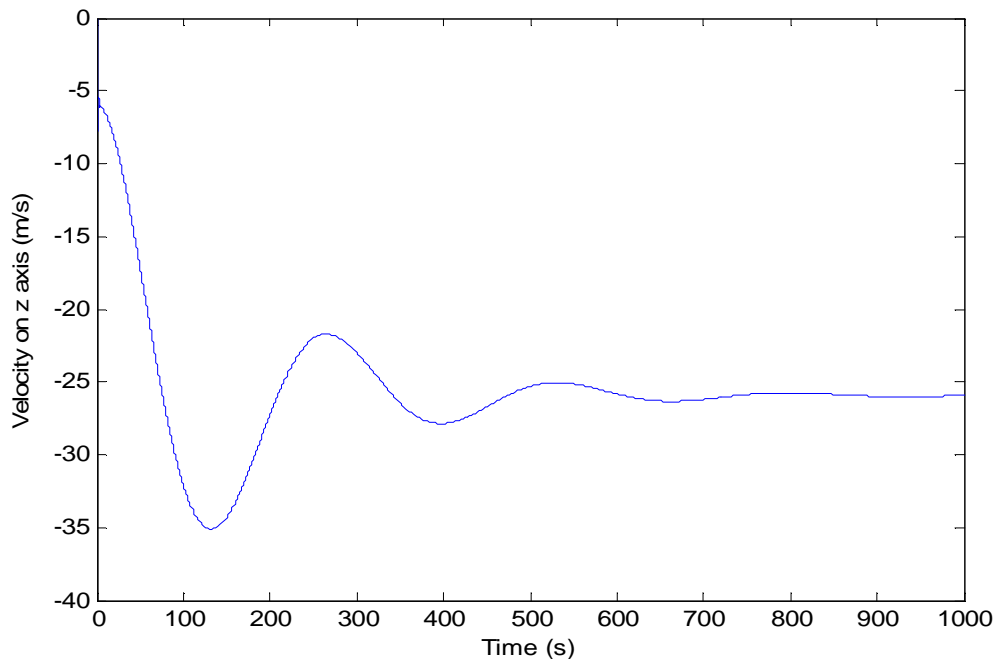
Fig 2.3   Velocity on z axis

Concerning the velocity on z-Axis, we can see the same kind of evolution as for the x-Axis. We also see that this speed is negative: it means that the plane is going higher and higher, because the z-Axis is in the direction of the ground.
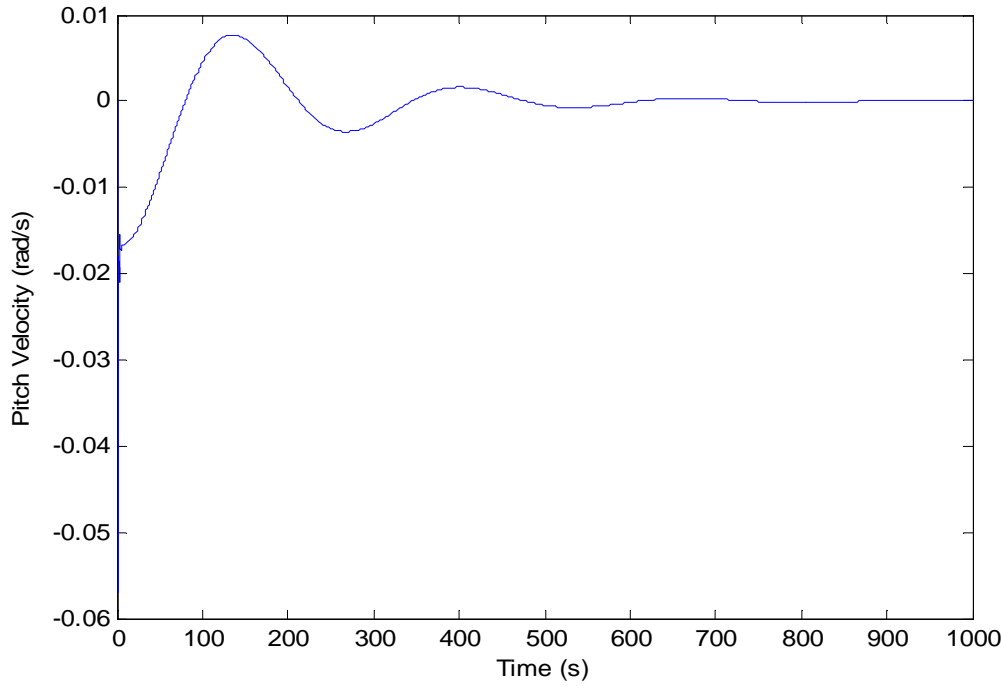
Fig 2.4   Pitch velocity

On this graph, we can see the action of the elevator: at the beginning, the pitch velocity has a big variation, and there are some oscillations. The final value of this velocity is null.
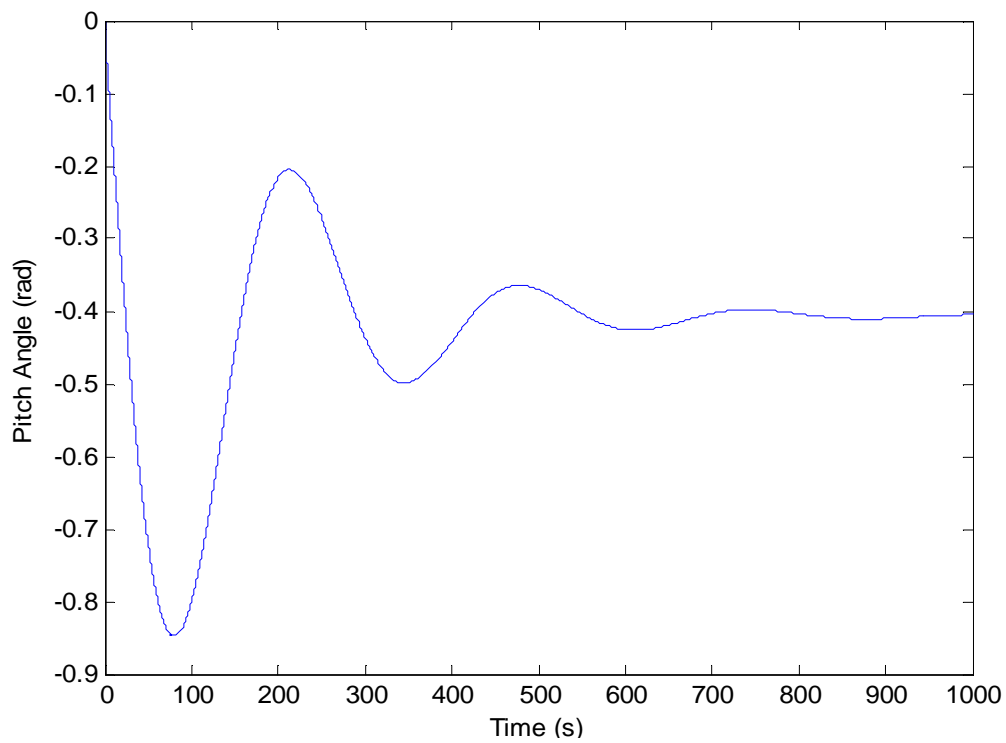
Fig 2.5   Pitch angle

As for the pitch velocity, we see a big variation of the pitch angle at the beginning, and then some decreasing oscillations. But here the final value is not null.

2.2. Lateral aircraft dynamics (linear)

2.2.1. Equations

The state space equation for lateral aircraft dynamics is the same as for longitudinal dynamics, but now the matrixes and vectors have different values:

$$A = \begin{bmatrix} Y_v & 0 & -1 & g/U_0 \\ L_\beta' & L_p' & L_r' & 0 \\ N_\beta' & N_p' & N_r' & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{And } B = \begin{bmatrix} Y_{\delta_A}{}^* & Y_{\delta_R}{}^* & -Y_v & 0 & 1 \\ L_{\delta_A}' & L_{\delta_R}' & -L_\beta' & -L_p' & -L_r' \\ N_{\delta_A}' & N_{\delta_R}' & -N_\beta' & -N_p' & -N_r' \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

With

$$L_\beta' = L_\beta + \frac{I_{xz}}{I_x}.N_\beta \qquad\qquad N_\beta' = N_\beta + \frac{I_{xz}}{I_z}.L_\beta$$

$$L_p' = L_p + \frac{I_{xz}}{I_x}.N_p \qquad\qquad N_p' = N_p + \frac{I_{xz}}{I_z}.L_p$$

$$L_r' = L_r + \frac{I_{xz}}{I_x}.N_r \qquad\qquad N_r' = N_r + \frac{I_{xz}}{I_z}.L_r$$

$$L_{\delta_A}' = L_{\delta_A} + \frac{I_{xz}}{I_x}.N_{\delta_A} \qquad\qquad N_{\delta_A}' = N_{\delta_A} + \frac{I_{xz}}{I_z}.L_{\delta_A}$$

$$L_{\delta_R}' = L_{\delta_R} + \frac{I_{xz}}{I_x}.N_{\delta_R} \qquad\qquad N_{\delta_R}' = N_{\delta_R} + \frac{I_{xz}}{I_z}.L_{\delta_R}$$

$$Y_{\delta_A}{}^* = \frac{Y_{\delta_A}}{U_0} \qquad\qquad\qquad Y_{\delta_R}{}^* = \frac{Y_{\delta_R}}{U_0}$$

$$x = \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} \quad \text{And } u = \begin{bmatrix} \delta_A \\ \delta_R \\ \beta_g \\ p_g \\ r_g \end{bmatrix}$$

x is the state vector. The variables in x are: β the yaw angle, p the roll velocity, r the yaw velocity and ϕ the roll angle.

u is the command vector. The variables used in u are: δ_A the command on the ailerons, δ_R the command on the rudder and β_g, p_g, r_g are about the wind speed.

2.2.2. Application

We need to use the equations above, and the values for the matrix of output equations are:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{And} \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
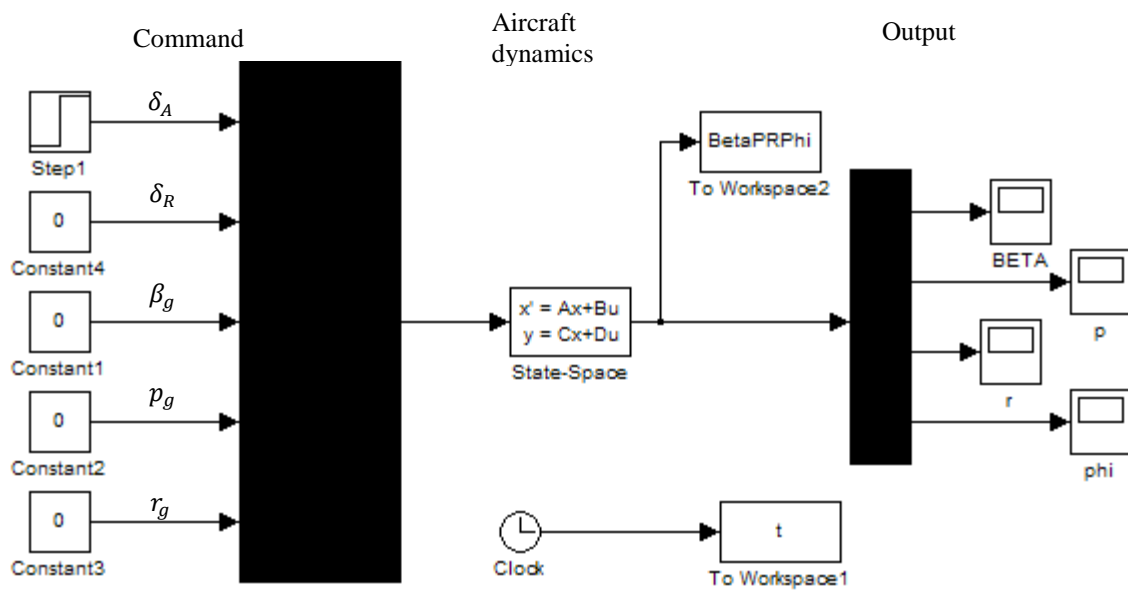
Fig 2.6   Simulink model used for the simulation

As we can see on this model, the input of our model is an action on the ailerons (step). Then this action should have a consequence on the roll angle.
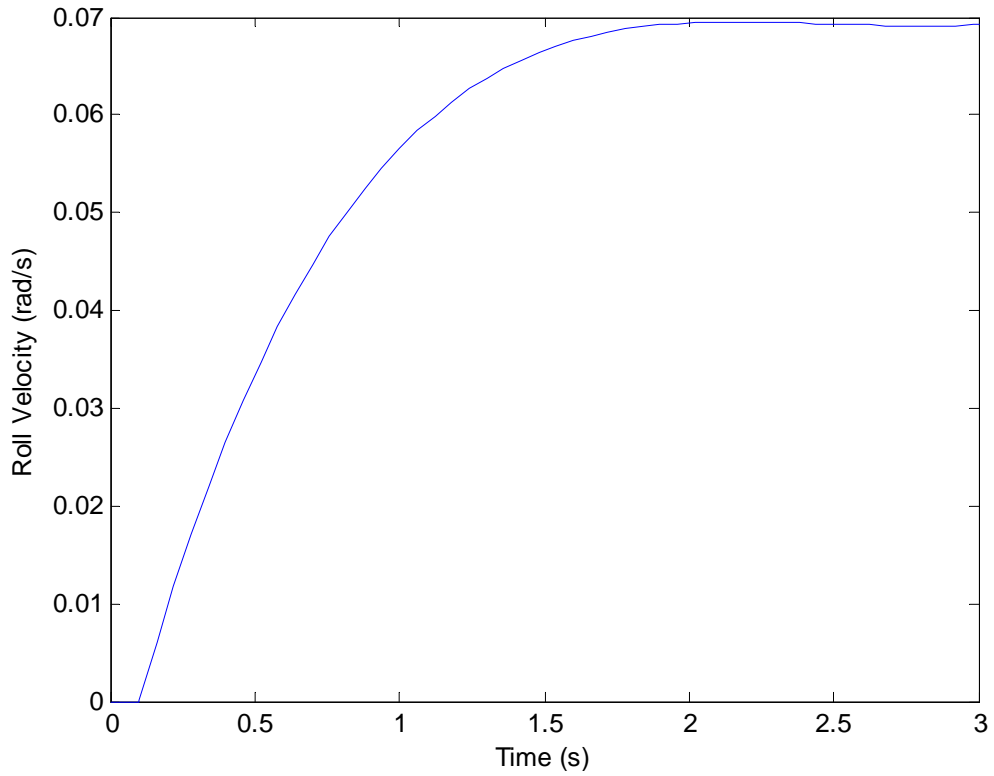
Fig 2.7   Roll velocity

The action on the ailerons has a direct consequence on roll velocity: at the beginning, this velocity is null, and then it increases until its final value.
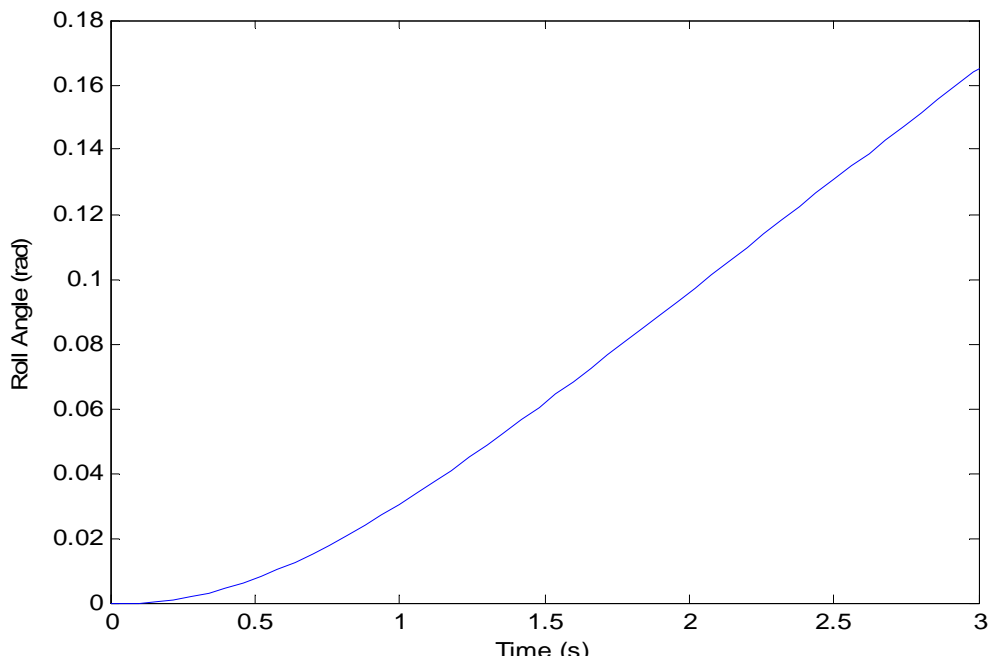
Fig 2.8   Roll angle

The roll angle is the integral of roll velocity, so it is normal to see and quite linear increase of its value.
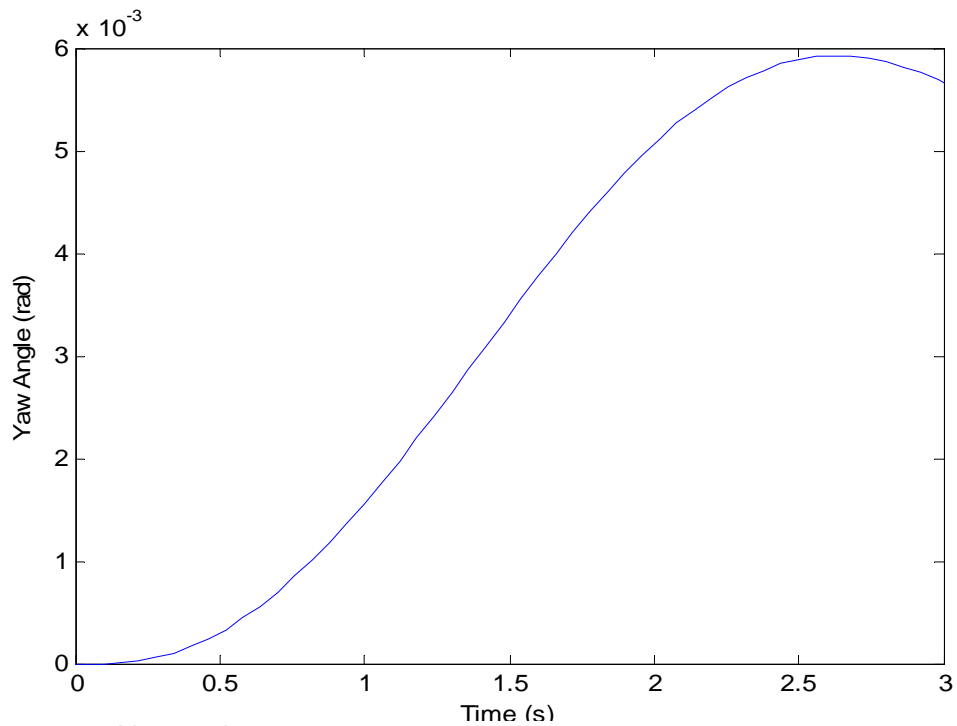
Fig 2.9   Yaw angle

The yaw angle increases a little. This is because of the adverse yaw effect: as the aircraft is no more horizontal, the lift is no more symmetric and there is a forward lift component on the descending wing. However, this angle is about 30 times smaller than the roll angle.
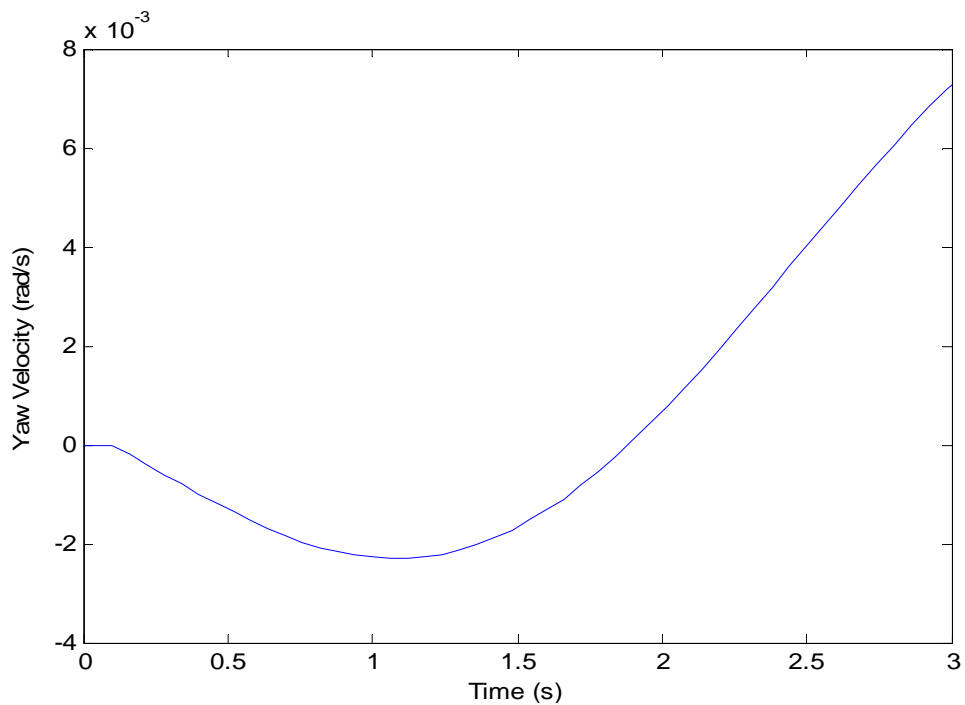
Fig 2.10   Yaw velocity

2.3. Non linear dynamics

After having tried the linear dynamics in two different cases of flight, we now have to study the non linear model of flight dynamics. This model is much more complex than the previous ones, but it is valid in each case of flight. It is even still valid when the flight is not strictly longitudinal or lateral, so we have more freedom with this model.

