HAW HAMBURG

DEPARTMENT FAHRZEUGTECHNIK UND FLUGZEUGBAU

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Flugzeugentwurf / Aircraft Design SS 2019
Partial Solution

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This solution only covers those questions for which the answer are (so far) less standard. Students experienced some problems answering them.

1.5) How can we find out from the „First Law of Aircraft Design" that a design is infeasible?

The design is infeasible, if the denominator of the „First Law of Aircraft Design" is zero or negative.

1.10) What is a typical length for the cockpit of passenger transport aircraft? Estimate the cabin length of a six-abreast aircraft seating 120 passengers!

Length of cockpit: 4 m

On average 1 m for each row (accounting for cross aisles, lavatory, galleys, ...). 6 seats in a row gives 120/6 = 20 rows and a cabin length of 20 m.

1.11) Determine together with results from 1.10 the length of the aircraft! The aircraft may have a fineness ratio $l_F/d_F = 8$. *You may need to derive an equation to find the result!*

$l_F/d_F = 8$ and $d_F = l_F / 8$

$l_F = 24\ m + 1.6\ d_F$

$l_F = 24\ m + 1.6\ l_F / 8$

$l_F = 24\ m + 0.2\ l_F$

$l_F\ (1 - 0.2) = 24\ m$

$l_F = 24\ m / 0.8 = 30\ m$

1.12) a) A container has a volume V. The container extends in aircraft x direction (aft) a distance x. Write down the equation to calculate the cross sectional area of the container.

$S = V / x$

b) The volume of the cargo V_C, baggage V_B, and overhead stowage V_{OS} is known. The aircraft overall length l_F is also known. Calculate the minimum cross sectional area of the cargo compartment!

We calculate the minimum cross sectional area of the cargo compartment S_{min} from the minimum volume of the cargo compartment V_{min}.

$V_{min} = V_C + V_B - V_{OS}$

$S_{min} = V_{min} / (0.4\ l_F)$

If n container are placed next to each other in the cargo compartment,

$n\ S \geq S_{min}$

with n usually 1 or 2.

1.16) Passenger aircraft are usually built to a given span according to ICAO/FAA classes. Assume that during design the wing area can be reduced by 50%. How does this change the aspect ratio? How does this change the tank volume? *Calculate two values!*

$A = b^2/S$ with wing area S reduced by 50% the aspect ration A is increased by a factor of 2.

We look at the proportionality of the tank volume $V \sim S^{1.5}\ A^{-1/2}$ based on equation (7.35):

$V \sim (1/2)^{1.5}\ 2^{-1/2}$

$V \sim (1/2)^{1.5}\ (1/2)^{1/2}$

$V \sim (1/2)^2 = 0.25$

Questions from the Evening Lectures

1.29) What does the Concorde flight engineer demonstrate in the picture with his right hand?

The flight engineer shows the cap that appeares in flight between the instrument panel and the wall due to the fact that the aircraft got longer from the high skin temperature at Mach 2.

1.30) When Concorde flies from Europe to the USA, a sun rise can be seen in the West! Why?

The sun appears to move from east to west over the earth. Concorde flies faster around the earth than the sun and catches up with it.

1.31) Technology readiness levels (TRLs) are a method for estimating the maturity of technologies. How many levels are defined?

9

1.32) MAMs can be used in aircraft to reduce low-frequency engine noise in the cabin. What does MAM stand for?

MAM: Mebrane-Type Accoustic Metamaterials

1.33) Write down the equation to calculate the range of a battery-electric aircraft!

$$R = \frac{m_{bat}}{m_{MTO}} \frac{1}{g} e_{bat} \eta_{elec} \eta_{prop} E$$

1.34) Who is the entrepreneur behind Space X? Who is the entrepreneur behind Blue Origin?

Space X: Elon Musk
Blue Origin: Jeff Bezos

1.35) We have a new buzz word: "disruptive technology". What does it mean?

"Disruptive Technologien sind Innovationen, die die Erfolgsserie einer bereits bestehenden Technologie, eines bestehenden Produkts oder einer bestehenden Dienstleistung ersetzen oder diese vollständig vom Markt verdrängen."

https://www.ipt.fraunhofer.de/de/kompetenzen/Technologiemanagement/disruptive-technologien.html

"In business theory, a disruptive innovation [technology] is an innovation that creates a new market and value network and eventually disrupts an existing market and value network, displacing established market-leading firms, products, and alliances."

https://en.wikipedia.org/wiki/Disruptive_innovation

Results to task 2.1

Please insert your results here! Do not forget the units!

- Wing loading from landing field length: 615 kg/m² (523 kg/m²)
- Thrust to weight ratio from take-off field length (at wing loading from landing): 0.299
- Glide Ratio in 2. Segment: 9.65
- Glide Ratio during missed approach maneuver: 8.40
- Thrust to weight ratio from climb requirement in 2. Segment: 0.255
- Thrust to weight ratio from climb requirement during missed approach maneuver: 0.238
- V_{CR}/V_{md}: 1,00
- Design point
  - Thrust to weight ratio : 0,299
  - Wing loading: 615 kg/m²
- Cruise altitude: 11126 m = 36501 ft
- maximum take-off mass: 69776 kg
- maximum landing mass: 59310 kg
- wing area: 113 m²
- thrust of one engine **in lb**: 22991 lb
- required tank volume **in m³**: 18,9 m³

Draw the matching chart and **indicate the design point in the matching chart!**

Label your line in the legend on the right of page 6. Here is your translation:

Durchstarten	=	missed approach
Start	=	take-off
Reiseflug	=	cruise
Landing	=	landing
Steigflug	=	climb (is not required here)

1.) Peliminary Sizing I

Calculations for flight phases approach, landing, tak-off, 2nd segment and missed approach

Bold blue values represent input data.
Values based on experience are light blue. Usually you should not change these values!
Results are marked **red**. Don't change these cells!
Interim values, constants, ... are in black!
"**<<<<**" marks special input or user action.

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Data: A220-300, see summer 2019

Approach

Factor	k_{APP}	1,70	$(m/s^2)^{0.5}$
Conversion factor		1,944	kt / m/s

Given: landing field length		**yes**		**<<<< Choose according to task**
Landing field length	s_{LFL}	**1509**	**m**	
Approach speed	V_{APP}	66,1	m/s	
Approach speed	V_{APP}	**128,5**	**kt**	$V_{APP} = k_{APP} \cdot \sqrt{s_{LFL}}$

Given: approach speed		**no**		
Approach speed	V_{APP}	**128,5**	**kt**	
Approach speed	V_{APP}	66,1	m/s	$s_{LFL} = \left(\frac{V_{APP}}{k_{APP}} \right)^2$
Landing field length	s_{LFL}	**1509**	**m**	

Landing

Landing field length	s_{LFL}	1509	m		
Temperature above ISA (288,15K)	ΔT_L	**0**	**K**		
Relative density	σ	1,000			
Factor	k_L	0,107	kg/m³	$k_L = 0{,}03694\, k_{APP}^2$	
Max. lift coefficient, landing	$C_{L,max,L}$	**3,24**			
Mass ratio, landing - take-off	m_{ML} / m_{TO}	**0,850**		$m_{ML} / S_W = k_L \cdot \sigma \cdot C_{L,max,L} \cdot s_{LFL}$	0,841
Wing loading at max. landing mass	m_{ML} / S_W	**523**	**kg/m²**		1,07%
Wing loading at max. take-off mass	m_{MTO} / S_W	**615**	**kg/m²**	$m_{MTO} / S_W = \frac{m_{ML} / S_W}{m_{ML} / m_{MTO}}$	