2.3.1. Equations

As given by **Scholz 1992**, the dynamic of the aircraft is described by 6 equations.
The forces (X, Y, Z) and moments (L, M, N) on the aircraft are equal to:

$$X = m\left(\dot{U} + QW - RV + g\,\sin\theta\right)$$
$$Y = m\left(\dot{V} + RU - PW - g\,\cos\theta\sin\varphi\right)$$
$$Z = m\left(\dot{W} + PV - QU - g\,\cos\theta\cos\varphi\right)$$
$$L = \dot{P}I_x - I_{xz}\left(\dot{R} + PQ\right) + \left(I_z - I_y\right)QR$$
$$M = \dot{Q}I_y - I_{xz}(P^2 - R^2) + (I_x - I_z)\,PR$$
$$N = \dot{R}I_z - I_{xz}\dot{P} + PQ\left(I_y - I_x\right) + I_{xz}QR$$

With these equations, we can easily calculate the derivatives of speeds ($\dot{U}, \dot{V}, \dot{W}, \dot{P}, \dot{Q}$ and $\dot{R}$):

- Derivative of speed on x axis: $\dot{U} = \frac{X}{m} - QW + RV - g\,\sin\theta$

- Derivative of speed on y axis: $\dot{V} = \frac{Y}{m} - RU + PW + g\,\cos\theta\sin\varphi$

- Derivative of speed on z axis: $\dot{W} = \frac{Z}{m} - PV + QU + g\,\cos\theta\cos\varphi - g$

- Derivative of roll speed: $\dot{P} = \frac{1}{I_x - I_{xz}^2/I_z}\left(L + \frac{I_{xz}}{I_z}\left[N - PQ\left(I_y - I_x\right) - I_{xz}\,QR\right] - I_{xz}\,PQ - \left(I_z - I_y\right)QR\right)$

- Derivative of pitch speed: $\dot{Q} = \frac{1}{I_y}\left(M + I_{xz}(P^2 - R^2) - (I_x - I_z)\,PR\right)$

- Derivative of yaw speed: $\dot{R} = \frac{1}{I_z - I_{xz}^2/I_x}\left(N + \frac{I_{xz}}{I_x}\left[L - PQ\,I_{xz} - \left(I_z - I_y\right)QR\right] - I_{xz}\,QR - PQ\left(I_y - I_x\right)\right)$

- Derivative of pitch angle: $\dot{\theta} = Q\,\cos\varphi - R\,\sin\varphi$
- Derivative of roll angle: $\dot{\varphi} = P + R\tan\theta\,\cos\varphi + Q\tan\theta\,\sin\varphi$

We also need these expressions that give us the inputs on the model:

- $\frac{X}{m} = X_u\,(U - U_0) + X_w\,W + X_{\delta_E}\delta_E + X_{\delta_F}\,\delta_F$
- $\frac{Y}{m} = Y_V\,V + Y_{\delta_A}\delta_A + Y_{\delta_R}\,\delta_R$

- $\frac{Z}{m} = Z_u \, (U - U_0) + Z_w \, W + Z_{\delta_E} \delta_E + Z_{\delta_F} \, \delta_F$

- $L = I_x \left(L_v \, V + L_p \, P + L_R \, R + L_{\delta_A} \, \delta_A + L_{\delta_R} \, \delta_R \right)$

- $M = I_y \left(M_u \, (U - U_0) + M_w \, W + M_{\dot{w}} \left[\frac{Z}{m} - PV + QU + g \, \cos \theta \, \cos \varphi - g \right] + M_q \, Q + M_{\delta_E} \delta_E + M_{\delta_F} \, \delta_F \right)$

- $N = I_z \left(N_v \, V + N_p \, P + N_R \, R + N_{\delta_A} \, \delta_A + N_{\delta_R} \, \delta_R \right)$

After adding these 6 expressions in the derivatives of angle speeds, we get these expressions:

- Derivative of roll speed:

$$
\begin{aligned}
\dot{P} = \frac{1}{I_x - I_{xz}{}^2 / I_z} &\left(I_x \left(L_v \, V + L_p \, P + L_R \, R + L_{\delta_A} \, \delta_A + L_{\delta_R} \, \delta_R \right) \right. \\
&+ \frac{I_{xz}}{I_z} \left[I_z \left(N_v \, V + N_p \, P + N_R \, R + N_{\delta_A} \, \delta_A + N_{\delta_R} \, \delta_R \right) - PQ \left(I_y - I_x \right) \right. \\
&\left. - I_{xz} \, QR \, \right] - I_{xz} \, PQ - \left(I_z - I_y \right) QR \bigg)
\end{aligned}
$$

- Derivative of pitch speed:

$$
\begin{aligned}
\dot{Q} = \frac{1}{I_y} \Big(I_y \Big(M_u \, (U - U_0) + M_w \, W \\
&+ M_{\dot{w}} \left[Z_u \, (U - U_0) + Z_w \, W + Z_{\delta_E} \delta_E + Z_{\delta_F} \, \delta_F - PV + QU + g \, \cos \theta \, \cos \varphi \right. \\
&\left. - g \right] + M_q \, Q + M_{\delta_E} \delta_E + M_{\delta_F} \, \delta_F \Big) + I_{xz}(P^2 - R^2) - (I_x - I_z) \, PR \Big)
\end{aligned}
$$

- Derivative of yaw speed:

$$
\begin{aligned}
\dot{R} = \frac{1}{I_z - I_{xz}{}^2 / I_x} &\left(I_z \left(N_v \, V + N_p \, P + N_R \, R + N_{\delta_A} \, \delta_A + N_{\delta_R} \, \delta_R \right) \right. \\
&+ \frac{I_{xz}}{I_x} \left[I_x \left(L_v \, V + L_p \, P + L_R \, R + L_{\delta_A} \, \delta_A + L_{\delta_R} \, \delta_R \right) - PQ \, I_{xz} \right. \\
&\left. - \left(I_z - I_y \right) QR \, \right] - I_{xz} \, QR - PQ \left(I_y - I_x \right) \bigg)
\end{aligned}
$$

Note: There is no change for the expressions of longitudinal speeds.

2.3.2. Application

We integrate these equations in a Matlab model:
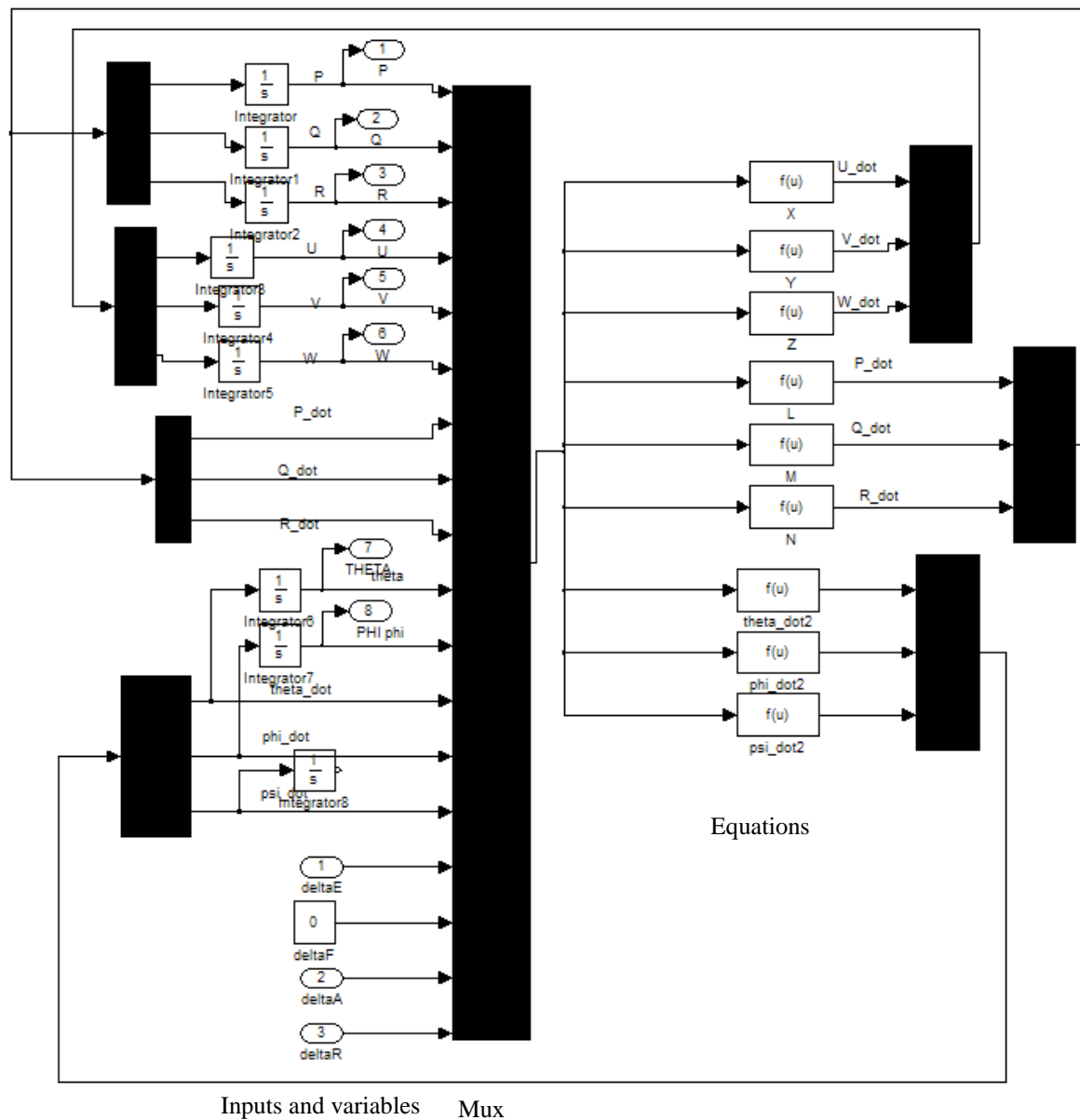
Fig 2.11   Non linear model of aircraft

As we can see, this model is really complex. The 9 equations are on the right of the model and all the variables needed for the equations are on the left. As we calculate derivatives of speeds, we need to integrate them to get the new value of speed on each axis. This is the reason why we use a lot of integrators.

We put this model into a box, to hide the complexity of it, and we get this model:
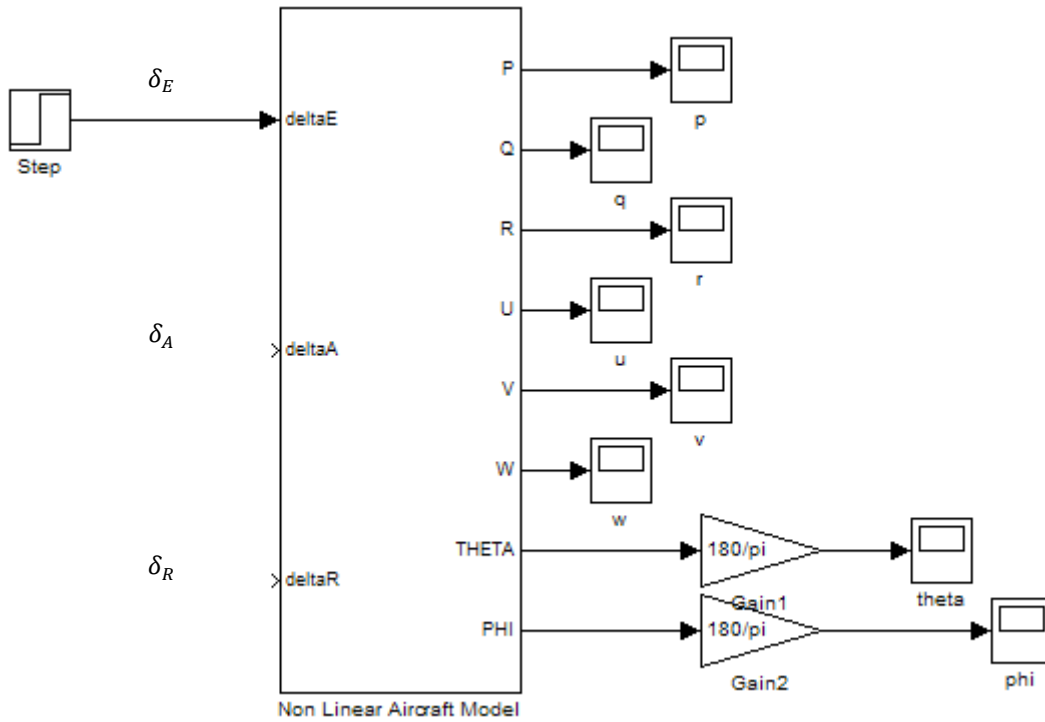
Fig 2.12   Non linear model with inputs and outputs

The model shown here is configured for longitudinal flight because there is only on input different from zero: δ_E which is the action given on the elevators.

But, unlike the linear model, we can easily change from a longitudinal flight to a lateral flight and even combine both, just by changing the input, whereas we needed to use two different models before.

Test for longitudinal dynamics
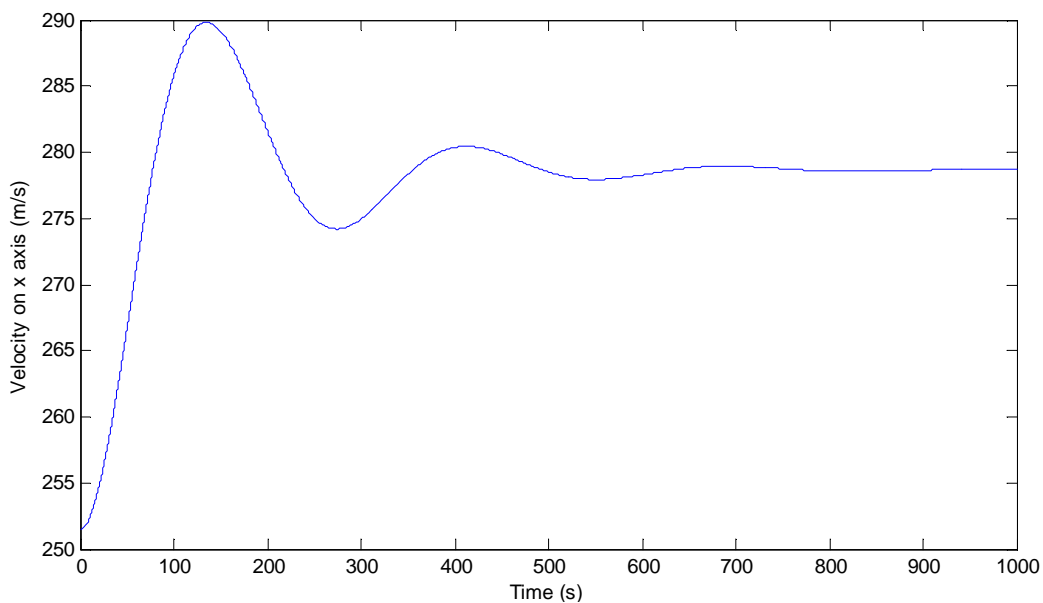
Fig 2.13   Evolution of the velocity on x axis
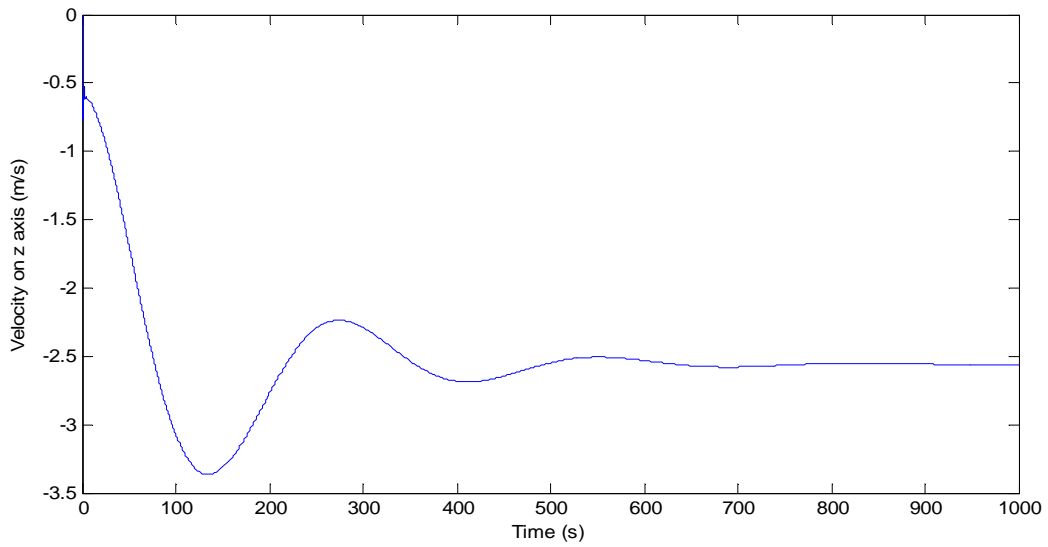
Fig 2.14   Evolution of the velocity on z axis
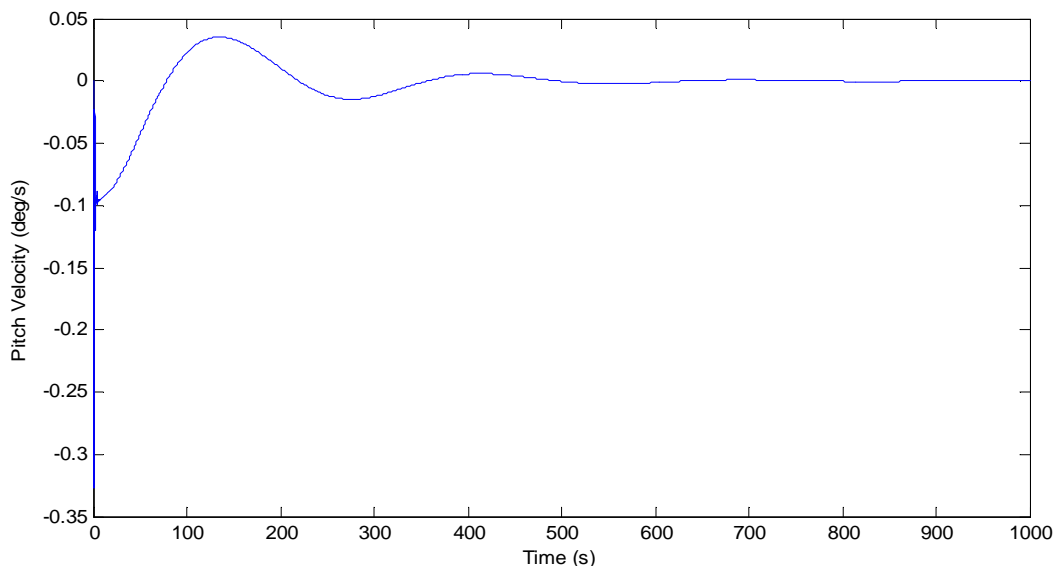
Fig 2.15   Evolution of the pitch velocity
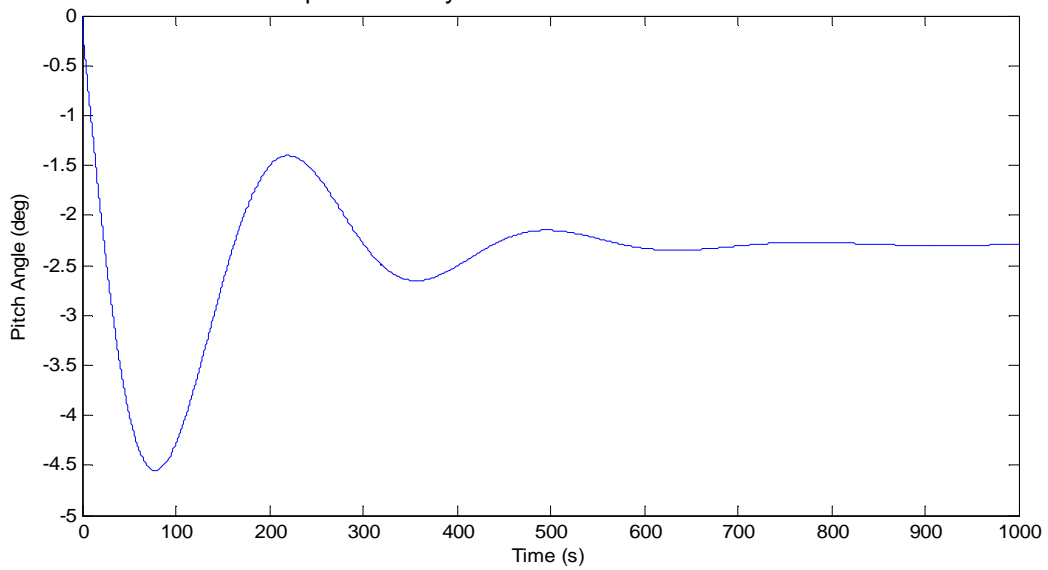
Fig 2.16   Evolution of the pitch angle

The evolution of velocities on x and z axis and the evolution of pitch angle are almost the same as for the linear system. The values are different because the initial conditions are not the same at the beginning, the inputs have not the same values and the outputs for the non linear system are displayed in degrees whereas we display the output of linear system in radians.

Test for lateral dynamics (action on the ailerons)

We change the input: there is now no action on the elevator, but a step on the ailerons.
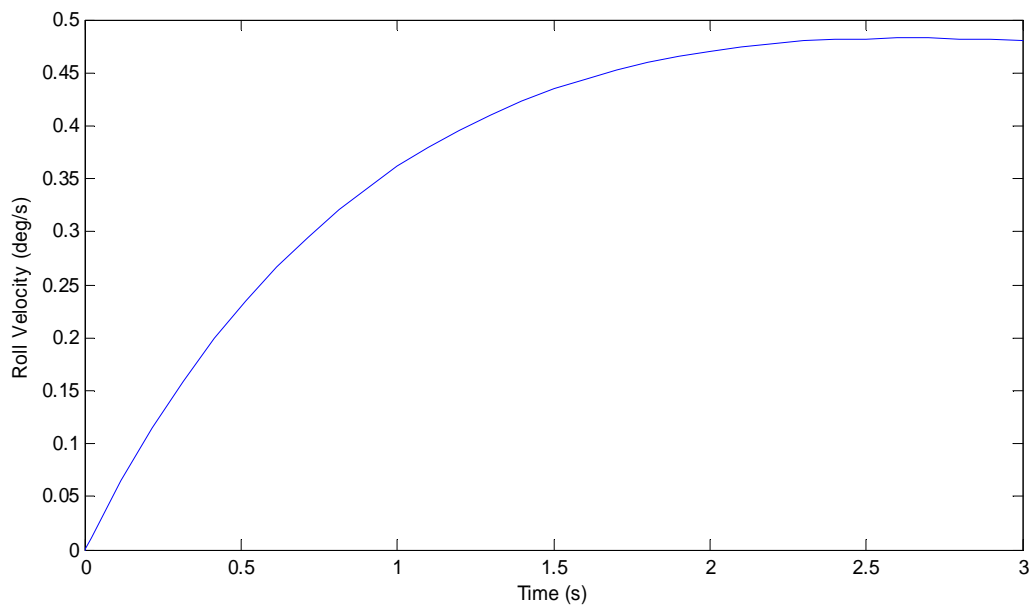
Fig 2.17   Evolution of the roll velocity
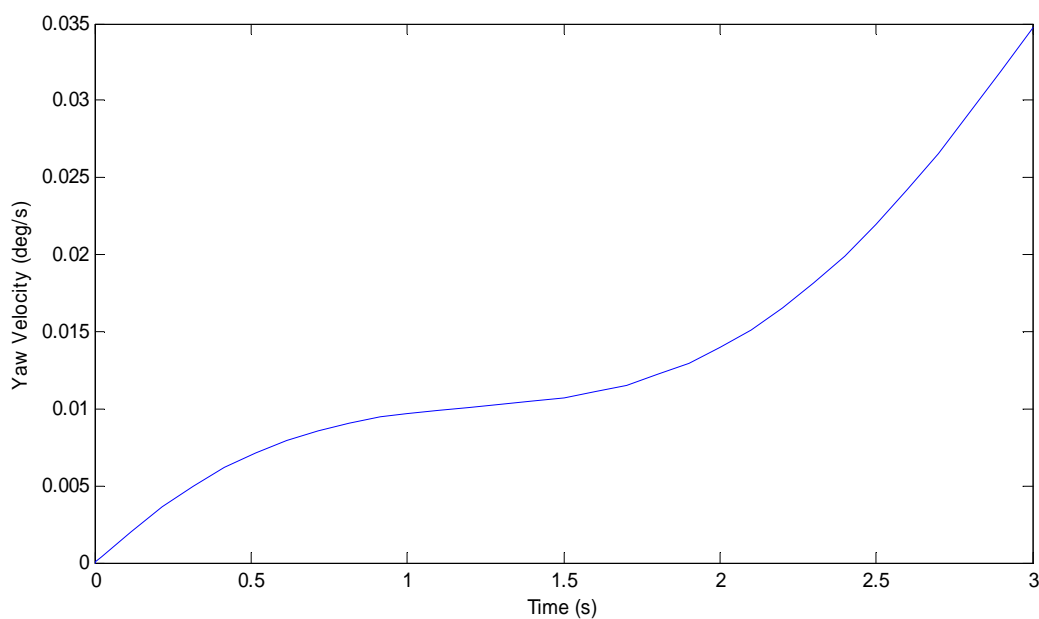
Fig 2.18   Evolution of the yaw velocity
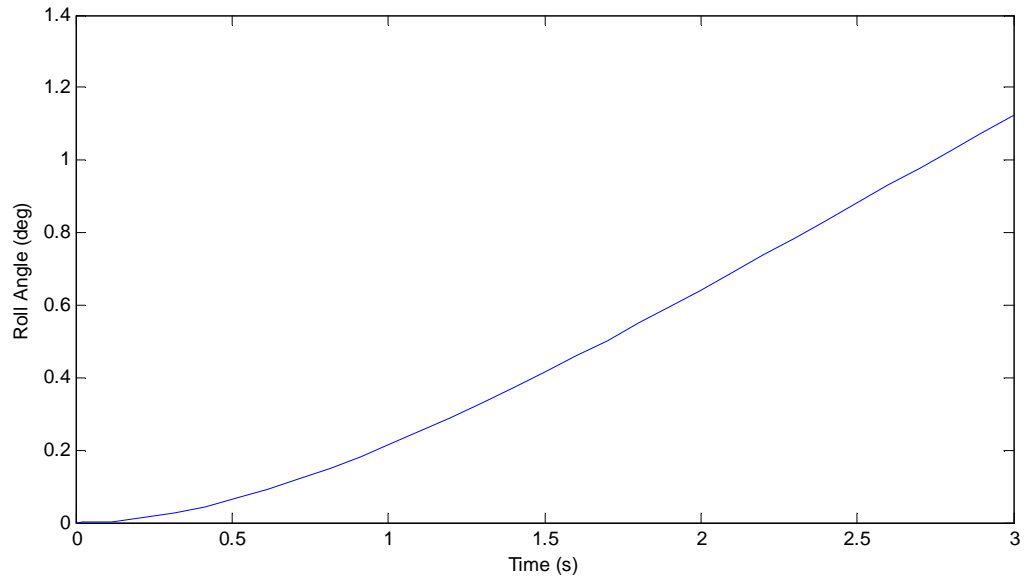
Fig 2.19   Evolution of the roll angle

As for the longitudinal test, the results are almost the same as for the linear model.

3. Flying qualities

We are now going to study the flying qualities of the aircraft, in order to know if it meets the specifications needed for stability. These specifications are given by a few criteria, so we need to calculate the values for the DC-8 and compare them with the acceptable values given in the extracts at the end of this report.