1.) Preliminary Sizing I

$$m_{MTO} / S_W = \frac{m_{ML} / S_W}{m_{ML} / m_{MTO}}$$

Take-off

Take-off field length	s_{TOFL}	**1890**	**m**
Temperatur above ISA (288,15K)	ΔT_{TO}	**0**	**K**
Relative density	σ	1,000	
Factor	k_{TO}	2,34	m³/kg
Exprience value for $C_{L,max,TO}$	0,8 * $C_{L,max,L}$	2,592	
Max. lift coefficient, take-off	$C_{L,max,TO}$	**2,55**	
Slope	a	**0,0004855**	**kg/m³**
Thrust-to-weight ratio	T_{TO}/m_{MTO}*g at m_{MTO}/S_W calculated from landing	**0,299**	

$$a = \frac{T_{TO} / (m_{MTO} \cdot g)}{m_{MTO} / S_W} = \frac{k_{TO}}{s_{TOFL} \cdot \sigma \cdot C_{L,max,TO}}$$

2nd Segment

Calculation of glide ratio

Aspect ratio	A	**10,97**	
Lift coefficient, take-off	$C_{L,TO}$	1,77	
Lift-independent drag coefficient, clean	$C_{D,0}$ (for calculation: 2. Segment)	0,020	
Lift-independent drag coefficient, flaps	$\Delta C_{D,flap}$	0,034	
Lift-independent drag coefficient, slats	$\Delta C_{D,slat}$	0,000	
Profile drag coefficient	$C_{D,P}$	0,054	
Oswald efficiency factor; landing configuration	e	0,7	
Glide ratio in take-off configuration	E_{TO}	9,65	

n_E	sin(γ)
2	0,024
3	0,027
4	0,030

Calculation of thrust-to-weight ratio

Number of engines	n_E	**2**	
Climb gradient	sin(γ)	0,024	
Thrust-to-weight ratio	T_{TO} / m_{MTO}*g	**0,255**	

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E_{TO}} + \sin\gamma\right)$$

Missed approach

Calculation of the glide ratio

Lift coefficient, landing	$C_{L,L}$	1,92
Lift-independent drag coefficient, clean	$C_{D,0}$ (for calculation: Missed Approach)	0,020
Lift-independent drag coefficient, flaps	$\Delta C_{D,flap}$	0,041
Lift-independent drag coefficient, slats	$\Delta C_{D,slat}$	0,000
Choose: Certification basis	CS-25	**no**
	FAR Part 25	**yes**
Lift-independent drag coefficient, landing gear	$\Delta C_{D,gear}$	0,015
Profile drag coefficient	$C_{D,P}$	0,076
Glide ratio in landing configuration	E_L	8,40

	CS-25	FAR Part 25
$\Delta C_{D,gear}$	0,000	0,015

<<<< Choose according to task

n_E	$\sin(\gamma)$
2	0,021
3	0,024
4	0,027

Calculation of thrust-to-weight ratio

Climb gradient	$\sin(\gamma)$	0,021
Thrust-to-weight ratio	T_{TO} / m_{MTO}*g	**0,238**

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E_L} + \sin\gamma\right) \cdot \frac{m_{ML}}{m_{MTO}}$$

2.) Max. Glide Ratio in Curise

Estimation of k_E by means of 1.), 2.) or 3.)

<u>1.) From theory</u>		
Oswald efficiency factor for k_E	e	**0,85**
Equivalent surface friction coefficient	$C_{f,eqv}$	**0,003**
Factor	k_E	14,9
<u>2.) Acc. to RAYMER</u>		
Factor	k_E	15,8
<u>3.) From own statistics</u>		
Factor	k_E	**???**

Estimation of max. glide ratio in cruise, E_{max}

Factor	$k_{E\ chosen}$	**17,0**	**<<<< Choose according to task**
Relative wetted area	S_{wet} / S_w	**6,1**	S_{wet} / S_w = 6,0 ... 6,2
Aspect ratio	A	10,97	(from sheet 1)
Max. glide ratio	E_{max}	**22,8**	
	or		
Max. glide ratio	$E_{max\ chosen}$	**22,80**	**<<<< Choose according to task**

3.) Preliminary Sizing II

Calculations for cruise, matching chart, fuel mass, operating empty mass and aircraft parameters m_{MTO}, m_L, m_{OE}, S_W, T_{TO}, ...

Parameter		Value	
By-pass ratio	BPR	12	
Max. glide ratio, cruise	E_{max}	22,80	(from part 2)
Aspect ratio	A	10,97	(from part 1)
Oswald eff. factor, clean	e	0,85	
Zero-lift drag coefficient	$C_{D,0}$	0,014	
Lift coefficient at E_{max}	$C_{L,md}$	0,64	
Mach number, cruise	M_{CR}	0,78	

$$C_{D,0} = \frac{\pi \cdot A \cdot e}{4 \cdot E_{max}^2}$$

$$C_{L,md} = \sqrt{C_{D,0} \cdot \pi \cdot A \cdot e}$$

Parameter	Value
V/V_{md}	1,00
$C_L/C_{L,md}$	1,000
C_L	0,642
E	22,800

Jet, Theory, Optimum: 1,316074013

$$C_L / C_{L,md} = 1/(V/V_{md})^2$$

$$E = E_{max} \cdot \frac{2}{\frac{1}{\left(\frac{C_l}{C_{l,md}}\right)} + \left(\frac{C_l}{C_{l,md}}\right)}$$

Constants			
Ratio of specific heats, air	γ	1,4	
Earth acceleration	g	9,81	m/s²
Air pressure, ISA, standard	p_0	101325	Pa
Euler number	e	2,718282	

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{(T_{CR}/T_{TO}) \cdot E}$$

$$\frac{m_{MTO}}{S_W} = \frac{C_L \cdot M^2}{g} \cdot \frac{\gamma}{2} \cdot p(h)$$

Altitude		Cruise				2nd Segment	Missed appr.	Take-off	Cruise	Landing
h [km]	h [ft]	T_{CR} / T_{TO}	T_{TO} / m_{MTO}*g	p(h) [Pa]	m_{MTO} / S_W [kg/m²]	T_{TO} / m_{MTO}*g	T_{TO} / m_{MTO}*g	T_{TO} / m_{MTO}*g	T_{TO} / m_{MTO}*g	T_{TO} / m_{MTO}*g
0	0	0,415	0,106	101325	2826	0,255	0,238	1,37	0,11	
1	3281	0,391	0,112	89873	2506	0,255	0,238	1,22	0,11	
2	6562	0,367	0,120	79493	2217	0,255	0,238	1,08	0,12	
3	9843	0,343	0,128	70105	1955	0,255	0,238	0,95	0,13	
4	13124	0,319	0,138	61636	1719	0,255	0,238	0,83	0,14	
5	16405	0,294	0,149	54015	1506	0,255	0,238	0,73	0,15	
6	19686	0,270	0,162	47176	1316	0,255	0,238	0,64	0,16	
7	22967	0,246	0,178	41056	1145	0,255	0,238	0,56	0,18	
8	26248	0,222	0,197	35595	993	0,255	0,238	0,48	0,20	
9	29529	0,198	0,222	30737	857	0,255	0,238	0,42	0,22	
10	32810	0,174	0,252	26431	737	0,255	0,238	0,36	0,25	
11	36091	0,150	0,293	22627	631	0,255	0,238	0,31	0,29	
12	39372	0,126	0,349	19316	539	0,255	0,238	0,26	0,35	
13	42653	0,102	0,432	16498	460	0,255	0,238	0,22	0,43	
14	45934	0,078	0,566	14091	393	0,255	0,238	0,19	0,57	
15	49215	0,053	0,821	12035	336	0,255	0,238	0,16	0,82	
					615					0
					616					0,5
Remarks:	1m=3,281 ft	T_{CR}/T_{TO}= f(BPR,h)	Gl.(5.27)	Gl. (5.32/5.33)	Gl. (5.34)	from sheet 1.)	from sheet 1.)	from sheet 1.)	Repeat for plot	from sheet 1.)