3.1. Longitudinal dynamics

In order to study the flying qualities of the aircraft, we need to calculate the eigenvalues of the matrix A (state space matrix). Using the example given by **Mc LEAN 1990**, we are going to solve this linear equation to find the eigenvalues:

$$|\lambda I - A| = 0$$

We solve this equation with the A matrix of a DC-8 in cruise conditions.

We find 4 solutions: $\quad \lambda_{1,2} = -1,0765 \mp 2,9575\,i$

And $\quad\quad\quad\quad \lambda_{3,4} = -0,0059 \mp 0,0236\,i$

The eigenvalues $\lambda_{1,2}$ are associated to the short period mode because the frequency is high (2,9575) rad/s. And the eigenvalues $\lambda_{3,4}$ are associated to the phugoid mode because their frequency is very low (0,0236).

With these eigenvalues, we can write:

$$(\ \lambda^2 + 2\,\xi_{sp}\,\omega_{sp}\,\lambda + \omega_{sp}{}^2\ )(\ \lambda^2 + 2\,\xi_{ph}\,\omega_{ph}\,\lambda + \omega_{ph}{}^2\ ) = 0$$

where ξ_{sp} and ξ_{ph} are the damping ratios associated to the short period (sp) and to the phugoid (ph), and where ω_{sp} and ω_{ph} are the angular frequencies associated to the short period and to the phugoid.

We have: $\quad (\ \lambda^2 + 2,1529\,\lambda + 9,9055\ )(\ \lambda^2 + 0,0117\,\lambda + 0,0006\ )$

3.1.1. Short period mode

$$\omega_{sp} = \sqrt{9,9055} = 3,1473\ rad/s \quad\quad\quad \text{and} \quad\quad\quad \xi_{sp} = \frac{2,1529}{2\,.\omega_{sp}} = 0,3420$$

We can also calculate the Control Anticipation Parameter (CAP) and N_{z_α}:

$$N_{z_\alpha} = -\frac{U_0}{g \cdot Z_w} \quad\quad\quad\quad\quad\quad CAP = \frac{\omega_{sp}{}^2}{N_{z_\alpha}}$$

We find:

$N_{z_\alpha} = 31,7981$ And $CAP = 0,3115$

Then we can see that the plane satisfies the short period damping ratio specification given by Mc Lean (p156): the flight phase category is B (cruise), so the minimum value for damping ratio is 0,3. The value of our aircraft is $\xi_{sp} = 0,3420 > 0,3$

Concerning the Control Anticipation Parameter, the value that we calculated satisfies the criterion for this kind of plane and flight phase: the level 1 criterion is:

$$0,085 < CAP = 0,3115 < 3,6$$

3.1.2. Phugoid mode

$\omega_{ph} = \sqrt{0,0006} = 0,0243 \; rad/s$ $\qquad$ And $\qquad$ $\xi_{ph} = \frac{0,0117}{2 \, . \, \omega_{ph}} = 0,2412$

So the damping ratio of the plane is 0,2412 which is upper than the minimum value for level-1 (0,04). Then, the plane satisfies the level 1 criterion for the phugoid mode qualities.

3.2. Lateral dynamics

As for the longitudinal dynamics, we have to solve this equation and find the eigenvalues:
$$|\lambda \, I - A| = 0$$
The eigenvalues we have are now:
$\lambda_{1,2} = -0,1187 \mp 1,4901i$
$\lambda_3 = -1,2544$
$\lambda_4 = -0,0040$

Now, we can write: $(\lambda^2 + 2 \, \xi_d \, \omega_d \, \lambda + \omega_d{}^2)(\lambda + e)(\lambda + f) = 0$
The values we have are: $(\lambda^2 + 0,2374 \, \lambda + 2,2346)(\lambda + 0,0040)(\lambda + 1,2544) = 0$

So we find:

$\omega_d = \sqrt{2,2346} = 1,4949 \; rad/s$ $\qquad$ And $\qquad$ $\xi_d = \frac{0,2374}{2 \, . \, \omega_d} = 0,0794$

The roll mode time constant is: $T_r = -\frac{1}{\lambda_3} = 0,7972 \; s$

The spiral mode time constant is: $T_s = -\frac{1}{\lambda_4} = 247,1148 \; s$

We can see that the value of ξ_d (0,0794) is a little under the minimum specification (0,08) for Dutch roll mode. There is the same problem with the product $\omega_d \, \xi_d$ (0,1187) instead of 0,15. But the value of ω_d is good: 1,4949 (specification : 0,4).

4. Simple models of linear aircraft control

In the second chapter, we built a model of the aircraft dynamics. The following steps will be to integrate this little model in a bigger one, representing the whole aircraft dynamics and control system. Before doing this, we have to know how we will control this aircraft, which means, how we build the loop. As we made in the second chapter, we will separate the linear and the non linear model, and then the longitudinal and lateral cases of flight. For each case, we will first do an open loop control, and then a closed loop control, and compare them.

4.1. Longitudinal dynamics for linear model

We want to control the pitch angle of the aircraft. The best command for controlling it is to make an action on the elevator. So the only input in our model is the action on the elevator: we compare two steps signals that have a different step time. We use this model in open loop.
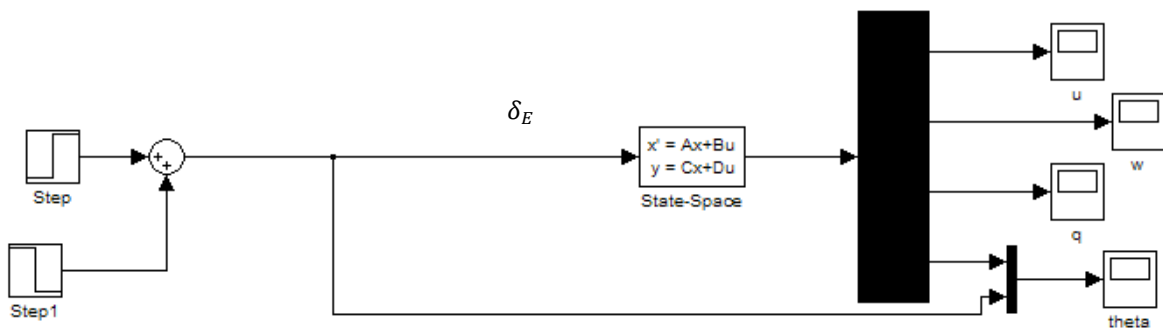
Fig 4.1   Simulink model used for the simulation in open loop
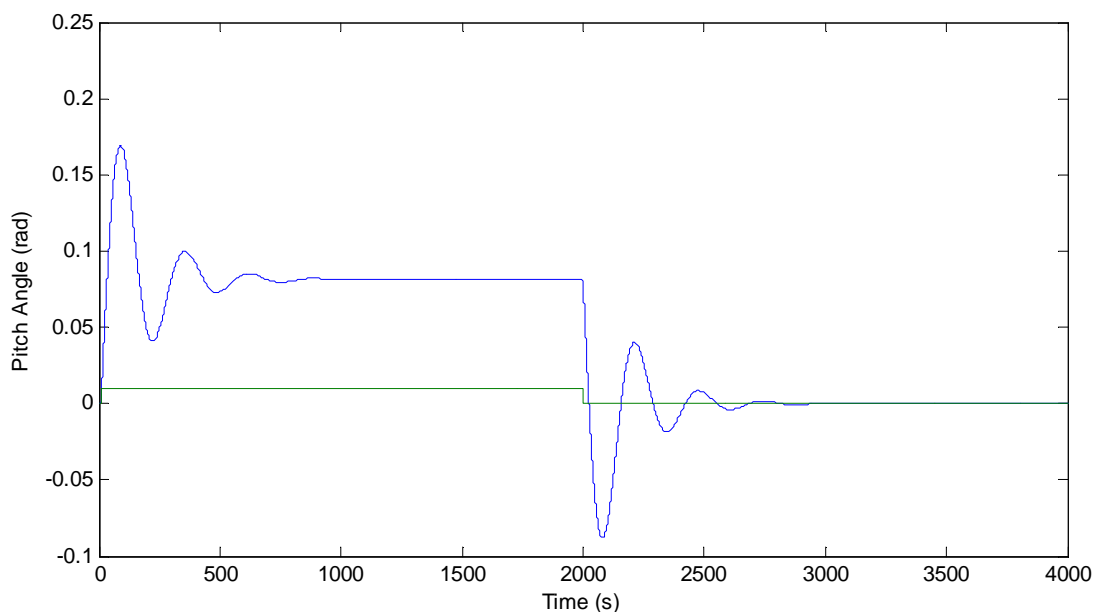
Fig 4.2   Temporal evolution of pitch angle

As we can see on this graph, the system has got oscillations and is really slow: it needs 1000 s to reach the final value. But it is stable: the oscillations decrease during the time. We also see that there is a big difference between the input and the final value of the output. We need to add a gain in the system to adapt the output to the input. When the input is $0,01$ the final value of the output is $0,08$. Then the value of the new gain that we add is $G = \frac{0,01}{0,08} = 1/8$. The new model is:
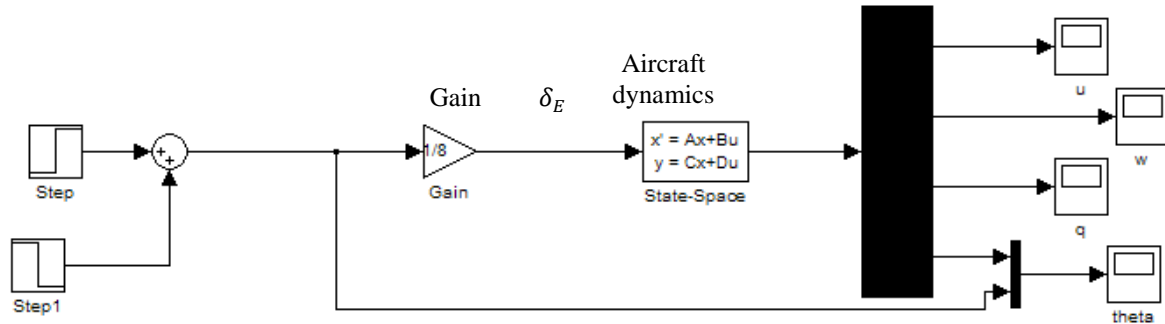
Fig 4.3   Simulink model with the new gain
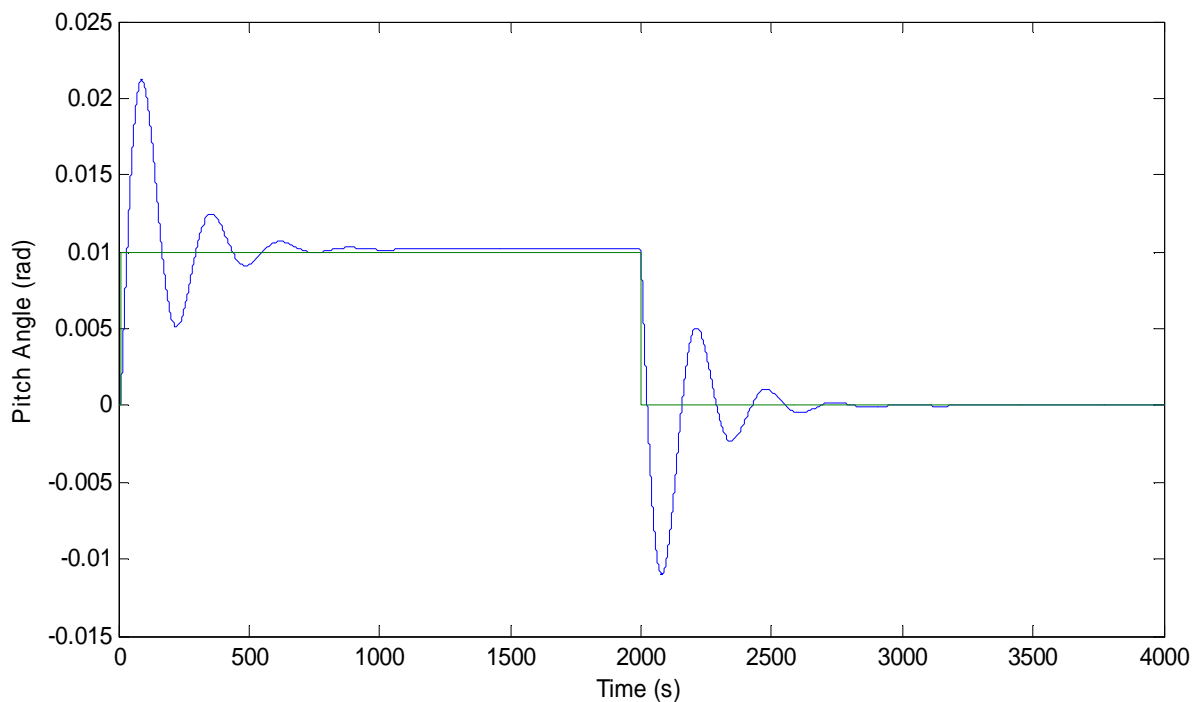
Fig 4.4   Temporal evolution of pitch after adding the gain

The final value of the system is now the equal to the input value: the response is adapted to the command. By adding this gain, we didn't change the fundamental behavior of the system: there are still decreasing oscillations, and the system is still stable.
So by adding a gain, we solved only one problem. If we want to solve the problem of the oscillations and the time of response, we need to <u>close the command loop</u>. We connect the output to the input, by comparing the input and output signals. We also remove the gain we added before, because it is no more needed.
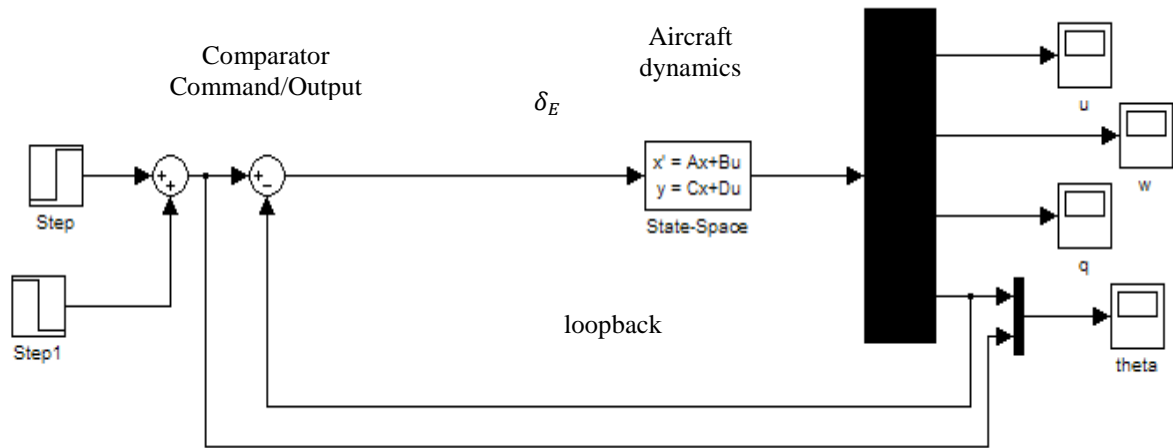
Fig 4.5   Simulink model with the closed loop
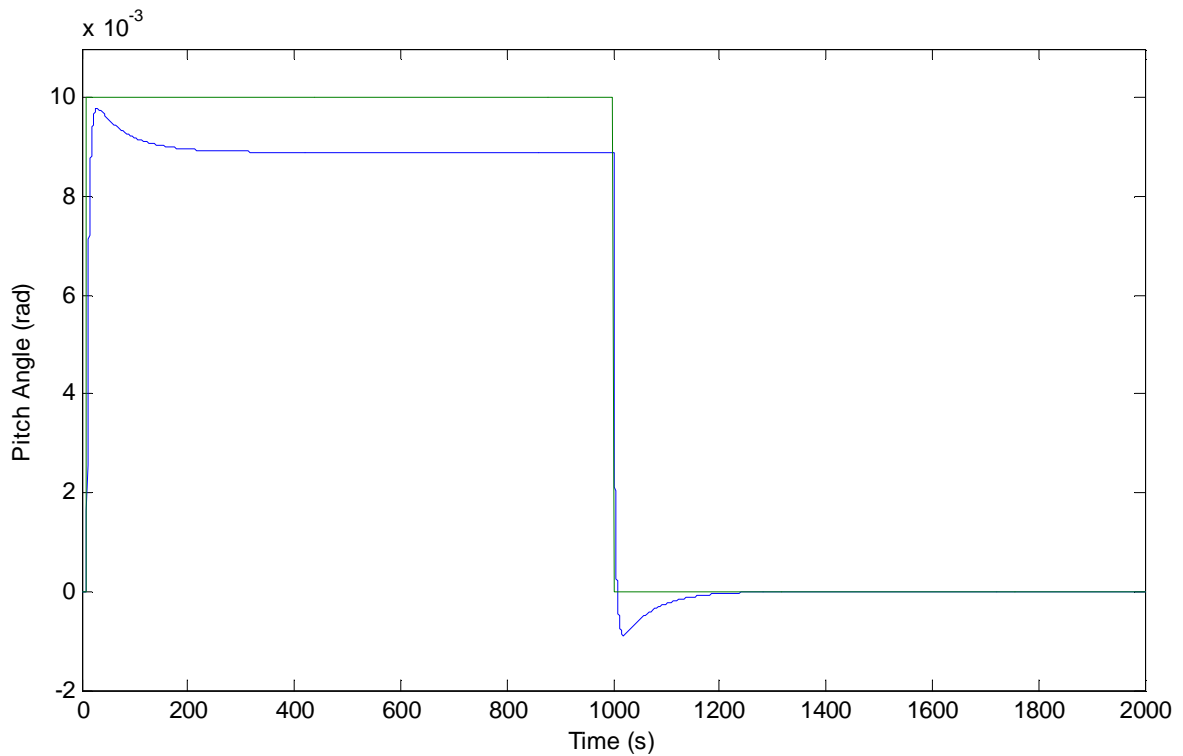
Fig 4.6   Temporal evolution in closed loop

The response in much more accurate and faster when we close the loop and the oscillations disappeared. There is a difference between the input and the output (about 10% of the input value). In order to correct it we add a gain in the loopback. The input value is 0.01 and the final value is 0.0088, so we choose a value equal to $G_L = 0.88$ for this new gain.

The new response is:
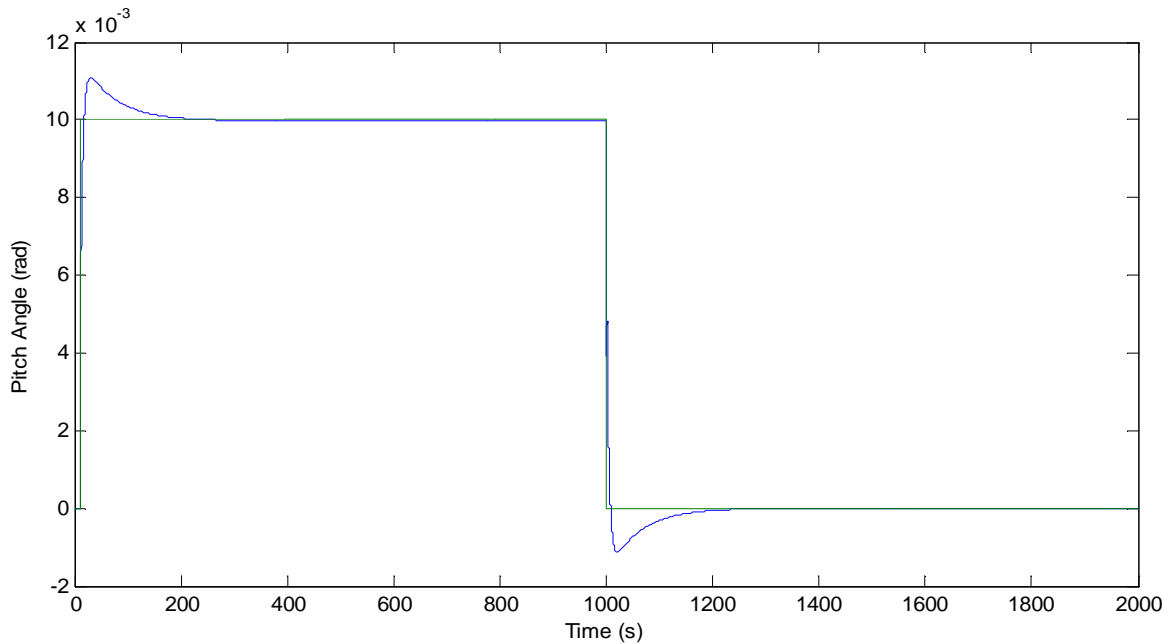
Fig 4.7 Temporal evolution of the pitch angle in closed loop after adding the gain

The response is now without any static error. The next improvement we could make is to add a corrector in order to get a faster response.

So, after closing the loop, the response is much faster than the response in open loop, and there is no oscillation. The final values are without errors in each case.

4.2. Longitudinal dynamics for non linear model

Now, we try to control the pitch angle of the aircraft but using the non linear model of the aircraft, in order to compare the both aircrafts models. At the beginning we do not use any gain and the model is in open loop. The response we get is:
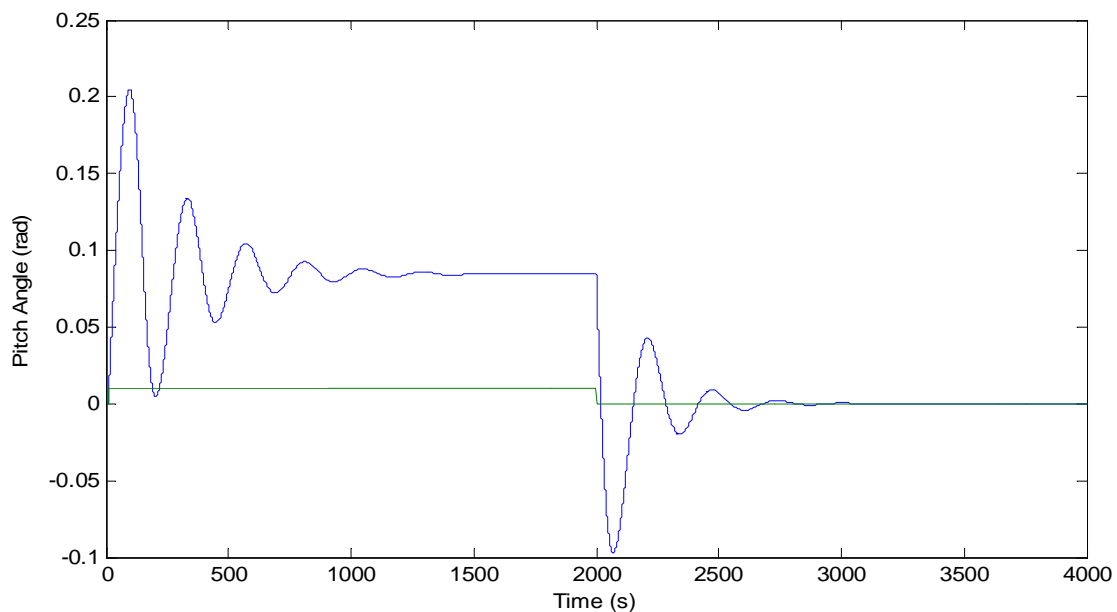
Fig 4.8 Temporal evolution of the pitch angle

This response is really slow and the static error is huge. There are also some oscillations with big amplitude at the beginning. Then we try to add a gain in order to remove the static error. As we want a final value equal to 0.01 and as the final value we get is equal to 0.0845, the value we use for this gain is: $G = \dfrac{1}{8.45}$
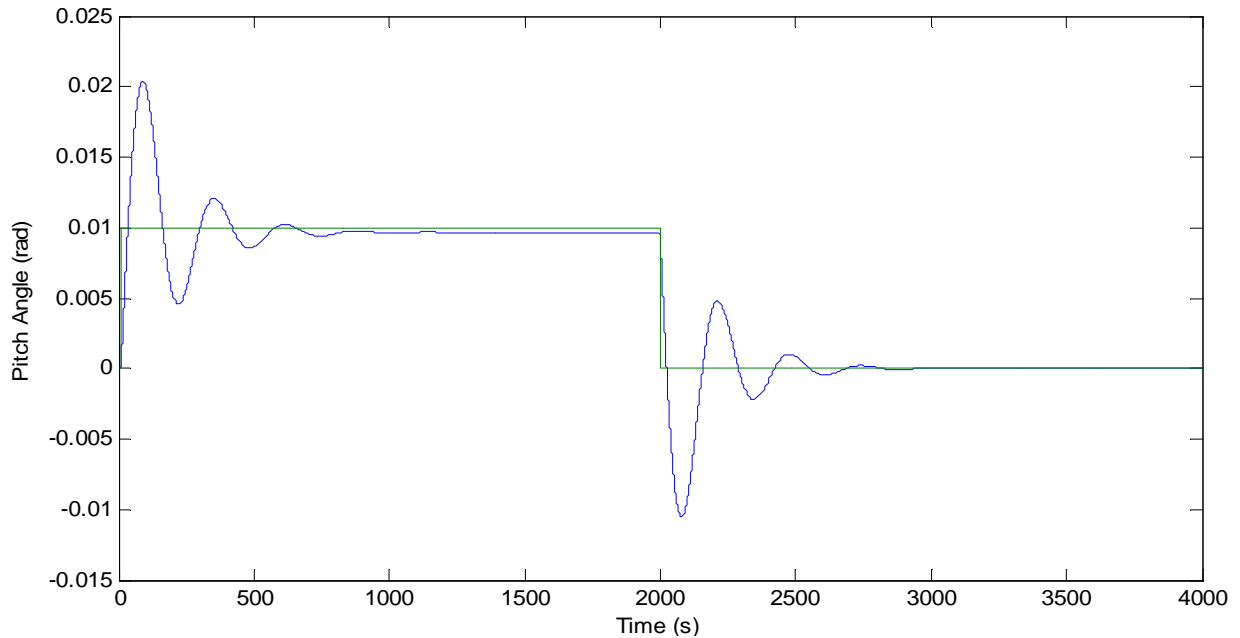
Fig 4.9   Temporal evolution of the pitch angle after adding the gain

The response is almost the same as the previous. The only difference we can see is that the final output value is equal to the input value. So the static error is null.

As we want a faster response, without oscillations, we close the loop. We connect the input and the output and compare their values. We remove the gain we added before. The response in closed loop is:
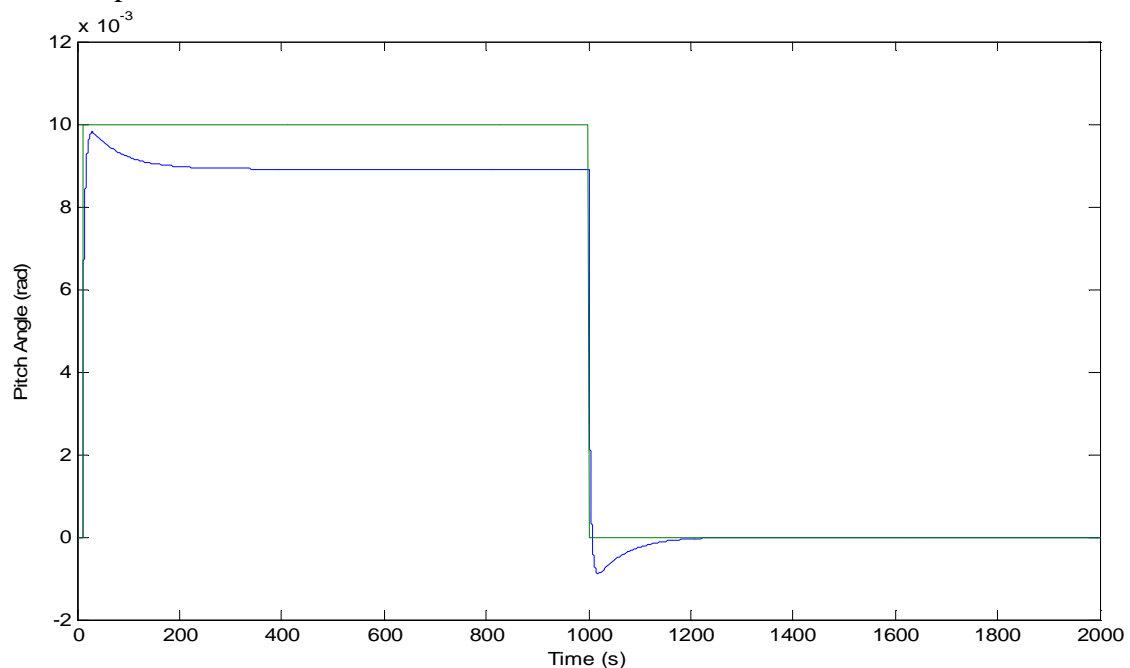
Fig 4.10   Temporal evolution of pitch angle in closed loop

The new response is really faster: the final value is reached after 400 seconds whereas it needed about 1000 seconds in open loop. The other difference is that there are no more oscillations. But there is a little static error. We remove it by adding a gain in the loopback. As the final value we want is 0.01and as the final value we get is 0.00875 the best value for this new gain is: $G_L = \frac{8.75}{10}$
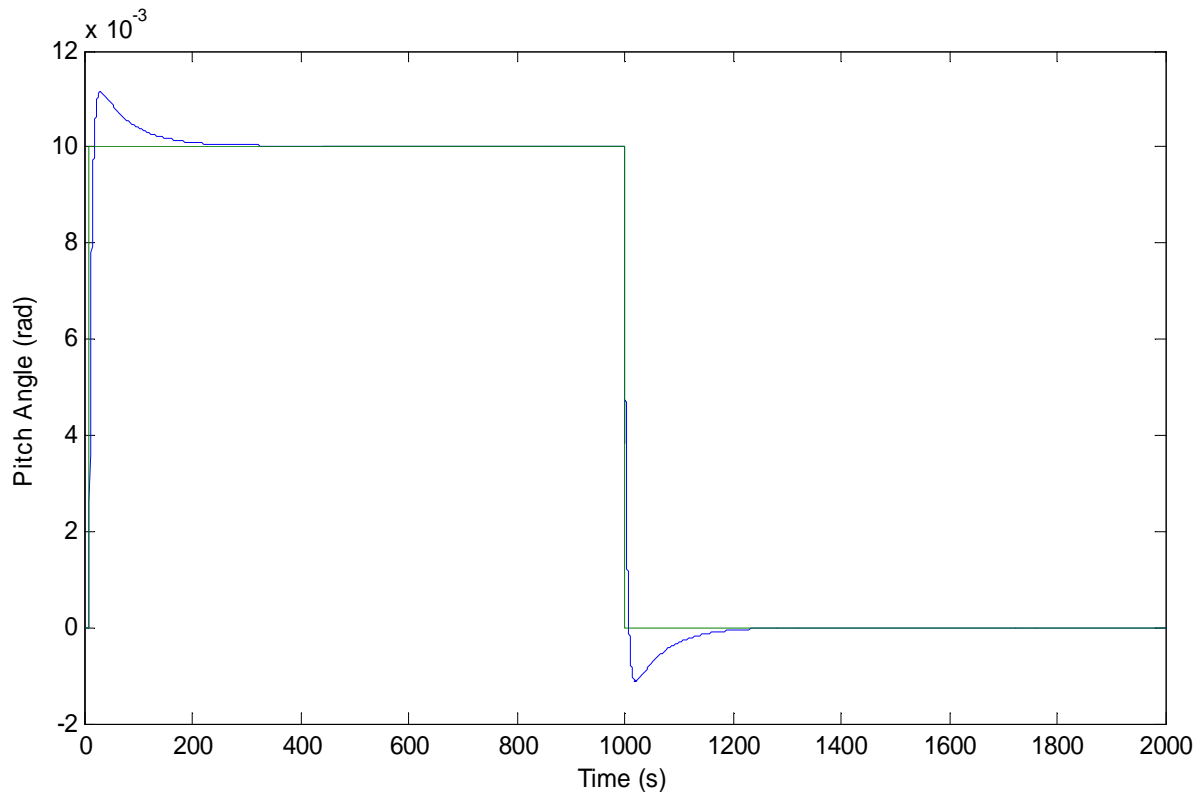
Fig 4.11   Evolution of pitch angle in closed loop after adding the gain in loopback

After adding this gain, the response is without static error. The next improvement would be to add a corrector in order to make the response a little faster.

As for the linear case, closing the loop is a good improvement for the response, because it makes it faster than in the open loop and without oscillation.

Now, if we compare the results we get using the linear or non linear model of aircraft, we realize that the behaviors are almost the same. There is a little difference concerning the values but the main aspect of the response is the same:
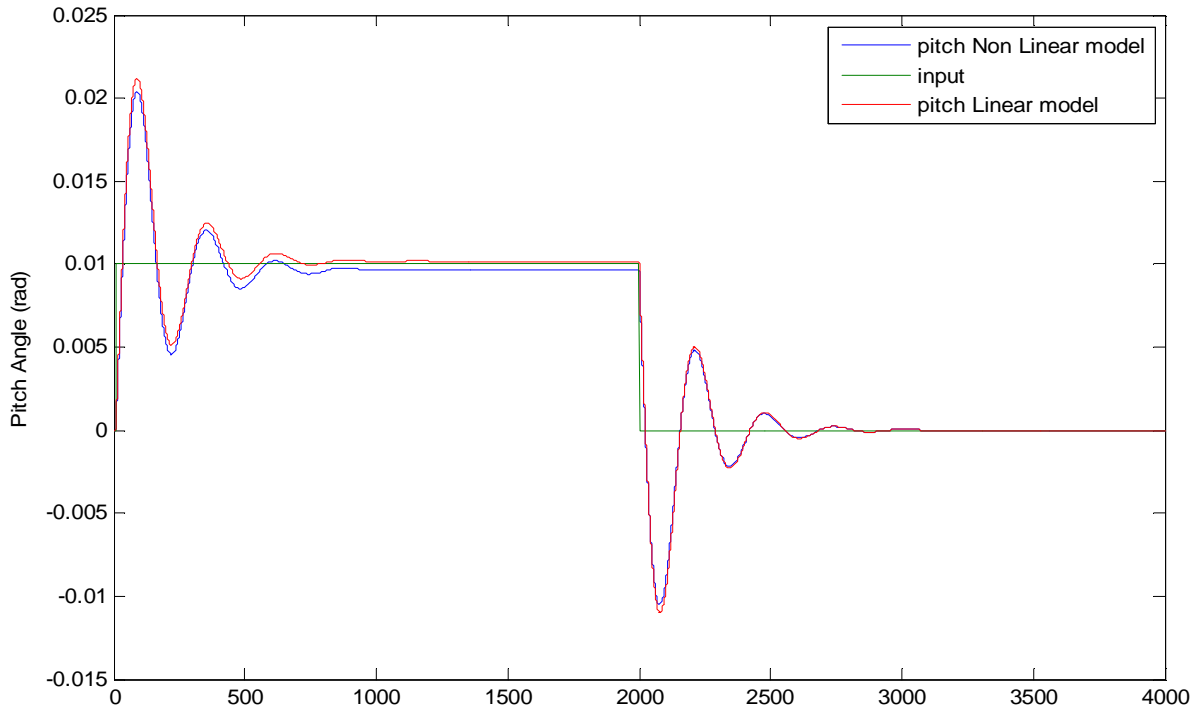
Fig 4.12   Comparison of both aircraft models in open loop after adding the gains
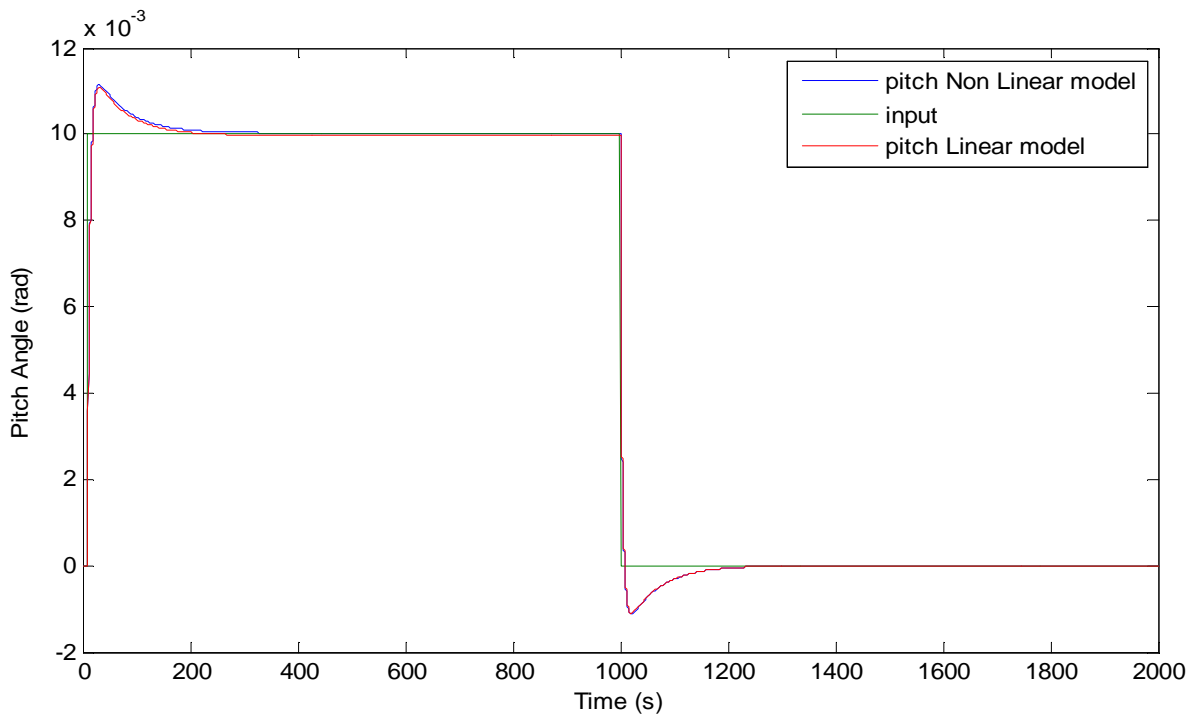
Fig 4.13   Comparison of both aircraft models in closed loop after adding the gains

As we can see, the responses are really close if we use the linear or the non linear model for the aircraft. As the simulation is really faster (between 50 and 100 times) when we use the linear model, we will not use the non linear model anymore because it is a waste of time and we get results that are almost the same.

4.3. Lateral dynamics

Concerning the lateral dynamics, we will try to control the roll angle φ. The action we use is an input on the ailerons. This model is used in open loop. We also add a gain to adapt the output to the input.
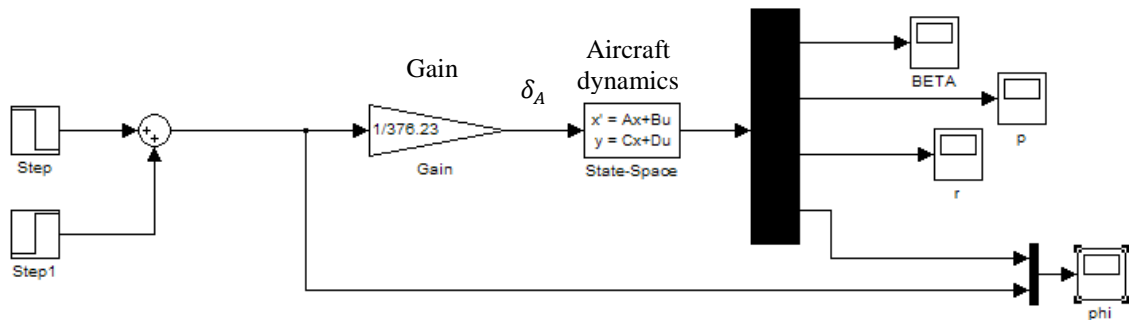
Fig 4.14   Simulink model with the gain, in open loop
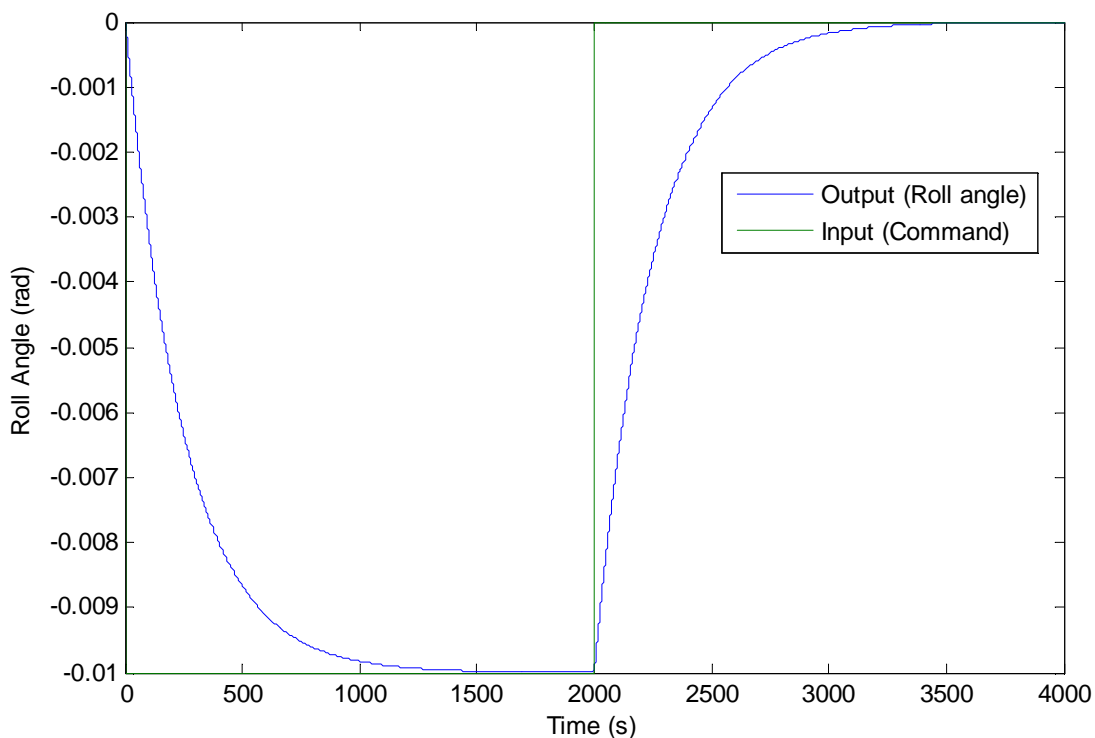
Fig 4.15   Temporal evolution of roll angle in open loop

As we can see, the roll angle depends on the input (action on the ailerons). The output has no oscillations. The final value is equal to the input value because of the gain we added. But the main problem is the time response which is really long: about 2000 seconds. We cannot use an airplane with a so long response time for the roll angle.

So, we need to close the loop to have a faster response. We close it by comparing the input value to the output value.
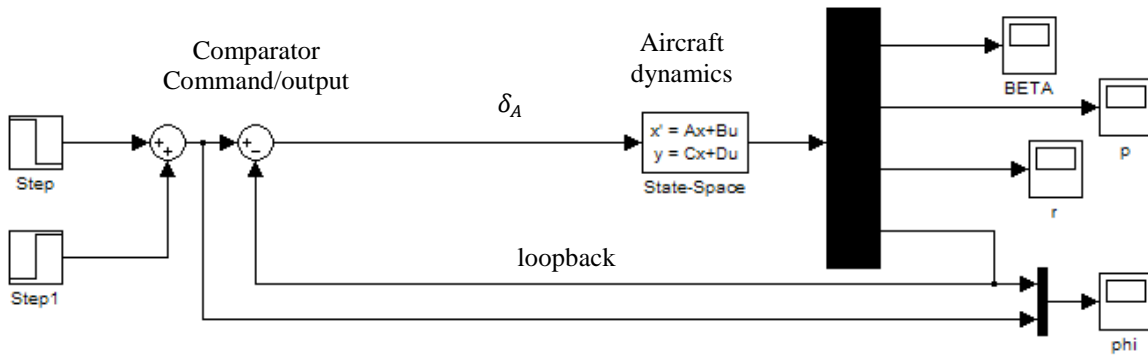
Fig 4.16   Simulink model in closed loop
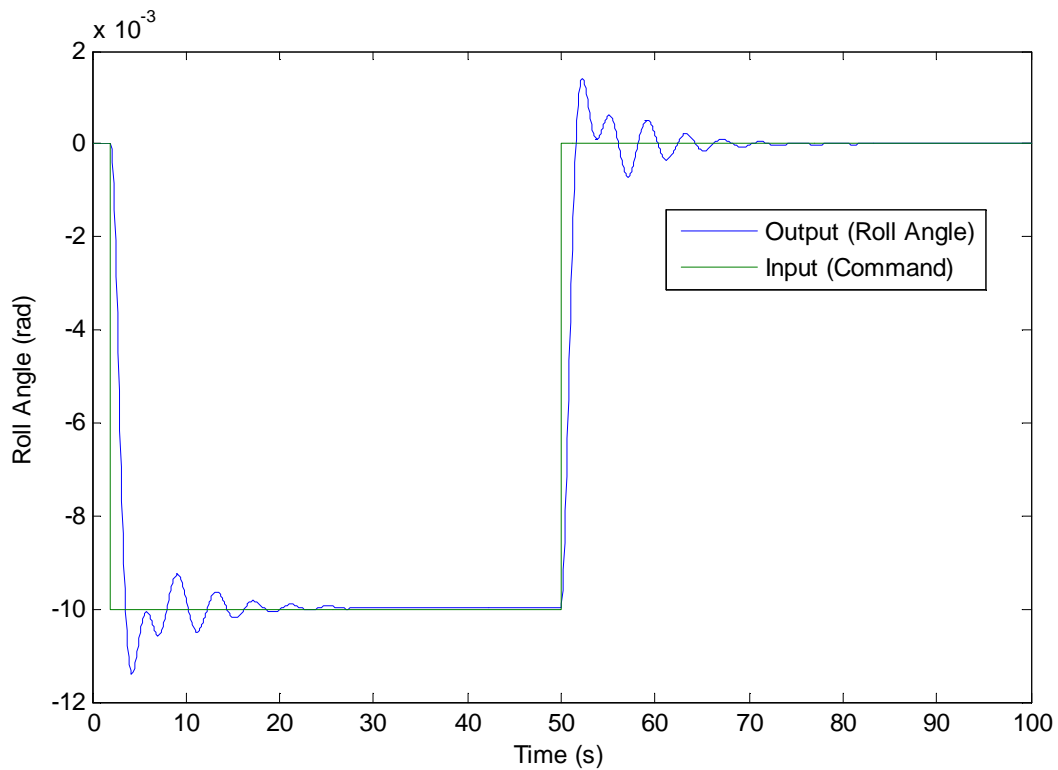
Fig 4.17   Temporal evolution of roll angle in closed loop

The response of the closed loop is much faster (30s instead of 2000) and there is no difference between the input and the output. But a problem appeared: there are oscillations. If we really wanted to remove them, we could use a controller.

5. Pilot models

Now we know how to make the control the aircraft. As we need a true model of the aircraft control, we have to integrate a pilot in this model.

We can find a lot of different pilot models. Some are easy, others are really complex. In this work, we are going to try 2 different ones given by **Mc Lean 1990**. The first one is called "Lead term and pure time delay" and the second one "Phase advance and pure time delay".

5.1. Lead term and pure time delay

The transfer function of this model is:

$$\frac{V_p(s)}{p_{comm}(s)} = K_p(1 + s\, T_L)e^{-s\tau}$$

Where $p_{comm}(s)$ is the command and $V_p(s)$ the pilot response. K_p is the gain that the pilot gives, T_L is a time constant and τ the delay of reaction of the pilot. These 3 values are parameters that are adjustable.

This is the Matlab model corresponding to this transfer function. The "s" of the equation in $(1 + s\, T_L)$ is a derivative block in the model, and the exponential $e^{-s\tau}$ is a delay which value is equal to τ.