3.) Preliminary Sizing II

Wing loading	m_{MTO} / S_W	**615**	**kg/m²**	**<<<< Read design point from matching chart!**
Thrust-to-weight ratio	$T_{TO} / (m_{MTO} \cdot g)$	**0,299**		<<<< Given data is correct when take-off and landing is sizing the aircraft at the same time.
Thrust ratio	$(T_{CR}/T_{TO})_{CR}$	0,147		
Conversion factor	m -> ft	0,305	m/ft	
Cruise altitude	h_{CR}	**11126**	**m**	
Cruise altitude	h_{CR}	**36501**	**ft**	
Temperature, troposphere	$T_{Troposphäre}$	215,83	K	$T_{Stratosphäre}$ 216,65 K
Temperature, h_{CR}	$T(h_{CR})$	216,65		
Speed of sound, h_{CR}	a	295	m/s	
Cruise speed	V_{CR}	**230**	**m/s**	
Conversion factor	NM -> m	1852	m/NM	
Design range	R	**3350**	**NM**	
Design range	R	6204200	m	
Distance to alternate	$s_{to\_alternate}$	200	NM	
Distance to alternate	$s_{to\_alternate}$	370400	m	
Chose: FAR Part121-Reserves?	domestic	**yes**		
	international	**no**		
Extra-fuel for long range		5%		
Extra flight distance	s_{res}	370400	m	
Spec.fuel consumption, cruise	SFC_{CR}	**1,12E-05**	**kg/N/s**	typical value 1,60E-05 kg/N/s
Breguet-Factor, cruise	B_s	47768010	m	
Fuel-Fraction, cruise	$M_{ff,CR}$	0,878		
Fuel-Fraction, extra fliht distance	$M_{ff,RES}$	0,992		
Loiter time	t_{loiter}	2700	s	
Spec.fuel consumption, loiter	SFC_{loiter}	**1,12E-05**	**kg/N/s**	
Breguet-Factor, flight time	B_t	207515	s	
Fuel-Fraction, loiter	$M_{ff,loiter}$	0,987		
Fuel-Fraction, engine start	$M_{ff,engine}$	**0,990**	**<<<< Copy**	
Fuel-Fraction, taxi	$M_{ff,taxi}$	**0,990**	**<<<< values**	
Fuel-Fraction, take-off	$M_{ff,TO}$	**0,995**	**<<<< from**	
Fuel-Fraction, climb	$M_{ff,CLB}$	**0,980**	**<<<< table**	
Fuel-Fraction, descent	$M_{ff,DES}$	**0,990**	**<<<< on the**	
Fuel-Fraction, landing	$M_{ff,L}$	**0,992**	**<<<< right !**	

Reserve flight distance:

FAR Part 121	s_{res}	
domestic	370400	m
international	680610	m

Extra time:

FAR Part 121	t_{loiter}	
domestic	2700	s
international	1800	s

Phase	M_{ff} per flight phases [Roskam]	
	transport jet	business jet
engine start	0,990	0,990
taxi	0,990	**0,995**
take-off	0,995	0,995
climb	0,980	0,980
descent	0,990	0,990
landing	0,992	0,992

3.) Preliminary Sizing II

Fuel-Fraction, standard flight	$M_{ff,std}$	0,841					
Fuel-Fraction, all reserves	$M_{ff,res}$	0,950					
Fuel-Fraction, total	M_{ff}	0,799					
Mission fuel fraction	m_F/m_{MTO}	0,201					
Realtive operating empty mass	m_{OE}/m_{MTO}	0,541		acc. to Loftin			
Realtive operating empty mass	m_{OE}/m_{MTO}	xxx		from statistics (if given)			
Realtive operating empty mass	m_{OE}/m_{MTO}	**0,531**		**<<<< Choose according to task**			
Choose: type of a/c	short / medium range	**yes**		**<<<< Choose according to task**			
	long range	**no**					