We add a gain ("Gain2") so that we can easily compare the input and the output, otherwise the output value would be really high in comparison with the input. (We choose the value 1 for all the others parameters).
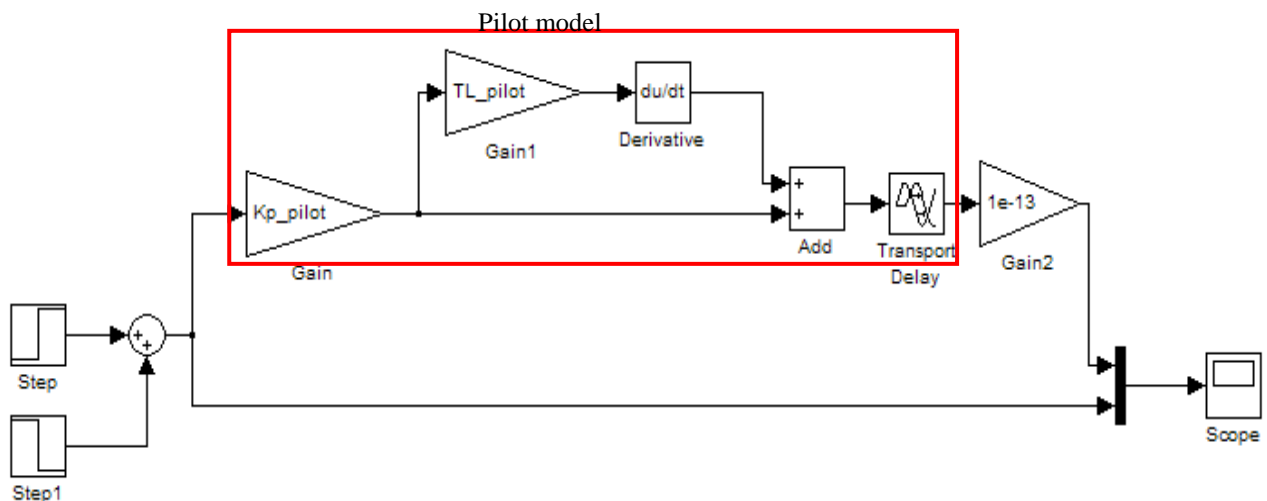
Fig 5.1 Simulink model for pilot: lead term and pure time delay
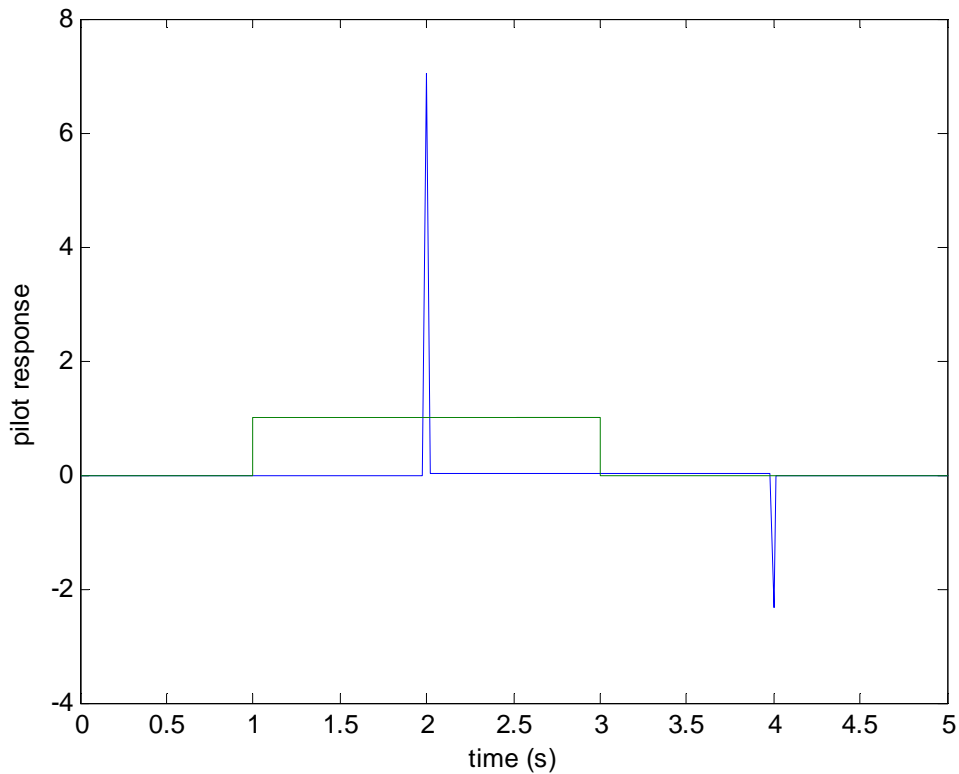
Fig 5.2   Temporal evolution of pilot response

As we can see on this graph, the response is a Dirac impulse with a delay of one second. This delay corresponds to the delay a man needs to take a decision and the delay between his decision and the effective action on his muscles. The value of this delay is the exact value of τ.

The real response is really bigger than the one we can see on this figure, because we used a gain with a really small value to have an easier comparison between the input and the output.

The most important point is that the pilot reaction to an event is really strong and a few delayed.

5.2. Phase advance and pure time delay

The transfer function is now:

$$\frac{V_p(s)}{p_{comm}(s)} = K_p \frac{(1+s\,T_L)}{(1+s\,T_1)} e^{-s\tau}$$

$p_{comm}(s)$ is the command and $V_p(s)$ is the pilot response. K_p is the gain that the pilot gives, T_L and T_1 are time constants and τ the delay of reaction of the pilot. These 4 values are parameters which are adjustable.
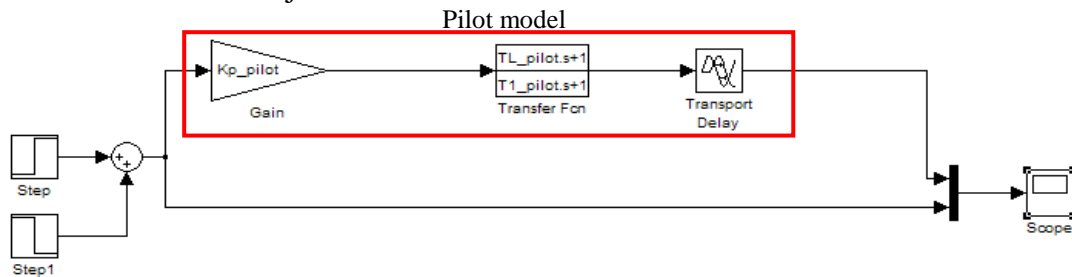
Fig 5.3   Simulink model for pilot: phase advance and pure time delay

As used in the model given by **Witte 2004**, we use a value of T_1 which is much smaller than T_L ($T_L = 0{,}583$ & $T_1 = 0{,}01$)

.
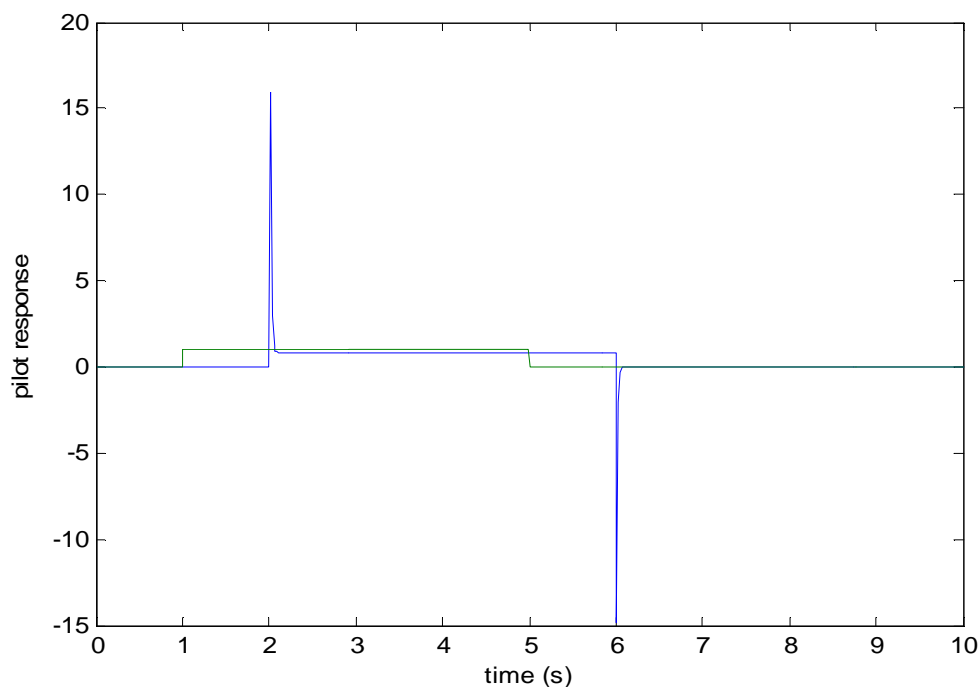
Fig 5.4   Temporal evolution of pilot response

The response is a bit like the one of the first model, but the difference is that the final value is not null: the first one was an only Dirac impulse, here we have a Dirac impulse and then a step. We can also see that there is a little difference between the final output value and the input

6. Aircraft control by the pilot

In order to simulate the aircraft controlled by the pilot, we need to add a pilot model in the controlled aircraft model. We saw the response of 2 different pilot models and we need to choose one of them. As the first one has a pure derivative block whereas the second has a derivative block with an integrator, we decide to use the second pilot model: "phase advance and pure time delay": the pure derivative block could create problems while running the simulation.

6.1. Longitudinal control
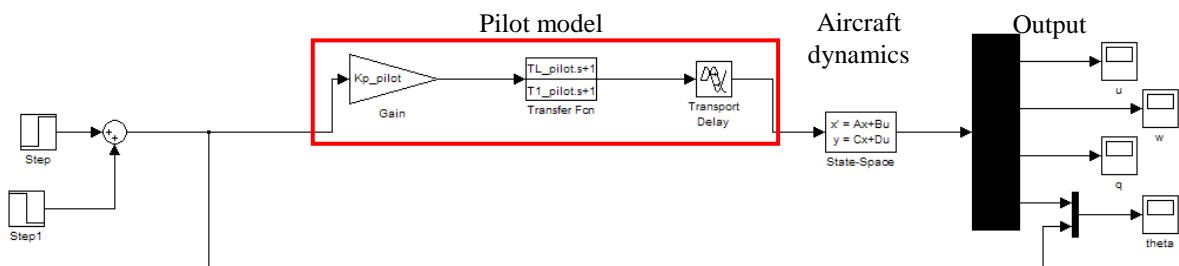
Fig 6.1   Simulink model of aircraft controlled by a pilot
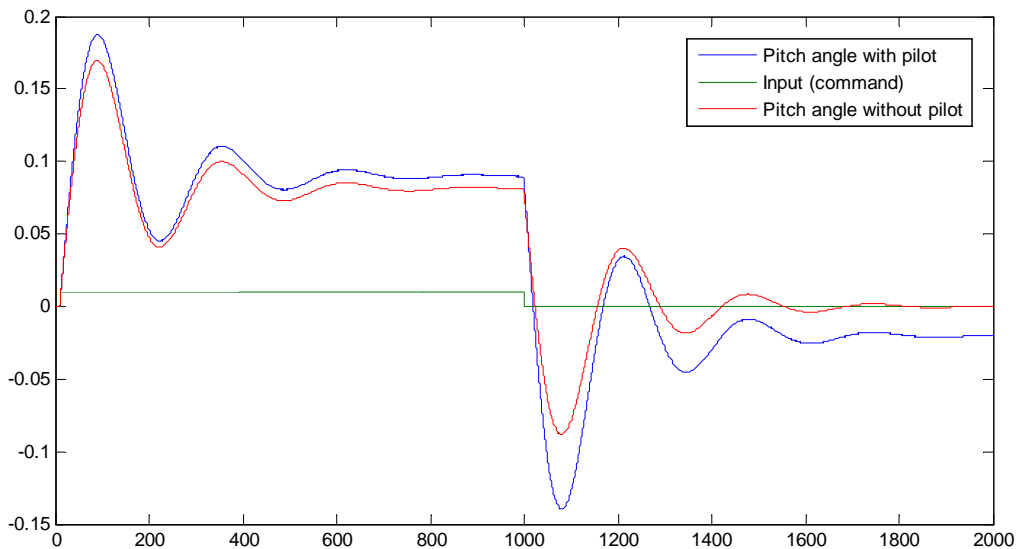
Fig 6.2   Comparison of pitch temporal evolution with or without pilot

As we can see on this figure, the final value is not the same with or without the pilot model. We also see that the oscillations are bigger, and that when we want a zero degree pitch, there is still an error.
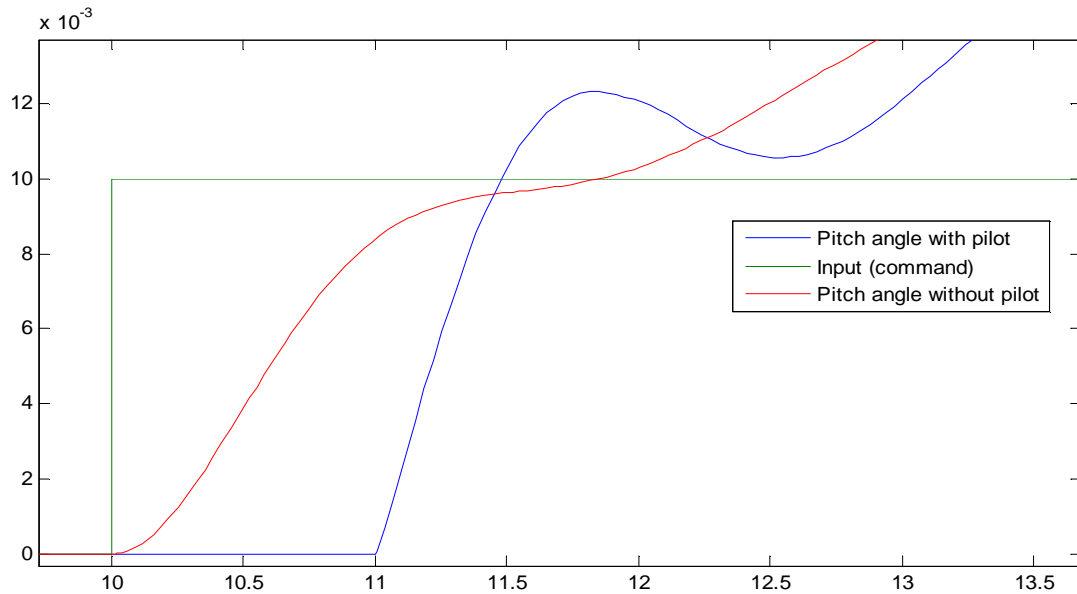
Fig 6.3   Focus on the beginning of the response

If we focus on the beginning, we also see that there is a delay between the step and the effective response on the pitch angle. This is because of the delay we added in the loop, to simulate the reaction time of the pilot. So, during one second, there is absolutely no action on the aircraft for the model with the pilot, whereas the reaction of the other model is instantaneous. Then, when the piloted aircraft starts to react, the other one has already started for a "long" time. The difference on pitch angles is already big.

If you close the loop by connecting the output (pitch angle) to the input and by comparing them, there is no improvement in the response.

6.2. Lateral control

As for the longitudinal model, we add the pilot model in the control aircraft model. We also add a gain to have a lower response.
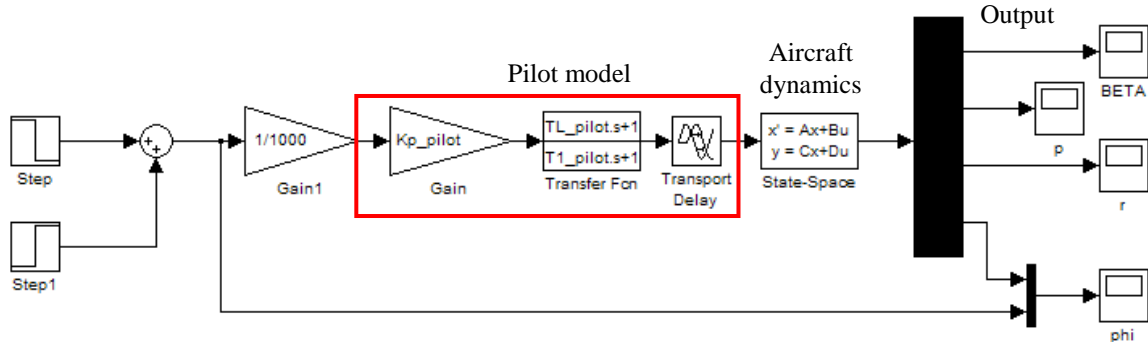
Fig 6.4   Model of aircraft controlled by a pilot
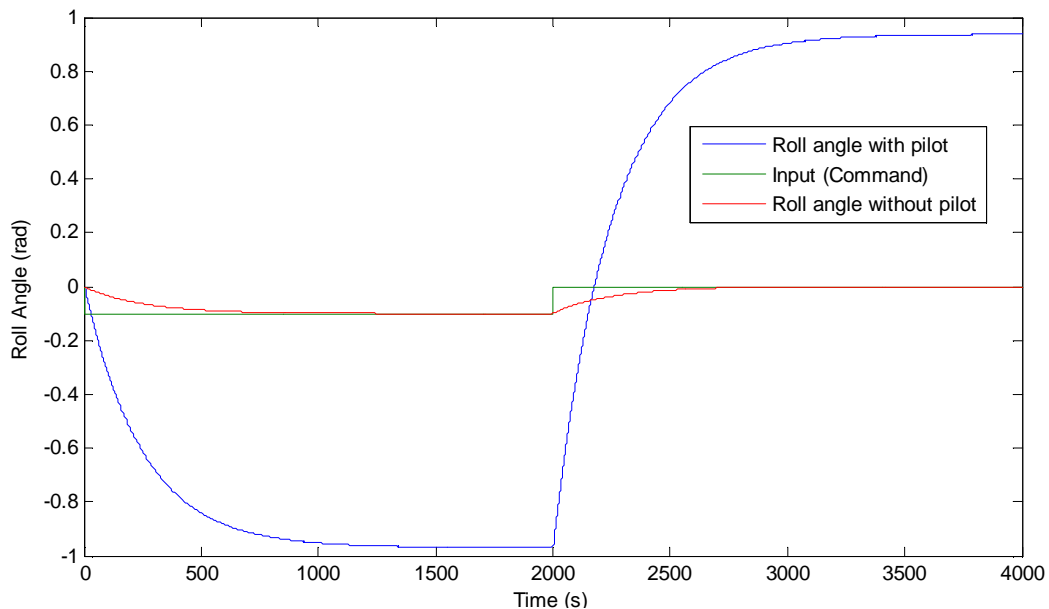
Fig 6.5   Comparison of the roll responses with or without the pilot in open loop

We see that the response with or without the pilot have the same behavior, but the values are totally different. For the model with pilot, there is a really big difference between the final output value and the input, whereas there is no one in the model without the pilot. We cannot remove this static error by adding a gain, because of the transfer function we used in the pilot model. If we want make this error lower, we need to close the loop in the model.

So we connect the output (roll angle) with the input and compare them.
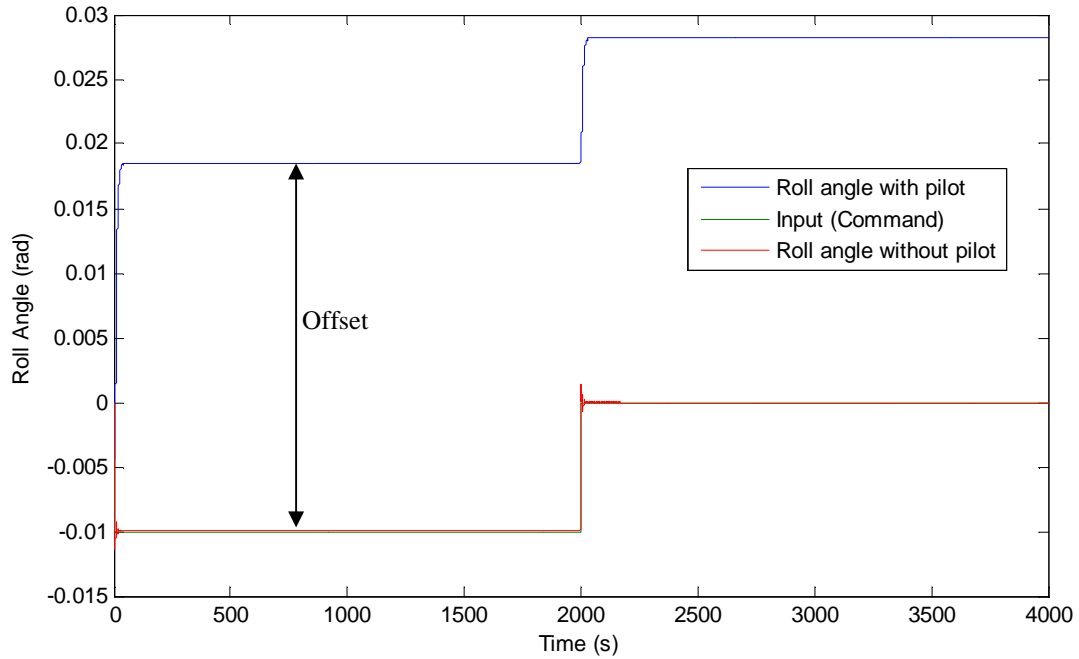
Fig 6.5   Comparison of the roll responses with or without the pilot in closed loop

The response is now really fast, and the error is very low, but there is an offset on the output value for the model with the pilot. The value of this offset is about 0,027 rad.

If we zoom on the beginning, we can see how fast the new response is:
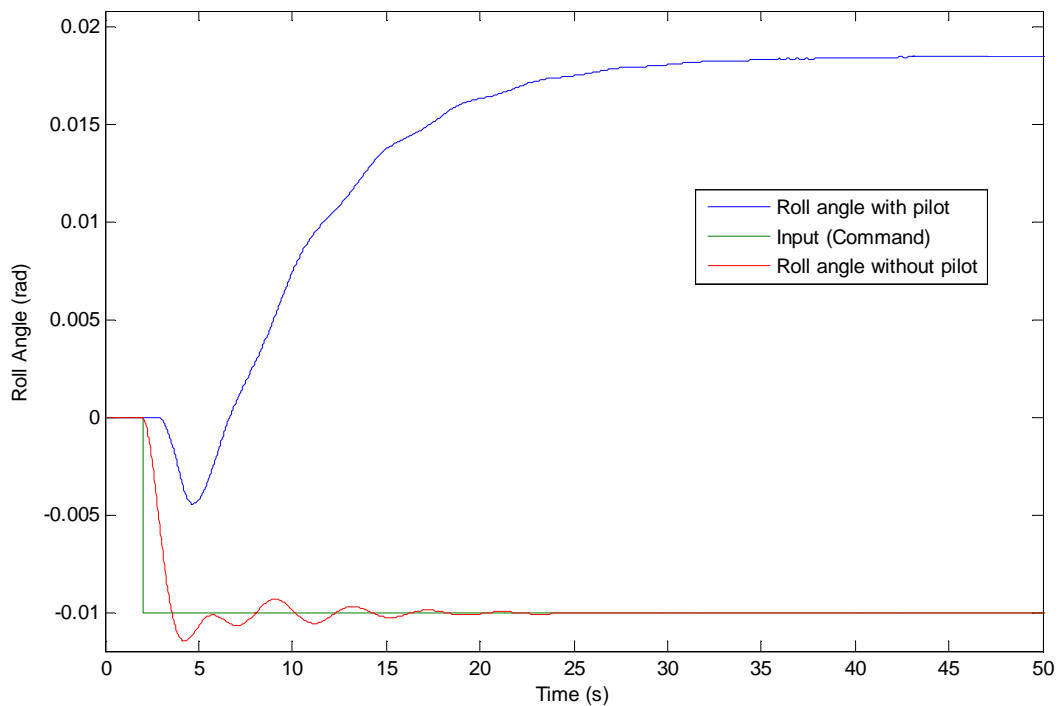
Fig 6.6   Zoom on the beginning of the response

7. Linear and non linear actuators models

The last little model we have to integrate into the big model is the model of an actuator. Indeed, we cannot represent truly the aircraft without the actuator model.

We can find two different models representing an actuator: the linear and the non-linear one. In this part, we will study both of them and compare them in order to select the best one.

7.1. Linear model

Every actuator receives a surface deflection command δ_c and transforms it into an actual surface deflection δ. The model of this actuator can be represented by this linear function:

$$\frac{\delta(s)}{\delta_c(s)} = K_{servo} \cdot \frac{1}{1 + T_p s}$$

Where T_p is the time constant of the actuator and K_{servo} the gain of the actuator.
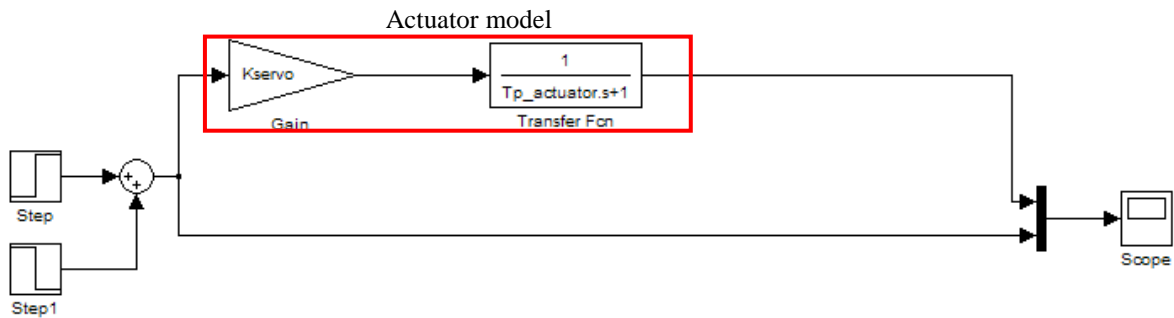
Fig 7.1   Simulink linear model of actuator
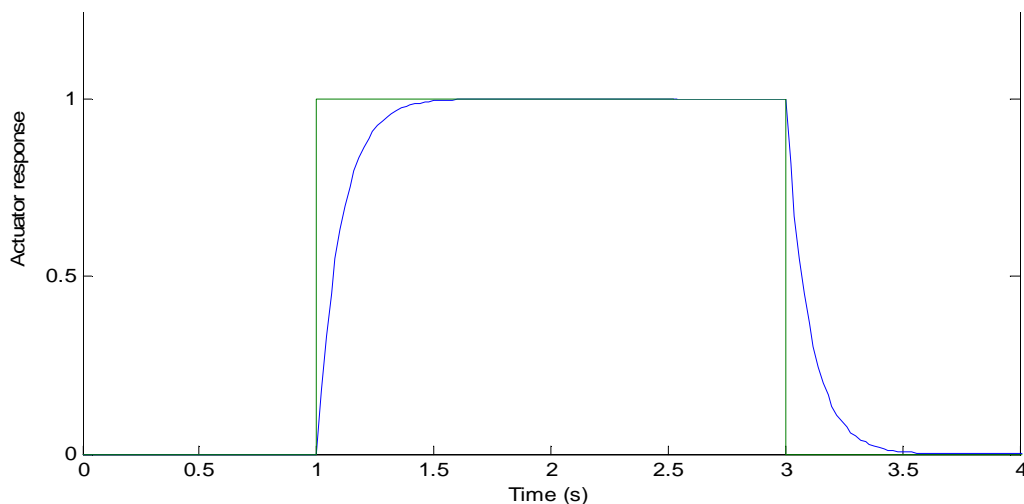
Fig 7.2   Temporal evolution of response for a step ($K_{servo} = 1$ & $T_p = 0,1\ s$)

The response of this model is a decreasing exponential response, also called first order response. The final value of the output is the same as the step input because we chose the value of 1 for the gain K_{servo}. We can also see that the response is fast: this is because of the small value of T_p.

7.2. Non linear model

Another way to simulate an actuator is to use a non linear model. For this, we use this closed loop model given by **Witte 2004** with a gain, an integrator and a saturation which is a non linear element.
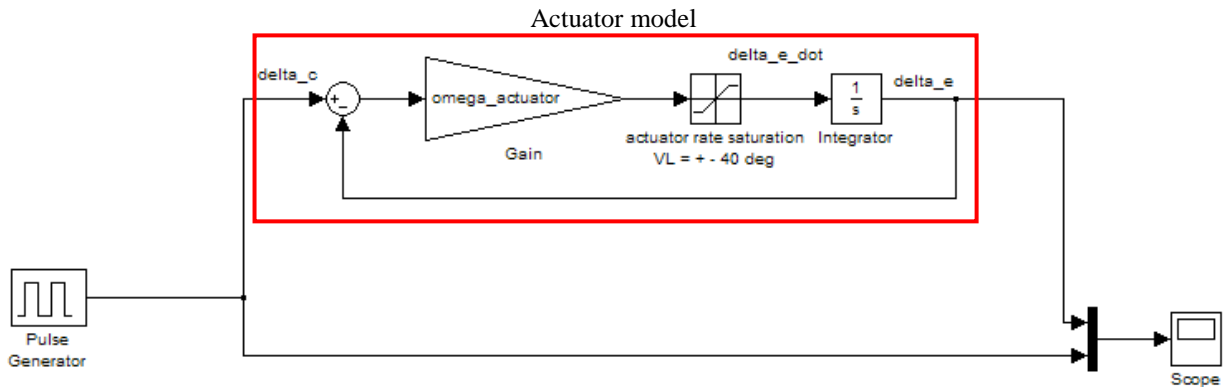
Fig 7.3   Simulink non linear model of actuator

The response to a step is quite the same as for the linear model:
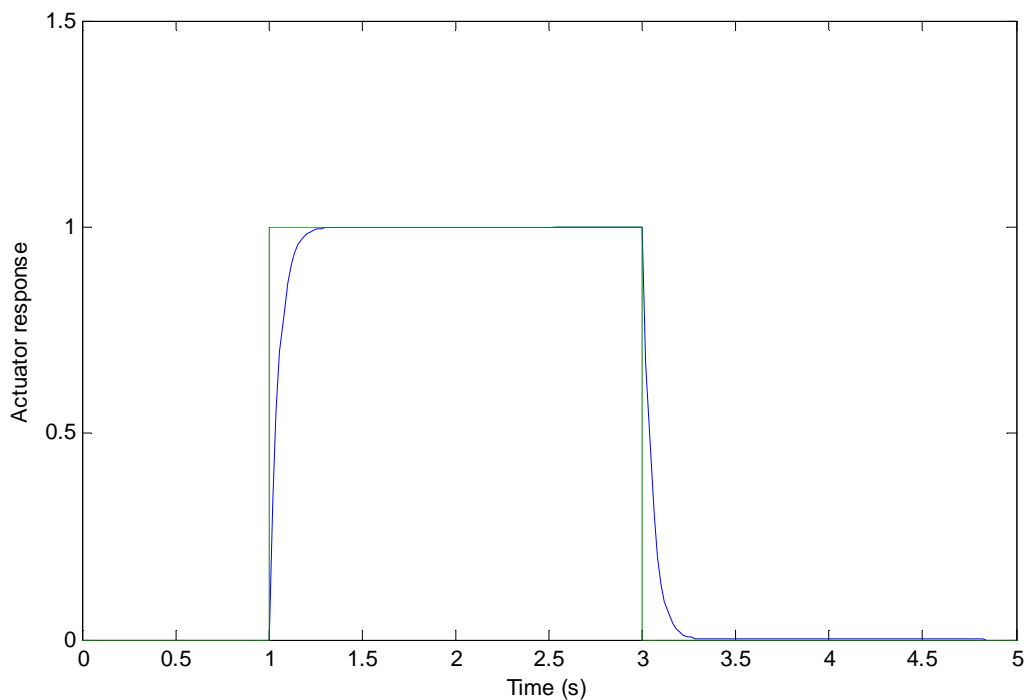
Fig 7.4   Temporal evolution of response for a step

The response is fast and without any static error, as for the linear model.

7.3. Comparison of linear/non linear actuators

We compare the actuators using a sinusoidal input. For the first test, the parameters for the sinusoid are: $\omega_i = 5\ rad/s$ and the amplitude $A = 10\ deg$
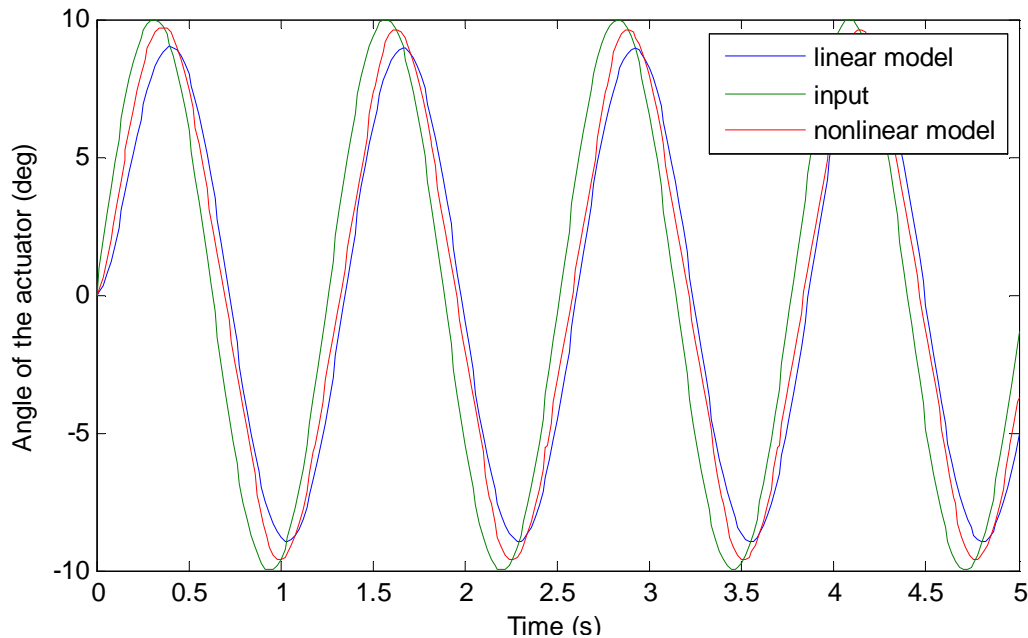
Fig 7.5   Comparison of temporal responses for linear and non linear model of actuators

In this case, we can see a little difference between both models, but the curves have the same aspect. The amplitude of the linear model is little smaller and seems to be a few delayed in comparison with the non linear model.

For the second test, we change the amplitude of the sinusoid: $A = 15\ deg$
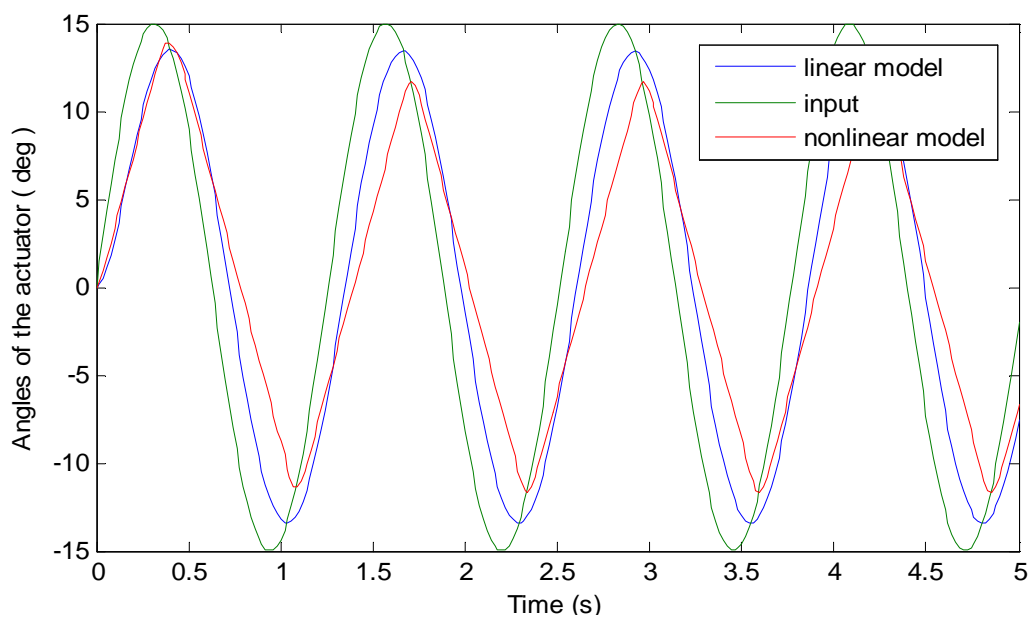
Fig 7.6   Comparison after changing the amplitude of the sinusoid

Now, we see that the non linear model has a quite triangular response. If we look at the actuator velocity before and after the saturation block this triangular response becomes obvious:
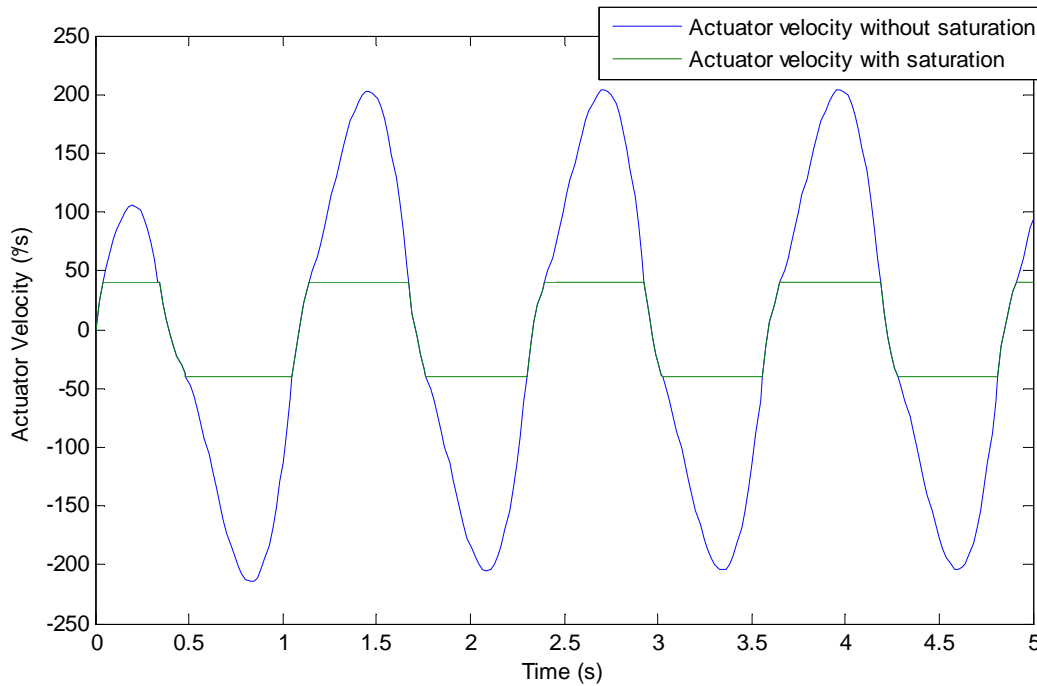
Fig 7.7   Comparison of actuator velocity with or without saturation

The reason of this triangular response is that the velocity is limited, so the slope of actuator angle response is limited to $\pm45°$/s. This is why the response looks more like a triangle than a sinusoid which needs higher slope values.

As the non-linear model of actuator is closer from the reality (because of the limitation of velocity), we will choose this one to put it in the aircraft model.

8. Pilot Induced Oscillations

8.1. Definition and systematic

Pilot Induced Oscillations (PIO) are defined by **Mc Ruer 1995** as:

> *„an inadvertent, sustained aircraft oscillation which is the consequence of an abnormal joint enterprise between the aircraft and the pilot"*

Klyde 1995 gives a more precise definition of this phenomenon: a complex interaction between the pilot and the intervention of the Automatic Flight Control System.

Mitchell 1995 has another definition in order to know when a PIO appears:

> *„A PIO exists when the airplane attitude, angular rate, or normal acceleration is 180 degrees out of phase with the pilot's control inputs"*

The definition on **Wikipedia 2009** is a mix of the previous definitions:

> *„Pilot-induced Oscillation occurs when the pilot of an aircraft inadvertently commands an often increasing series of corrections in opposite directions, each an attempt to cover the aircraft's reaction to the previous input with an overcorrection in the opposite direction"*
> *„Pilot-induced oscillations may be the fault of the aircraft, the pilot, or both. It is a common problem for inexperienced, and especially student pilots"*

The phases of flight when this phenomenon can occur are phases which need a rapid reaction of the pilot or which require to keep a precise altitude of flight:

- Aerial refueling
- Formation flying
- Landing
- Unexpected and sudden change of flying configuration that can perturb the Automatic Flight Control System

All these phases of flight require a high need of amplification gain in the loop, in order to perform the exact action wanted by the pilot.

A basic representation of the Flight Control System is:
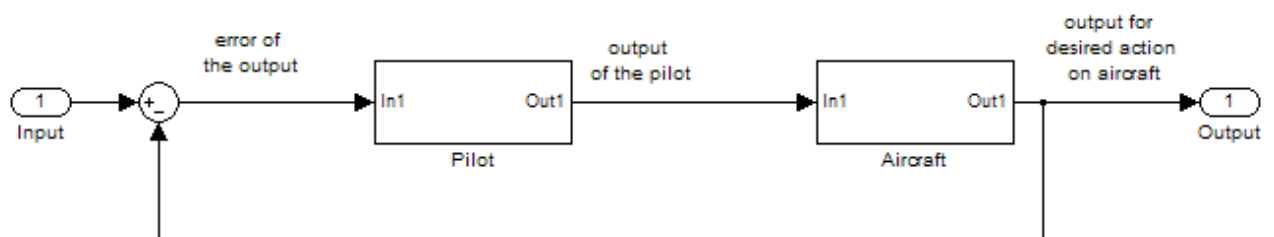
Fig 8.1 Basic model of Flight Control System

The input of this model could be:
- A desired value of pitch/roll/yaw angle
- A value of pitch/roll/yaw angle velocity
- A value of speed on x/y/z axis
- The value of altitude the aircraft has to reach

PIO phenomena of small scale can be treated by linear theory, if we stay in a domain which disturbance is small. If the amplitude of oscillation is higher, it can be treated with the non linear theory (Mc Ruer 1995).

When flight qualities are reduced, this high-frequency lags can occur, then the emergence of the phenomenon is expected. But these conditions are hardly predicted, and the current theory, based on criteria, simulations and tests can not cover all cases that may arise.

8.2. History

Pilot induced oscillations caused the destruction of a few aircraft in the past.

In February 1989 a Saab JAS-39 Gripen prototype crashed in Linköping (Sweden) during the landing. PIO, as a result of an over-sensitive yet slow-response steering system was determined to be the cause. The video of this accident is available here: http://www.youtube.com/watch?v=IRk4vfq9AZ4 (2009-06-29)

Pilot-induced oscillation was blamed for the 1992 crash of the prototype F/A-22 Raptor, landing at Edwards Air Force Base (USA) in California. This crash was linked to actuator rate limiting causing the pilot to over-compensate for pitch fluctuations. The video of this crash can be watched here: http://www.youtube.com/watch?v=faB5bIdksi8 (2009-06-29)

These PIO occurred also while the landing of space shuttle prototype Enterprise in October 1977. The space shuttle entered in a PIO along roll and pitch axis, but fortunately the space shuttle didn't crash. The cause of the problem was a 270-millisecond time delay in the fly-by-wire system: http://www.dfrc.nasa.gov/Gallery/Movie/STS/HTML/EM-0084-02.html (2009-06-29)

It appears that the Pilot-induced Oscillations never caused the crash of an airliner.

8.3. Simulation

For this simulation, we use all the models we used before. We have to connect a pilot model, an actuator, and the flight dynamics block.

We use the second pilot model (called Phase advance and pure time delay) as we used in the previous simulations. We add the non linear actuator model and the block concerning the flight dynamics to this pilot model.

8.3.1. Longitudinal dynamics

For the case of a longitudinal flight, we are going to control the pitch angle θ. The input will be a comparison between two steps, with different step times. As we need to close the loop in order to control the aircraft, we need to compare this input with an output value. The output value will be the pitch angle θ because this is the parameter we want to control. We also add a gain in the loopback. We choose to set the value of 0,7 for this gain.
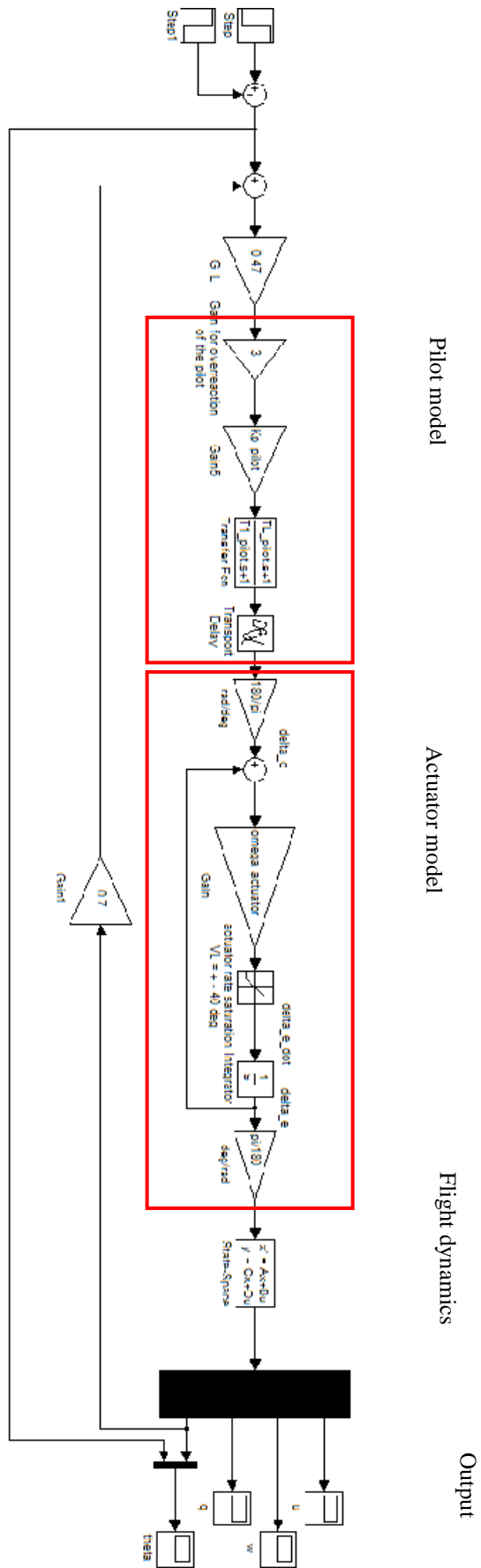
Fig 8.2   Model of aircraft controlled by a pilot with the non linear actuator model, in closed loop
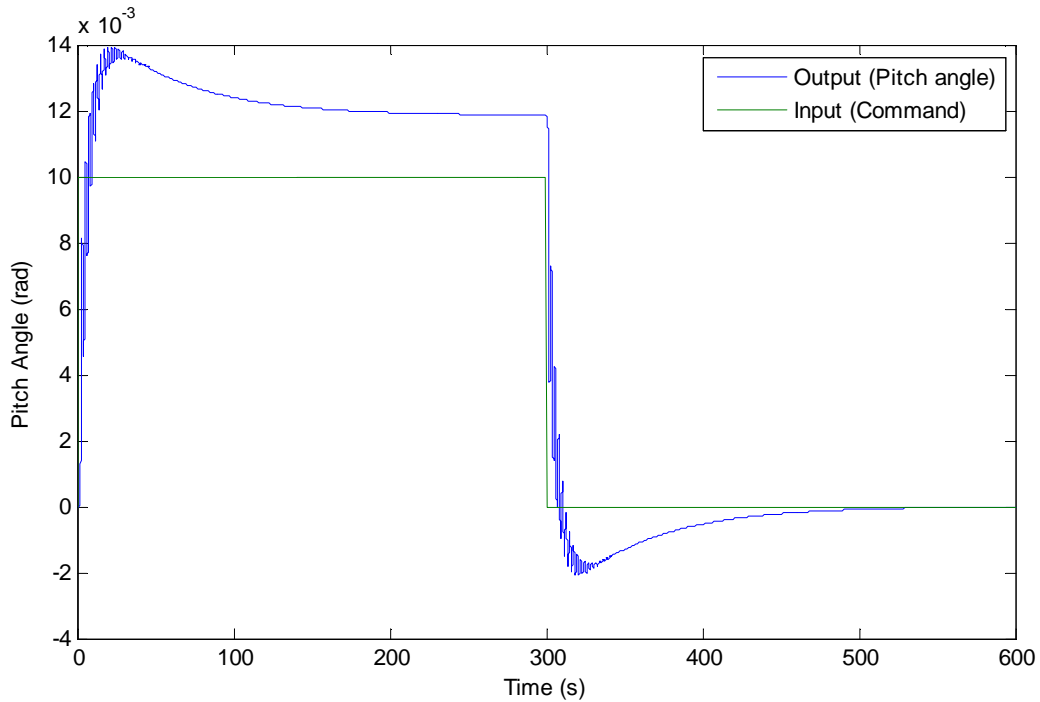
Fig 8.3   Temporal evolution of the pitch angle

With this graph, we can see that the system has a few oscillations and that the final value is not the one we want, but it is stable. There is also a little delay at the beginning and end of the step, but we can't do anything against it. This delay is because of the delay introduced by the pilot model.

If we want that the systems reaches the final value, we need to add a gain in the loop. We add a gain with a value of 0,47. The new response is:
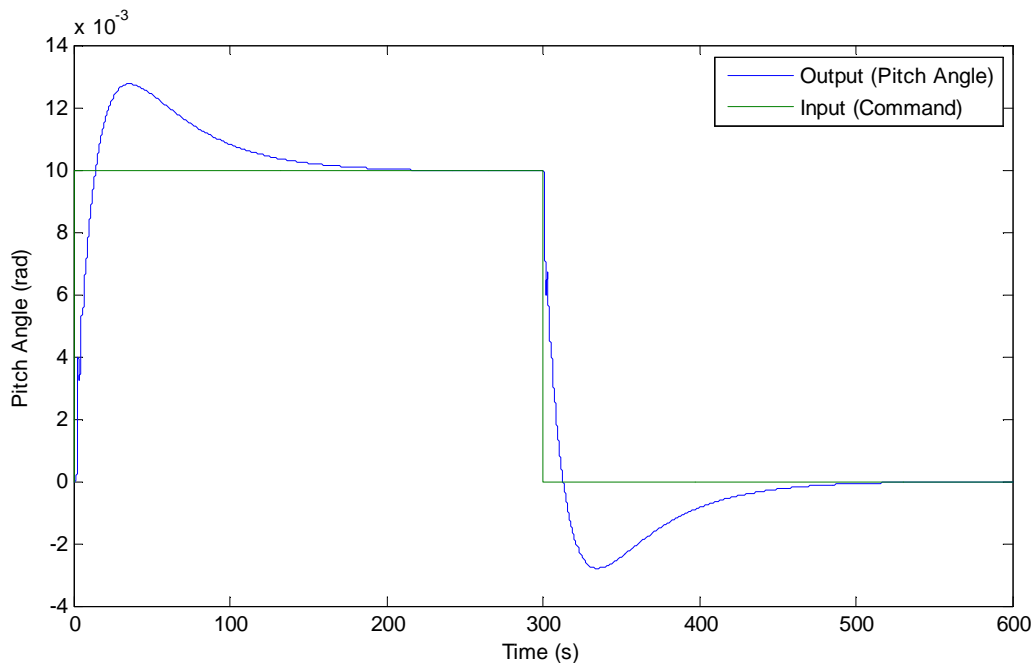
Fig 8.4   Temporal evolution of the pitch angle after adding the gain in the loop

There are less oscillations and the final value is reached. But the system is still very slow. If we wanted to make it faster, we should use a controller.