in kg	Short- and Medium Range	Long Range
m_{PAX}	93,0	97,5

Mass: Passengers, including baggage	m_{PAX}	93,0	kg				
Number of passengers	n_{PAX}	**160**					
Cargo mass	m_{cargo}	**3831**	**kg**				
Payload	m_{PL}	**18711**	**kg**	18711	kg	0,00%	Source: https://en.wikipedia.org/wiki/Airbus_A220
Max. Take-off mass	m_{MTO}	**69776**	**kg**	69853	kg	-0,11%	
Max. landing mass	m_{ML}	**59310**	**kg**	58740	kg	0,97%	
Operating empty mass	m_{OE}	**37051**	**kg**	37081	kg	-0,08%	
Mission fuel fraction, standard flight	m_F	**14014**	**kg**	17726	kg	-20,94%	This table calculates for max. payload!
Wing area	S_w	**113**	**m²**	112,3	m²	0,96%	
Take-off thrust	T_{TO}	204546	N	**all engines together**			
T-O thrust of ONE engine	T_{TO} / n_E	**102**	**kN**	103,6	kN	-1,28%	
T-O thrust of ONE engine	T_{TO} / n_E	22991	lb	**one engine**			
Fuel mass, needed	$m_{F,req}$	15124	kg				
Fuel density	ρ_F	800	kg/m³				
Fuel volume, needed	$V_{F,req}$	**18,9**	**m³**	(check with tank geometry later on)			
Max. Payload	m_{MPL}	**18711**	**kg**				
Max. zero-fuel mass	m_{MZF}	55762	kg				
Zero-fuel mass	m_{ZF}	55762	kg				
Fuel mass, all reserves	$m_{F,res}$	3471	kg				

Check of assumptions	check:	m_{ML}	>	$m_{ZF} + m_{F,res}$	**?**
		59310 kg	>	59233 kg	
			yes		
			Aircraft sizing finished!		

Task 2.2

Wing span	b_W	35,3 m	$b_W = (A*S)^{(1/2)}$	35,1 m	0,47%

Comment: The wing span is less than 36 m and fits as such in ICAO Aerodrome Reference Code C

Task 2.3 a)

Take-off:

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{k_{TO}}{s_{TOFL} \cdot \sigma \cdot C_{L,max,TO}} \cdot \frac{m_{MTO}}{S_W}$$

Landing:

$$\frac{m_{MTO}}{S_W} = \frac{k_L \cdot \sigma \cdot C_{L,max,L} \cdot s_{LFL}}{m_{ML}/m_{MTO}}$$

$$\boxed{\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{k_{TO} \cdot k_L}{\frac{m_{ML}}{m_{MTO}} \cdot \frac{C_{L,max,TO}}{C_{L,max,L}} \cdot \frac{s_{TOFL}}{s_{LFL}}}}$$

b) We desire a low thrust-to-weight ratio.

c) For b) we desire a large ratio m_{ML}/m_{MTO}.

d) Landing field length s_{LFL} is fixed from its requirements or from required approach speed V_{APP}. s_{TOFL} should be as large as possible, i.e. a factor $\frac{s_{TOFL}}{s_{LFL}} \gg 1$. It may be questioned why $s_{TOFL} > s_{LFL}$. With this: $s_{TOFL} = s_{LFL}$

e) $C_{L,max,TO}/C_{L,max,L}$ should be as large as possible, but is limited by 2. segment requirements. I.e.: $C_{L,max,TO}/C_{L,max,L} \leq 1$ often it is 0.8.

Move to Task 2.3

$$k_{TO} \cdot k_L = 2.34 \, m^3/kg \cdot 0.107 \, kg/m^3 = 0.25$$

Table 5.3

range	m_{ML}/m_{MTO}
short	0,93
medium	0,89
long	0,78
ultra long	0,71

A/C	s_{TOFL}/s_{LFL}
Airbus	≈ 1,5
Boeing	≈ 1.5
DC/MD	≈ 1.5
IL/Tu	≈ 1.25
Regio Jets	≈ 1.25

Source: Jenkinson A/C Data evaluated

→ See Excel: Thrust-to-Weight Ratio_from_Basic_Parameters .xlsx
PDF: .pdf

Thrust-toWeight Ratio from Basic Parameters

mML/mMTO	C,L,max,TO/C,L,max,L	STOFL/SLFL	TTO/(MMTO g)
0,93	0,8	1,5	0,224
0,89	0,8	1,5	0,234
0,78	0,8	1,5	0,267
0,71	0,8	1,5	0,293
0,93	0,8	1,0	0,336
0,89	0,8	1,0	0,351
0,78	0,8	1,0	0,401
0,71	0,8	1,0	0,440
0,93	1,0	1,5	0,179
0,89	1,0	1,5	0,187
0,78	1,0	1,5	0,214
0,71	1,0	1,5	0,235
0,93	1,0	1,0	0,269
0,89	1,0	1,0	0,281
0,78	1,0	1,0	0,321
0,71	1,0	1,0	0,352
		average	0,286

Task 2.4

$$b_s = b / \cos \varphi_{50} \qquad (10.2)$$

$$m_W = m_{MZF} \cdot 4.9 \cdot 10^{-3} \cdot b_s^{0,75} \cdot n_{ult}^{0,55} \left(\frac{b_s}{t_r}\right)^{0.3} \cdot \left(\frac{m_{MTO}}{S_W}\right)^{-0.3} \qquad (10.5)$$

↑ ignoring the small quantity $\left(1 + \sqrt{b_{ref}/b_s}\right)$

t_r = const

S_W = const

m_{MTO}/S_W = const m_{MTO} = const, also

assume: m_{MZF} = const

$$\frac{m_{W1}}{m_{W2}} = \left(\frac{b_{S1}}{b_{S2}}\right)^{1,05}$$

If $\cos \varphi_{50}$ = const :

$$\frac{m_{W1}}{m_{W2}} = \left(\frac{b_1}{b_2}\right)^{1,05} \qquad A = \frac{b^2}{S} \qquad b = (AS)^{1/2}$$

$$\boxed{\frac{m_{W1}}{m_{W2}} = \left(\frac{A_1}{A_2}\right)^{0,525} \approx \sqrt{\frac{A_1}{A_2}}}$$

i.e. wing mass increases with the square root of the aspect ratio

- If we assume that $\varphi_{25} = \text{const}$:

$$\tan \varphi_{50} = \tan \varphi_{25} - \frac{4}{A}\left[\frac{50-25}{100} \cdot \frac{1-\lambda}{1+\lambda}\right]$$

with $\lambda = 1$

$$\tan \varphi_{50} = \tan \varphi_{25}$$

and we have the same result as before.

- Check assumption : $m_{MZF} = \text{const}$?

$$m_{MTO} = m_{OE} + m_F + m_{MPL}$$

$$= m_{MZF} + m_F$$

$$m_{MZF} = m_{OE} + m_{MPL}$$

A larger aspect ratio A can lead to a heavier wing and as such a larger m_{OE}, reducing fuel mass at the same time. Therefore, m_{MZF} may increase with m_W. However, $m_W \ll m_{MZF}$ and changes in m_{MZF} are small.

Task 2.5: Snow Ball Factor

DmL	1	kg			
mMTO	**73000**	kg			
mOE/mMTO	**0,5**				
mOE	36500	kg			
mF/mMTO	**0,25**				
mF	18250	kg			
mMPL	18250	kg	Check, mMTO	73000	kg

Iteration

	Column D		
=mMPL+mOE+DmL+mF	73001	kg	Line 14
=mMPL+mOEmMTO*D14+DmL+mFmMTO*D14	73001,75	kg	Line 15
	73002,3125	kg	Line 16
=mMPL+mOEmMTO*D16+DmL+mFmMTO*D16	73002,73438	kg	Line 17
	73003,05078	kg	
	73003,28809	kg	
	73003,46606	kg	
	73003,59955	kg	
	73003,69966	kg	
	73003,77475	kg	
	73003,83106	kg	
	73003,87329	kg	
	73003,90497	kg	
	73003,92873	kg	
	73003,94655	kg	
	73003,95991	kg	
	73003,96993	kg	
	73003,97745	kg	
mMTO,final - mMTO,initial	3,97744916	kg	
k_SB	3,98	Snow Ball Factor, k_SB	