Now, we add another gain in the loop that simulates a strong action of the pilot. We choose to triple the action of the pilot. The result on the plane is:
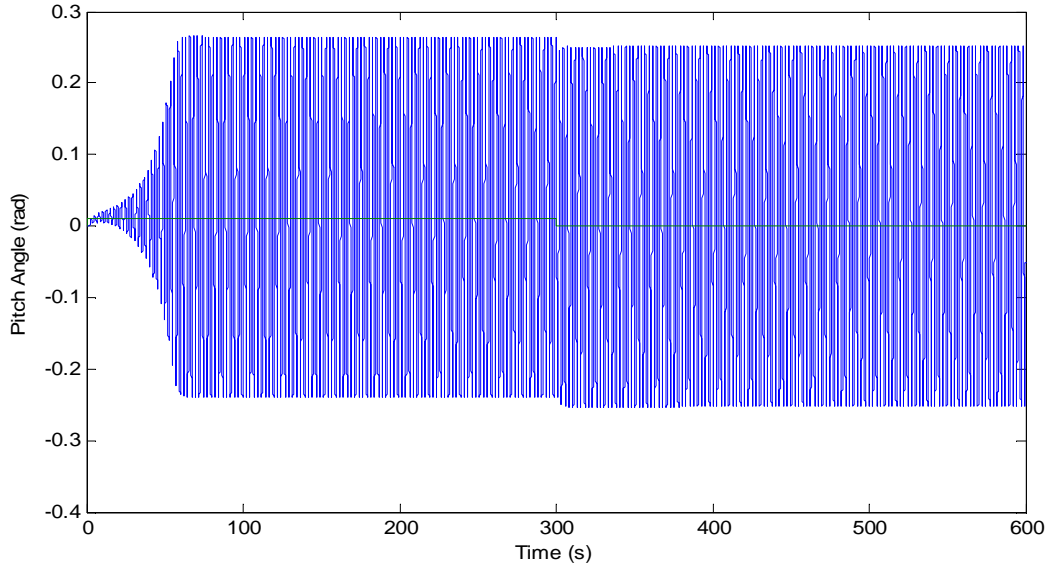
Fig 8.5   Temporal evolution of pitch after adding the new gain

As we can see, the system has now really big oscillations that are growing until they reach their maximum value. The system is unstable. This is a case of Longitudinal Pilot Induced Oscillations (Longitudinal PIO).
So if we only add a gain representing an overreaction of the pilot, we can create oscillations.
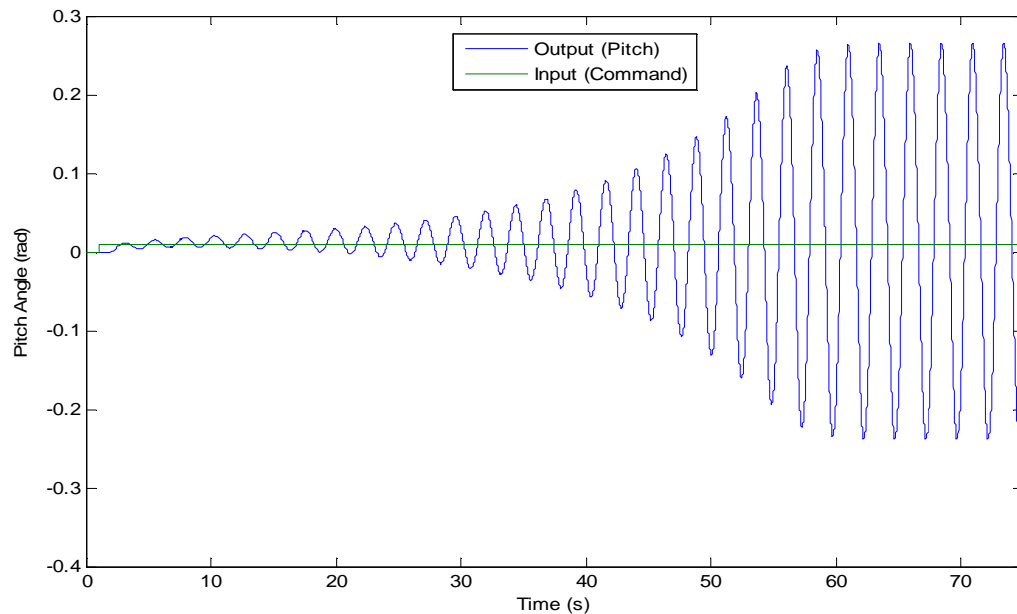
Fig 8.6   Zoom on the beginning of the response

With this zoom, we can easily see the growing oscillations, before they reach their maximum value.

8.3.2. Lateral dynamics

In the case of a flight with lateral movements, we are going to control the roll angle φ. The best action we can do in order to control the roll angle is to move the ailerons. So the input of our model will be a command on these ailerons. Then we have to connect the pilot model with the actuator model and with the aircraft dynamics block.

As for the longitudinal flight, the command we set in the input of our model is a comparison of two steps signals. We also close the loop. As we want to control the roll angle, we connect the output roll angle of the aircraft to the input and compare it with the command. We also add a gain in the loopback. For the longitudinal dynamics, after a few tries it was easy to know which was the best value for this loopback gain, but for the lateral dynamics choosing the right value of this gain is much harder. Then we will compare the different responses we have using different values (G_L) for this gain, and find out what is the best value for this gain.
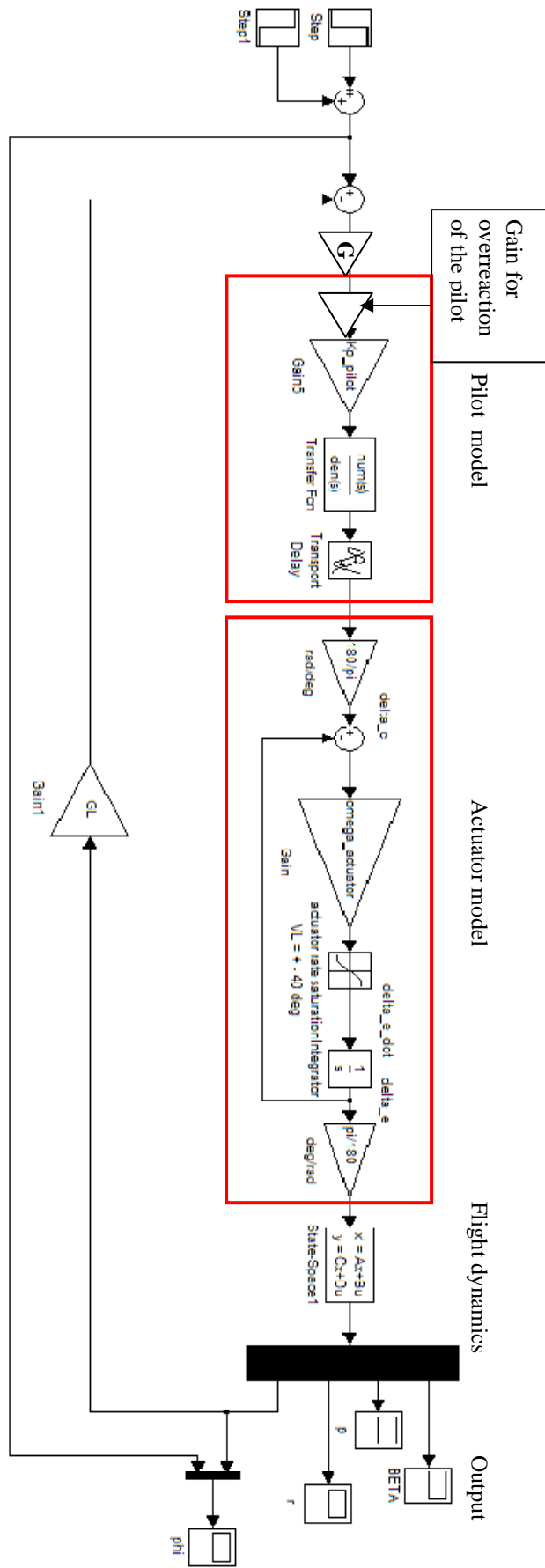
Fig 8.7   Model of aircraft controlled by a pilot with the non linear actuator model, in closed loop
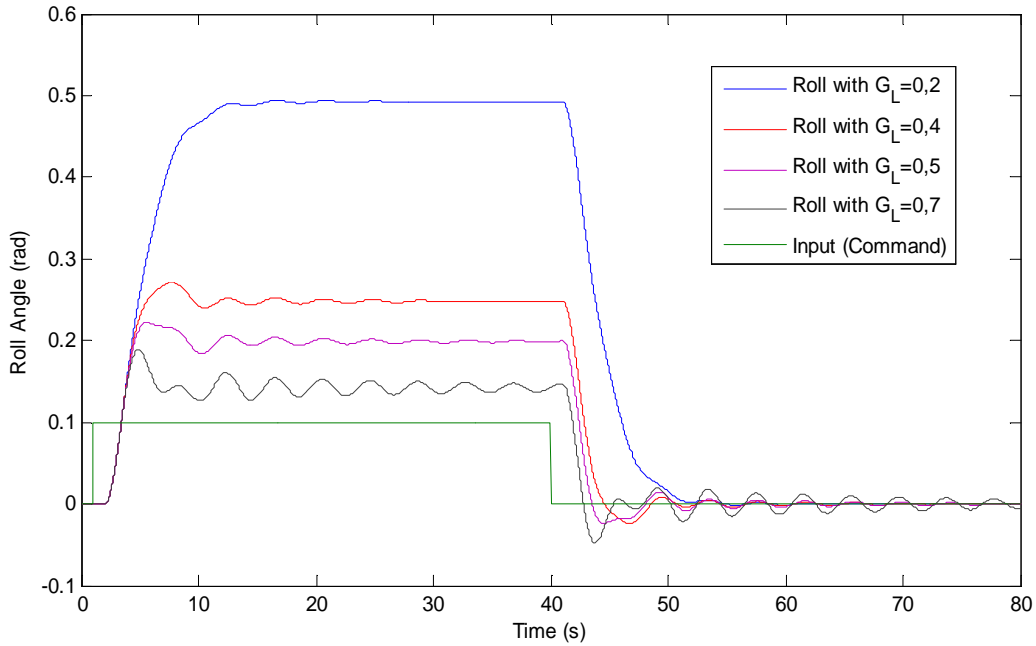
Fig 8.8   Comparison of roll temporal evolution with different values of G_L

If we compare the 4 results we have, we see that when the gain is high, the static error is low, but there are some oscillations. When the gain is low, the static error is very high, but there is almost no oscillation.

If we add a gain in the main loop in order to adapt the output to the input, there is no interesting change. For example, we try this on the model with $G_L = 0{,}2$.
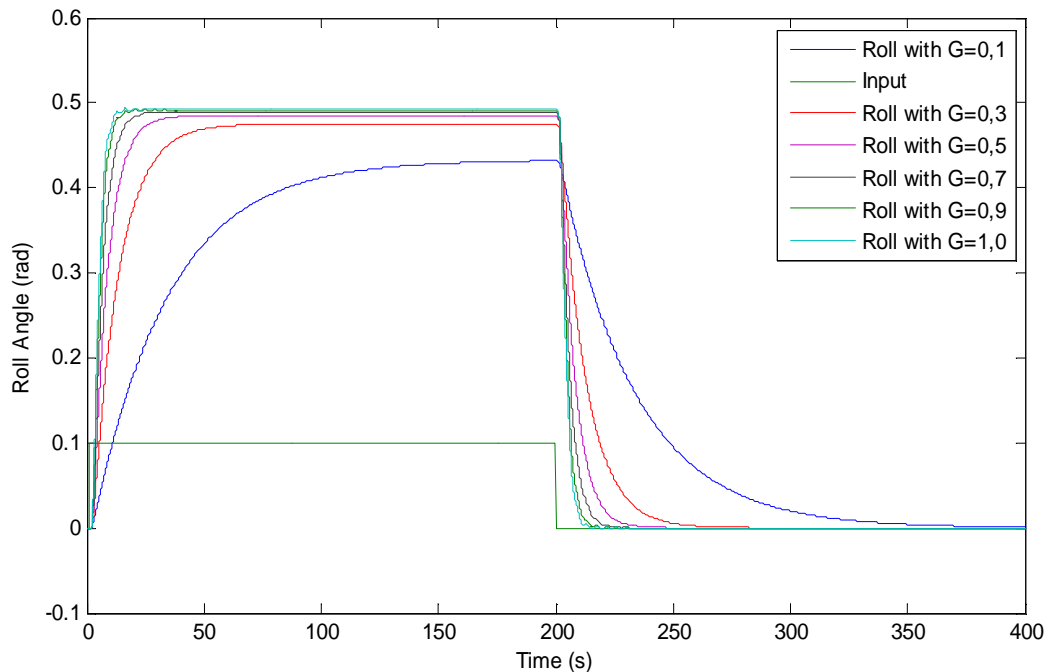
Fig 8.9   Comparison of roll temporal evolution with different values of G

When we add a low gain, the static error is a very few lower and the system is really slower. We remove the new gain (G) we added in the loop because it is useless and we only keep the gain in the loopback (G_L). We need to choose the right value for it, so we use the figure 8.7.

The best value seems to be $G_L = 0,4$ because the oscillations are lower than for higher values of G_L and the error is lower than when $G_L = 0,2$.

Then the response of our system is:
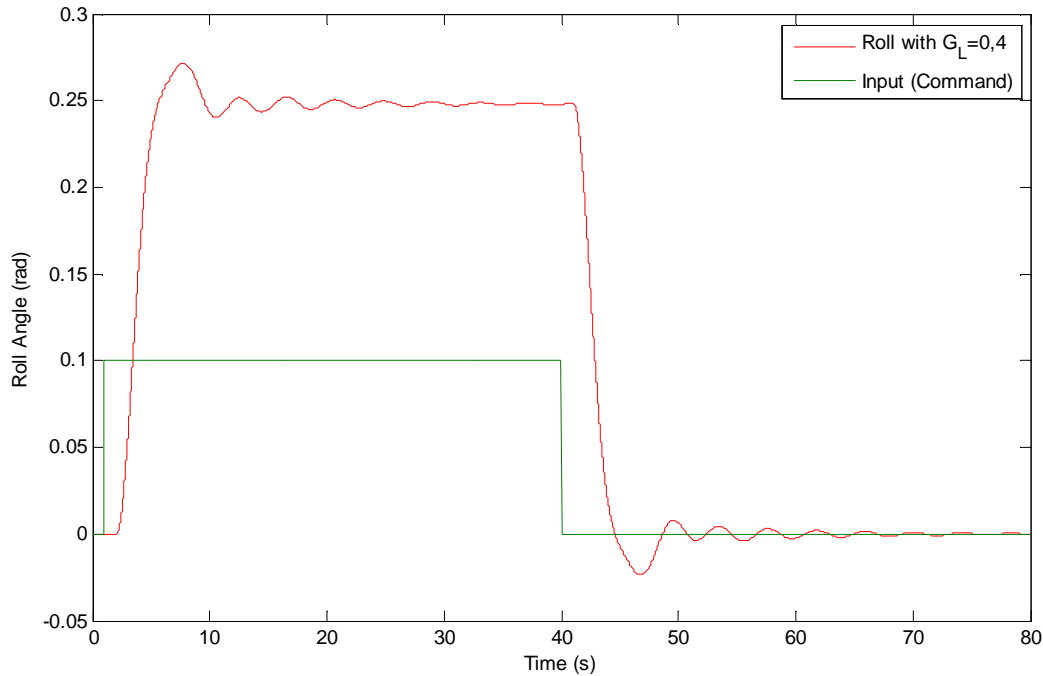
Fig 8.10   Evolution of roll with $G_L = 0,4$

There are a few oscillations but they are not really high. The main problem is the error on the final value. We know that we cannot correct it by changing the value of the gain, because this will create huge oscillations. The other way would be to use a PI or PID controller chosen with the good values to optimize the response of the aircraft.

Now, we want to simulate an overreaction of the pilot, so we need to add a gain in the loop. If we double the action of the pilot (gain = 2), the oscillations become bigger and longer, but the system is still stable:
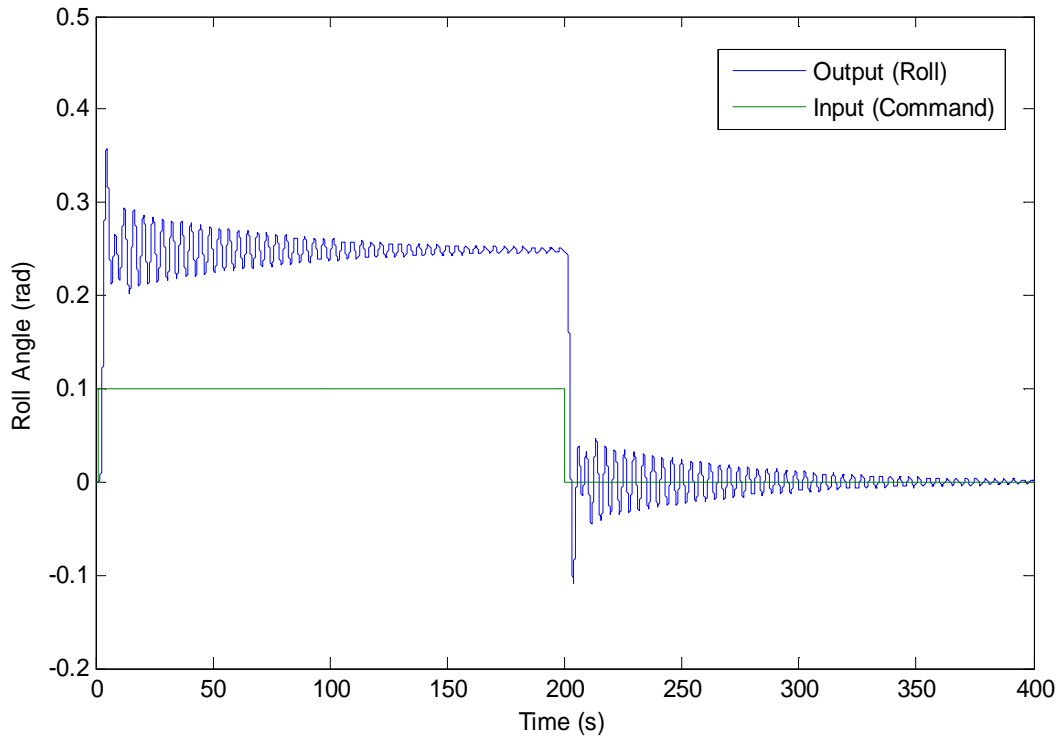
Fig 8.11   Temporal evolution of roll

But if we chose to triple the gain:
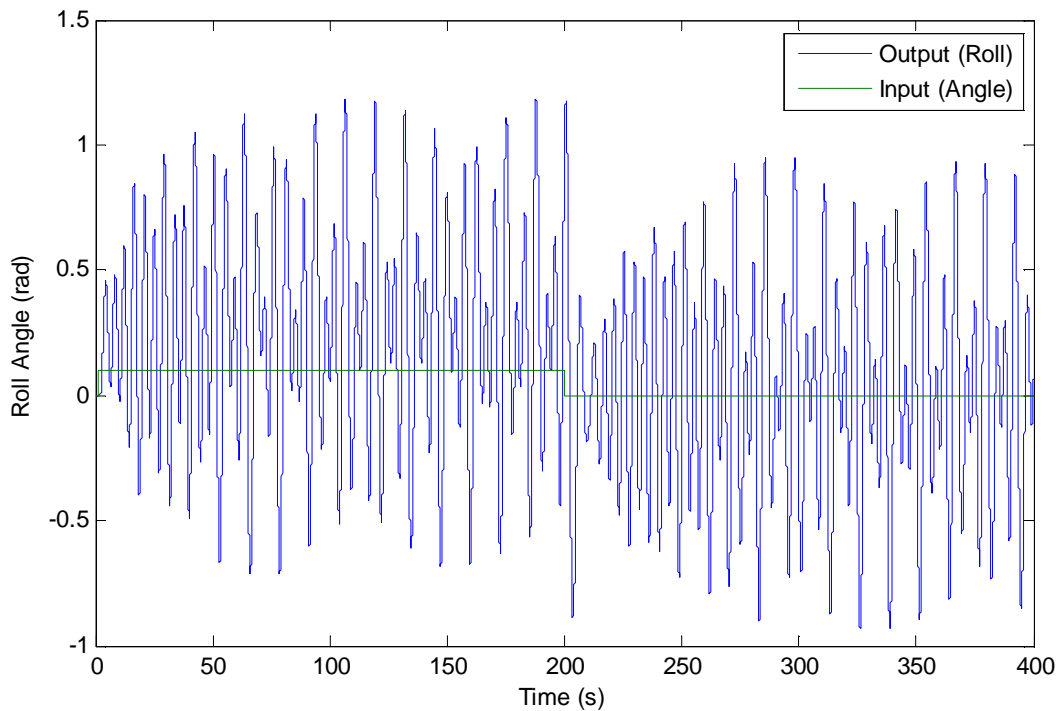
Fig 8.12   Temporal evolution of roll after adding the new gain

The response of the system has now oscillations that don't decrease. The system is totally unstable. This is another case of Pilot Induced Oscillations: the output has no final value, but oscillates without ending. In this case, it is a lateral PIO.

8.4. Prediction

With the previous simulations, we realized that the apparition of a PIO depends on the gain that is in the loop. If we double or triple the gain of a stable system, we can create instabilities. So the question we would like to answer is: "what is the minimum gain we need to create instabilities in our system?"

In order to solve this problem, we would need to draw the Nyquist diagram and the need use the Nyquist stability criterion to know easily if the system is stable and what is the gain for it. But before drawing this diagram, we need to transform the open loop of our system into a transfer function and because of the non-linearities in the system, the software cannot do this.

Then, the only solution would be to find the expression of this transfer function using the describing function.

Witte 2004 explains how to use this describing function. But this work is really long that is why I could not do it. Then, it could be the subject of a new project.

Summary

In this study, we learned how to build the model of an automatic flight control system (AFCS). At the beginning we had only the model of aircraft dynamics and by adding other models, we designed an entire AFCS.

Besides, by comparing the linear and non-linear models of aircraft dynamics, we realized that their responses are almost the same when we put the same input (small perturbation). The main difference between theses both models is the time needed to perform the calculation: the non linear model needs almost 100 more time than the linear one, which is not convenient when we try to work or "play" with the model. This is why we only used the linear model after having compared these both models.

In the last part, we showed that once the entire AFCS model is built, it is quite easy to cause a Pilot-induced oscillation: the only element needed is a high amplification in the loop. Indeed, once this gain is added, the aircraft pitch or roll angle oscillates more and more before reaching a maximal value.

Then, with this work we now have the model of an aircraft in order to know if it will be stable under certain conditions. This work could be the beginning of another work: the new subject would be to study the stability of the aircraft in different phases of flight, by changing the values inside the file of aircraft's characteristics. Another subject could be to compare the stability of different comparable aircrafts. Finally, another study could be to add the PI correctors in the model, in order to have a faster and more accurate response, and then see if the PIO occurs in the same conditions, or if we need a higher or lower gain than we used in this work.

In conclusion, we could say that this work is only the first step of a bigger PIO study, in which we could compare different aircrafts in different phases of flight…

References

Witte 2004 WITTE, Joel B. : *An Investigation Relating Longitudinal Pilot Induced Oscillation Tendency Rating to Describing Function Predictions for Rate-Limited Actuators*. Ohio: Wright Patterson Air Force Base, 2004.

Scholz 1992 SCHOLZ, Dieter: *Simple Linear and Nonlinear Simulation of Aircraft Dynamics*. Hamburg : Deutsche Aerospace Airbus, 1992 (TN-EV52-551/92)

Mc Lean 1990 MCLANE, Donald: *Automatic Flight Control Systems.* New York : Prentice Hall International, 1990.

LBM 2009 Laboratory of Biological Modeling, National Institute of Health, USA URL: http://mrb.niddk.nih.gov/glossary/glossary.html (2009-06-30)

Wikipedia 2009 http://www.wikipedia.com

Klyde 1995 KLYDE, David H.: *Unified Pilot Induced Oscillation Theory, Volume 1: PIO Analysis with Linear and Nonlinear Effective Vehicle Characteristics, Including Rate Limiting.* WL-TR-96-3028. Air Force Research Laboratories. Wright-Patterson AFB-OH, 1995.

Mitchell 1995 MITCHELL, David G.: HOH, Roger H.: *Development of a Unified Method to Predict Tendencies for Pilot-Induced Oscillations*. WL-TR-95-3049. Air Force Research Laboratories, Wright- Patterson AFB OH, 1995.

Mc Ruer 1995 MCRUER, Duane T.: *Pilot Induced Oscillations and Human Dynamic Behaviour*. NASA Contractor Report 4683, 1995

Appendix

Derivative data of Douglas DC-8 (**Mc Ruer 1995**):

A. Geometrical and inertial parameters[a]

	Flight condition			
	8001 (approach)	8002 (holding)	803 (cruise)	8004 V_{NE}
h (ft)	0	15,000	33,000	33,000
M (−)	0.219	0.443	0.84	0.88
a (ft/sec)	1,117	1,058	982	982
ρ (slugs/ft^3)	0.002378	0.001496	0.000795	0.000795
V_{r_0} (ft/sec)	243.5	468.2	824.2	863.46
$\bar{q} = \rho V^2/2$ (lb/ft^2)	71.02	163.97	270.0	296.36
W (lb)	190,000	190,000	230,000	230,000
m (slugs)	5,900	5,900	7,143	7,143
I_x (slug-ft^2)	3,090,000	3,110,000	3,770,000	3,770,000
I_y (slug-ft^2)	2,940,000	2,940,000	3,560,000	3,560,000
I_z (slug-ft^2)	5,580,000	5,880,000	7,130,000	7,130,000
I_{zz} (slug-ft^2)	28,000	−64,500	45,000	53,700
$x_{c.g.}/\bar{c}$	0.15	0.15	0.15	0.15
θ_0 (degrees)	0	0	0	0
U_0 (ft/sec)	243.5	468.2	824.2	863.46
W_0 (ft/sec)	0	0	0	0
δ_F (degrees)	35	0	0	0

B. Longitudinal dimensional derivatives[b]

	Flight condition			
	8001	8002	8003	8004
h (ft)	0	15,000	33,000	33,000
M (−)	0.219	0.443	0.84	0.88
T_u (1/sec)	−0.000595	−0.0000846	0.000599	0.000733
$X_{u_{aero}}$ (1/sec)	−0.02851	−0.00707	−0.0145	−0.0471
X_u (1/sec)	−0.0291	−0.00714	−0.014	−0.0463
X_w (1/sec)	0.0629	0.0321	0.0043	−0.0259
X_{δ_e} [(ft/sec^2)/rad]	0	0	0	0
$Z_{u_{aero}}$ (1/sec)	−0.2506	−0.1329	−0.0735	0.0622
Z_u (1/sec)	−0.2506	−0.1329	−0.0735	0.0622
$Z_{\dot{w}}$ (−)	0	0	0	0
Z_w (1/sec)	−0.6277	−0.756	−0.806	−0.865
Z_{δ_e} [(ft/sec^2)/rad]	−10.19	−23.7	−34.6	−38.6
$M_{u_{aero}}$ (1/sec-ft)	−0.0000077	−0.000063	−0.000786	−0.00254
M_u (1/sec-ft)	−0.0000077	−0.000063	−0.000786	−0.00254
$M_{\dot{w}}$ (1/ft)	−0.001068	−0.00072	−0.00051	−0.00052
M_w (1/sec-ft)	−0.0087	−0.0107	−0.0111	−0.0139
M_q (1/sec)	−0.7924	−0.991	−0.924	−1.008
M_{δ_e} (1/sec^2)	−1.35	−3.24	−4.59	−5.12

	D. Elevator longitudinal transfer function factors		E. Aileron lateral transfer function factors		F. Rudder lateral transfer function factors		G. Lateral coupling numerators		
		Flt. cond. 4701 (approach)		Flt. cond. 4701 (approach)		Flt. cond. 4701 (approach)			Flt. cond. 4701 (approach)
Mach No., M ($-$)		0.122		0.122		0.122			0.122
Altitude, h (ft)		1000		1000		1000			1000

Column D — Δ_{long}:
- $1/T_{sp_1}$ = 1.25
- $1/T_{sp_2}$ = 3.31
- ζ_p = 0.201
- ω_p = 0.201

$N_{\delta_e}^{\theta}$:
- A_θ = -4.55
- $1/T_{\theta_1}$ = 0.114
- $1/T_{\theta_2}$ = 1.19
- d-c gain = -3.64

$N_{\delta_e}^{u}$:
- A_u = -0.082
- $1/T_{u_1}$ = 2.40
- $1/T_{u_2}$ = -920
- d-c gain = 1074

Column E — Δ_{lat}:
- $1/T_s$ = -0.043
- $1/T_R$ = 6.57
- ζ_d = 0.323
- ω_d = 1.10

$N_{\delta_a}^{p}$:
- A_p = 4.06
- $1/T_{p_1}$ = 0
- ζ_p = 0.312
- ω_p = 1.02
- d-c gain = 0

$N_{\delta_a}^{\varphi}$:
- A_φ = 4.06
- ζ_φ = 0.312
- ω_φ = 1.02
- d-c gain = -12.4

Column F — Δ_{lat}:
- $1/T_s$ = -0.043
- $1/T_R$ = 6.57
- ζ_d = 0.323
- ω_d = 1.10

$N_{\delta_r}^{p}$:
- A_p = 0.0726
- $1/T_{p_1}$ = 0
- $1/T_{p_2}$ = 0.779
- $1/T_{p_3}$ = -36.3
- d-c gain = 0

$N_{\delta_r}^{\varphi}$:
- A_φ = 0.0726
- $1/T_{\varphi_1}$ = 0.779
- $1/T_{\varphi_2}$ = -36.3
- d-c gain = 6.02

Column G — $N_{\delta_a\delta_r}^{p\ \beta}$:
- $A_{p\beta}$ = 0.164
- $1/T_{p\beta_1}$ = 0
- $1/T_{p\beta_2}$ = 24.4

$N_{\delta_a\delta_r}^{r\ \beta}$:
- $A_{r\beta}$ = 0.00191
- $\zeta_{r\beta}$ = -0.238
- $\omega_{r\beta}$ = 22.0

$N_{\delta_a\ \delta_r}^{p/s\ r} = N_{\delta_a\delta_r}^{\varphi\ r}$:
- $A_{\varphi r}$ = -3.89
- $1/T_{\varphi r}$ = 0.0169

$N_{\delta_a\delta_r}^{\beta\ a_y}$:
- $A_{\beta a_y}$ = -0.260
- $1/T_{\beta a_{y_1}}$ = 0.377
- $1/T_{\beta a_{y_2}}$ = -31.1
- c.g. $1/T_{\beta a_{y_3}}$ = —

D. Lateral dimensional derivatives[b]

	Flight condition			
	8001	8002	8003	8004
h (ft)	0	15,000	33,000	33,000
M ($-$)	0.219	0.443	0.84	0.88
Y_v (1/sec)	-0.1113	-0.1008	-0.0868	-0.0931
Y_β [(ft/sec²)/rad]	-27.1	-47.2	-71.5	-80.4
Y_{δ_a} [(ft/sec²)/rad]	0	0	0	0
$Y_{\delta_a}^*$ [(1/sec)/rad]	0	0	0	0
Y_{δ_r} [(ft/sec²)/rad]	5.79	13.48	18.33	20.12
$Y_{\delta_r}^*$ [(1/sec)/rad]	0.0238	0.0288	0.0222	0.0233
L_β (1/sec²)	-1.335	-2.68	-4.43	-5.05
L_p (1/sec)	-0.95	-1.233	-1.18	-1.289
L_r (1/sec)	0.612	0.391	0.336	0.35
L_{δ_a} (1/sec²)	-0.726	-1.62	-2.11	-2.3
L_{δ_r} (1/sec²)	-0.1848	0.374	0.559	0.63
L'_β (1/sec²)	-1.328	-2.71	-4.41	-5.02
L'_p (1/sec)	-0.951	-1.232	-1.181	-1.29
L'_r (1/sec)	0.609	0.397	0.334	0.346
L'_{δ_a} (1/sec²)	-0.726	-1.62	-2.11	-2.3
L'_{δ_r} (1/sec²)	0.1813	0.392	0.549	0.612
N_β (1/sec²)	0.763	1.271	2.17	2.47
N_p (1/sec)	-0.1192	-0.048	-0.01294	-0.00744
N_r (1/sec)	-0.268	-0.252	-0.23	-0.252
N_{δ_a} (1/sec²)	-0.0496	-0.0365	-0.0519	-0.0615
N_{δ_r} (1/sec²)	-0.39	-0.86	-1.168	-1.282
N'_β (1/sec²)	0.757	1.301	2.14	2.43
N'_p (1/sec)	-0.124	-0.0346	-0.0204	-0.01715
N'_r (1/sec)	-0.265	-0.257	-0.228	-0.25
N'_{δ_a} (1/sec²)	-0.0532	-0.01875	-0.0652	-0.0788
N'_{δ_r} (1/sec²)	-0.389	-0.864	-0.01164	-1.277

E. Aileron lateral transfer function factors

		Flight condition			
		8001	8002	8003	8004
Mach No., M (—)		0.219	0.443	0.84	0.88
Altitude, h (ft)		0	15,000	33,000	33,000
c.g. ($\% \bar{c}$)		15	15	15	15
Weight, W (lb)		190,000	190,000	230,000	230,000
Δ_{lat}	$1/T_s$	-0.013	0.00649	0.00404	0.00447
	$1/T_R$	1.121	1.329	1.254	1.356
	ζ_d	0.1096	0.1061	0.0793	0.0855
	ω_d	0.996	1.197	1.495	1.589
$N_{\delta_a}^p$	A_p	-0.726	-1.62	-2.11	-2.30
	$1/T_{p_1}$	0	0	0	0
	ζ_p	0.223	0.1554	0.1072	0.1094
	ω_p	0.943	1.166	1.515	1.620
	d-c gain	0	0	0	0
$N_{\delta_a}^{\varphi}$	A_φ	-0.726	-1.62	-2.11	-2.30
	ζ_φ	0.223	0.1554	0.1072	0.1094
	ω_φ	0.943	1.166	1.515	1.620
	d-c gain	44.5	-177.9	-428	-395
$N_{\delta_a}^r$	A_r	-0.0532	-0.01875	-0.0652	-0.0788
	$1/T_{r_1}$	0.998	1.589	1.644	1.757
	ζ_r	-0.656	-0.727	-0.392	-0.345
	ω_r	1.242	2.23	1.323	1.269
	d-c gain	5.66	-12.0	-16.57	-14.59
$N_{\delta_a}^\beta$	A_β	0.0532	0.01875	0.0652	-0.0788
	$1/T_{\beta_1}$	-2.75	-7.9	-1.036	-0.704
	$1/T_{\beta_2}$	0.203	0.197	0.291	0.404
	$1/T_{\beta_3}$	—	—	—	—
	d-c gain	2.05	-2.35	-1.733	-1.467
$N_{\delta_y}^{a}$ c.g.	A_{a_y}	$-1.19 \cdot 10^{-7}$	-0.885	$-9.54 \cdot 10^{-7}$	-6.33
	$1/T_{a_{y_1}}$	-2.75	-7.9	-1.036	-0.704
	$1/T_{a_{y_2}}$	0.203	0.197	0.291	0.404
	$1/T_{a_{y_3}}$	—	—	—	—
	$1/T_{a_{y_4}}$	$1.21 \cdot 10^7$	—	$4.89 \cdot 10^6$	—
	d-c gain	-55.6	111.1	123.9	117.9

F. Rudder lateral transfer function factors

		Flight condition			
		8001	8002	8003	8004
Mach No., M (—)		0.219	0.443	0.84	0.88
Altitude, h (ft)		0	15,000	33,000	33,000
c.g. (% $\bar{c}$)		15	15	15	15
Weight, W (lb)		190,000	190,000	230,000	230,000
Δ_{lat}	$1/T_s$	−0.013	0.00649	0.00404	0.00447
	$1/T_R$	1.121	1.329	1.254	1.356
	ζ_d	0.1096	0.1061	0.0793	0.0855
	ω_d	0.996	1.197	1.495	1.589
$N_{\delta_r}^p$	A_p	0.1813	0.392	0.545	0.612
	$1/T_{p_1}$	0	0	0	0
	$1/T_{p_2}$	1.028	1.85	2.43	2.57
	$1/T_{p_3}$	−2.13	−2.56	−3.01	−3.150
	d-c gain	0	0	0	0
$N_{\delta_r}^\varphi$	A_φ	0.1813	0.392	0.545	0.612
	$1/T_{\varphi_1}$	1.028	1.85	2.43	2.57
	$1/T_{\varphi_2}$	−2.13	−2.56	−3.01	−3.15
	d-c gain	27.5	−150.5	−353	−324
$N_{\delta_r}^r$	A_r	−0.389	−0.864	−1.165	−1.277
	$1/T_{r_1}$	1.124	1.335	1.276	1.377
	ζ_r	−0.0743	−0.0451	−0.0619	−0.0475
	ω_r	0.339	0.330	0.323	0.323
	d-c gain	3.46	−10.19	−13.68	−12.00
$N_{\delta_r}^\beta$	A_β	0.0238	0.0288	0.0222	0.0233
	$1/T_{\beta_1}$	−0.0559	−0.01475	−0.00726	−0.00637
	$1/T_{\beta_2}$	1.141	1.297	1.217	1.323
	$1/T_{\beta_3}$	16.47	30.2	52.6	55.0
	d-c gain	1.725	−1.346	−0.912	−0.707
$N_{\delta_r}^{a_y}$	A_{a_y}	5.79	13.48	18.33	20.1
	$1/T_{a_{y_1}}$	−0.819	−0.0347	−0.01883	−0.01746
	$1/T_{a_{y_2}}$	−0.1077	1.535	1.122	1.231
	$1/T_{a_{y_3}}$ (ζ_{a_y})	(0.994)	−1.157	−1.418	−1.494
c.g.	$1/T_{a_{y_4}}$ (ω_{a_y})	(1.078)	1.147	1.723	1.819
	d-c gain	−41.0	77.0	83.5	76.9

G. Lateral coupling numerators

		Flight condition			
		8001	8002	8003	8004
Mach No., M (−)		0.219	0.443	0.84	0.88
Altitude, h (ft)		0	15,000	33,000	33,000
$N^{p\;\beta}_{\delta_a\delta_r}$	A_{p_β}	−0.01727	−0.0466	−0.0470	−0.0536
	$1/T_{p_{\beta_1}}$	0	0	0	0
	$1/T_{p_{\beta_2}}$	17.21	30.5	53.4	56.0
$N^{r\;\beta}_{\delta_a\delta_r}$	A_{r_β}	−0.00127	−0.00054	−0.00145	−0.001837
	ζ_{r_β}	−0.0672	−0.0656	0.0317	0.0507
	ω_{r_β}	5.52	13.39	8.2	7.79
$N^{p/s\;r}_{\delta_a\;\delta_r}$ $N^{\varphi\;r}_{\delta_a\;\delta_r}$	A_{φ_r}	0.292	1.408	2.49	2.99
	$1/T_{\varphi_r}$	0.061	0.0567	0.0439	0.0464
$N^{\beta\;ay}_{\delta_a\;\delta_r}$ c.g.	$A_{\beta_{ay}}$	0.308	0.253	1.195	1.586
	$1/T_{\beta_{ay1}}$	−2.75	−7.90	−1.036	−0.704
	$1/T_{\beta_{ay2}}$	0.203	0.1967	0.291	0.404
	$1/T_{\beta_{ay3}}$	—	—	—	—
$N^{r\;ay}_{\delta_a\;\delta_r}$ c.g.	$A_{r_{ay}}$	−0.308	−0.253	−1.195	−1.586
	$1/T_{r_{ay_1}}$	−1.532	−2.94	−1.281	−1.192
	$\zeta_{r_{ay}}$	0.359	0.317	0.593	0.645
	$\omega_{r_{ay}}$	1.1	1.86	1.519	1.536
$N^{p\;ay}_{\delta_a\;\delta_r}$ c.g.	$A_{p_{ay}}$	−4.21	−21.8	−38.7	−46.3
	$1/T_{p_{ay_1}}$	0	0	0	0
	$1/T_{p_{ay_2}}$	−0.871	−1.184	−1.413	−1.482
	$1/T_{p_{ay_3}}$	1.18	1.445	1.651	1.744
$N^{\varphi\;\beta}_{\delta_a\delta_r}$	A_{φ_β}	−0.01727	−0.0466	−0.0470	−0.0536
	$1/T_{\varphi_\beta}$	17.21	30.5	53.4	56.0
$N^{\varphi\;ay}_{\delta_a\;\delta_r}$ c.g.	$A_{\varphi_{ay}}$	−4.21	−21.8	−38.7	−46.3
	$1/T_{\varphi_{ay_1}}$	−0.871	−1.184	−1.413	−1.482
	$1/T_{\varphi_{ay_2}}$	1.18	1.445	1.651	1.744

[a] The data in A are for body-fixed stability axes. $S = 2600$ ft²; $b = 142.3$; $c = 23$ ft; $\gamma_0 = 0$ degrees.

[b] The data in B and D are for body-fixed stability axes.

Aircraft stability criteria (**Mc Lean 1990**) :

Dutch roll mode specification

Flight phase category	Class	Level								
		1			**2**			**3**		
		ζ_D rad/s	$\zeta_D \omega_D$ rad/s	ω_D	ζ_D rad/s	$\zeta_D \omega_D$ rad/s	ω_D	ζ_D rad/s	$\zeta_D \omega_D$ rad/s	ω_D
A	I, IV	0.19	0.35	1.0	0.02	0.05	0.4	0.02	-	0.4
A	I, III	0.19	0.35	0.4	0.02	0.05	0.4	0.02	-	0.4
B	all	0.08	0.15	0.4	0.02	0.05	0.4	0.02	-	0.4
C	I,IV	0.08	0.15	1.0	0.02	0.05	0.4	0.02	-	0.4
C	II, III	0.08	0.15	0.4	0.02	0.05	0.4	0.02	-	0.4

Note: Minimum values are specified. The governing damping requirement equals the largest value of ζ_D obtained from either column labelled ζ_D and $\zeta\omega_D$.
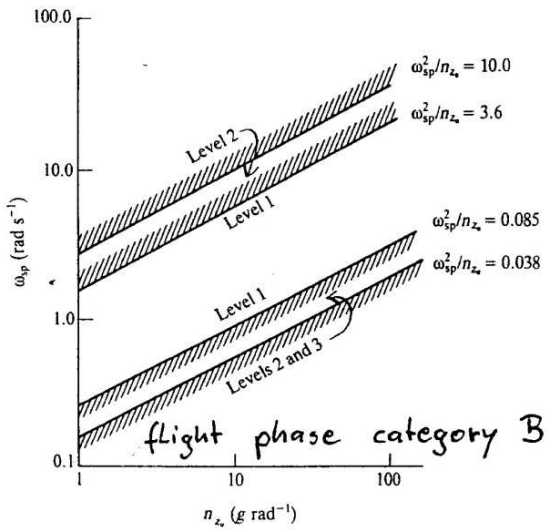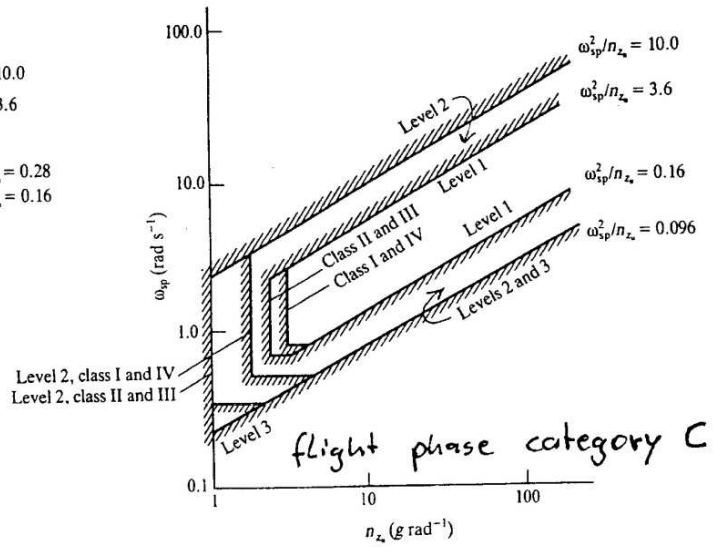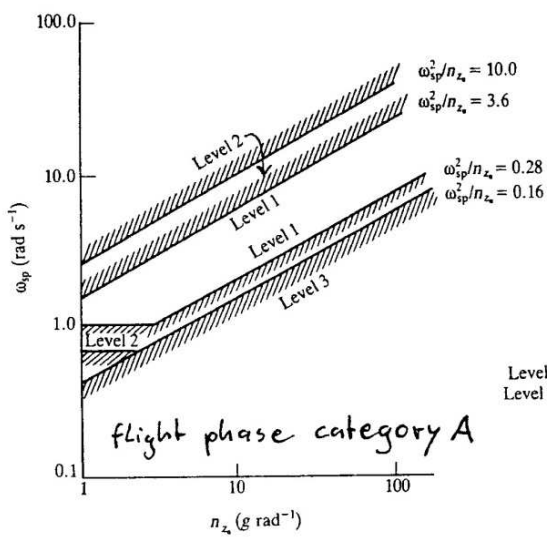
TABLE 4.7
Classification of airplanes

Class I	Small, light airplanes, such as light utility, primary trainer, and light observation craft
Class II	Medium-weight, low-to-medium maneuverability airplanes, such as heavy utility/search and rescue, light or medium transport/cargo/tanker, reconnaissance, tactical bomber, heavy attack and trainer for Class II
Class III	Large, heavy, low-to-medium maneuverability airplanes, such as heavy transport/cargo/tanker, heavy bomber and trainer for Class III
Class IV	High-maneuverability airplanes, such as fighter/interceptor, attack, tactical reconnaissance, observation and trainer for Class IV

TABLE 4.8
Flight phase categories

Nonterminal flight phase

Category A	Nonterminal flight phases that require rapid maneuvering, precision tracking, or precise flight-path control. Included in the category are air-to-air combat ground attack, weapon delivery/launch, aerial recovery, reconnaissance, in-flight refueling (receiver), terrain-following, antisubmarine search, and close-formation flying
Category B	Nonterminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in the category are climb, cruise, loiter, in-flight refueling (tanker), descent, emergency descent, emergency deceleration, and aerial delivery.

Terminal Flight Phases:

Category C	Terminal flight phases are notmally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this category are takeoff, catapult takeoff, approach, wave-off/go-around and landing.

Phugoid mode flying qualities

Level	Damping ratio of phugoid mode
1	≥ 0.04
2	≥ 0.0
3	An undamped oscillatory mode with a time to double amplitude of at least 55 s

Short period damping ratio specification

Flight phase category	Level 1 min.	Level 1 max.	Level 2 min.	Level 2 max.	Level 3 min.	Level 3 max.
A and C	0.35	1.3	0.25	2.0	0.15	-
B	0.3	2.0	0.2	2.0	0.15	-

Roll mode time constant specification

Flight phase category	Class	T_R [s] Level 1	T_R [s] Level 2	T_R [s] Level 3
A	I,IV	1.0	1.4	10.0
A	II, III	1.4	3.0	10.0
B	all	1.4	3.0	10.0
C	I, IV	1.0	1.4	10.0
C	II, III	1.4	3.0	10.0

Bank angle specification

Class	Flight phase category	Bank angle in fixed time Level 1	Bank angle in fixed time Level 2	Bank angle in fixed time Level 3
I	A	60° in 1.3s	60° in 1.7s	60° in 2.6s
I	B	60° in 1.7s	60° in 2.5s	60° in 3.4s
I	C	30° in 1.3s	30° in 1.8s	30° in 2.6s
II	A	45° in 1.4s	45° in 1.9s	45° in 2.8s
II	B	45° in 1.9s	45° in 2.8s	45° in 3.8s
II	C	30° in 1.8s	30° in 2.5s	30° in 3.6s
III	A	30° in 1.5s	30° in 2.0s	30° in 3.0s
III	B	30° in 2.0s	30° in 3.3s	30° in 5.0s
III	C	30° in 2.5s	30° in 4.0s	30° in 6.0s
IV	A	90° in 1.3s	90° in 1.7s	90° in 2.6s
IV	B	90° in 1.7s	90° in 2.5s	90° in 3.4s
IV	C	30° in 1.1s	30° in 1.3s	30° in 2.0s

The time required to reach the bank angle has to be _less_ than the specified maximum value.

Spiral mode stability specification

Flight phase category	Level 1	Level 2	Level 3
A and C	12s	8s	4s
B	20s	8s	4s