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Tutorial Questions with Solutions

Flight Mechanics

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1 Introduction to Flight Mechanics and the ISA

1.1 An aircraft cruises at a calibrated airspeed of 320 kt in FL 200. The outside air temperature is -23 °C.

a) Calculate the air pressure *p* in FL 200.
b) Calculate the air density ρ in FL 200 under given conditions.
c) Determine the equivalent airspeed EAS from a suitable diagram.
d) Calculate the equivalent airspeed EAS.
e) Calculate the true airspeed TAS.
f) Calculate the Mach number.

Solution

Given: $h_p = 20000 \text{ ft}$

$\text{CAS} = \text{V}_\text{C} = 320 \text{ kt}$

$\text{t} = \text{-}23\,°\text{C} \qquad T = 250.15 \text{ K}$

a) $$p = p_0\left(1-\frac{L}{T_0}\cdot h_p\right)^{5.25588} = 101325\text{Pa}\left(1\text{-}\frac{1.9812\cdot 10^{-3}\,\text{K}}{\text{ft}\cdot 288.15\text{K}}\cdot 20000\text{ft}\right)^{5.25588} = 46563\text{Pa}$$

Answer: $p = 466 \text{ hPa}$.

b) $$\rho = \frac{p}{R\cdot T} = \frac{46563\text{N}\cdot\text{kg}\cdot\text{K}}{\text{m}^2\cdot 287.053\text{Nm}\cdot 250.15\text{K}} = 0.6485\,\frac{\text{kg}}{\text{m}^3}$$

Answer: $\rho = 0.649\,\frac{\text{kg}}{\text{m}^3}$.

c) $\Delta V_C = 9.5 \text{ kt} \qquad V_E = V_C - \Delta V_C = 310.5 \text{ kt}$

Answer: $\text{EAS} = \text{V}_\text{E} = 311 \text{ kt}$.

d) $\delta = p / p_0 = 46563 / 101325 = 0.45954$

$$\text{V}_\text{E} = a_0\sqrt{5\delta\left\{\left[\frac{1}{\delta}\left\{\left[1+0.2\left(\frac{V_C}{a_0}\right)^2\right]^{3.5}-1\right\}+1\right]^{\frac{1}{3.5}}-1\right\}}$$

$$V_E = 661.48\ \text{kt} \sqrt{5 \cdot 0.45954 \cdot \left\{ \left[\frac{1}{0.45954} \left\{ \left[1 + 0.2 \left(\frac{320}{661.48} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right]^{\frac{1}{3.5}} - 1 \right\}} = 310.5\ \text{kt}$$

Answer: $\text{EAS} = V_E = 311\ \text{kt}$.

e) $\sigma = \rho / \rho_0 = 0.6485 / 1.225 = 0.5294$

$V = V_E / \sqrt{\sigma} = 310.5\ \text{kt} / \sqrt{0.5294} = 426.7\ \text{kt}$

Answer: $\text{TAS} = V = 427\ \text{kt}$.

f) $a = \sqrt{\gamma R T} = \sqrt{1.4 \cdot 287.053 \frac{\text{Nm}}{\text{kg} \cdot \text{K}} \cdot 250.15\ \text{K}} = 317.06\ \text{m/s} = 1141.4\ \text{km/h} = 616.32\ \text{kt}$

$M = \frac{V}{a} = \frac{426.7}{616.32} = 0.6923$

Answer: $M = 0.69$.

1.2 The map shows the elevation of a mountain: 10000 ft. An airplane passes an airport near the mountain in FL 100. A manual indicates the elevation of the airport: 4000 ft. The *Automatic Terminal Information Service* (ATIS) of the airport reports a temperature of 0°C and QNH 993 hPa.

How many feet is the airplane above or below the mountain top?

Solution

Given:	mountain:	$h = 10000\ \text{ft}$		
	airplane:	$h_p = 10000\ \text{ft}$		
	airport:	$h = 4000\ \text{ft}$,	$t = 0\ °\text{C}$,	$T = 273.15\ \text{K}$
	mean sea level:	$h = 0\ \text{ft}$,	$p = 993\ \text{hPa}$	

- Airport data used to check if ISA conditions exist:

$$H = \frac{r_{earth} \cdot h}{r_{earth} + h} = 3999\ \text{ft} \qquad \text{with} \quad r_{earth} = 2.0902 \cdot 10^7\ \text{ft}$$

(In the future we could save time. In most cases $H \approx h$ will be of enough accuracy.)

$$T_{ISA}(h = 4000 \text{ ft}) = \text{T}_0 - L \cdot H = 288.15 \text{ K} - 1.9812 \cdot 10^{-3} \text{ K / ft} \cdot 3999 \text{ ft} = 280.22 \text{ K}$$

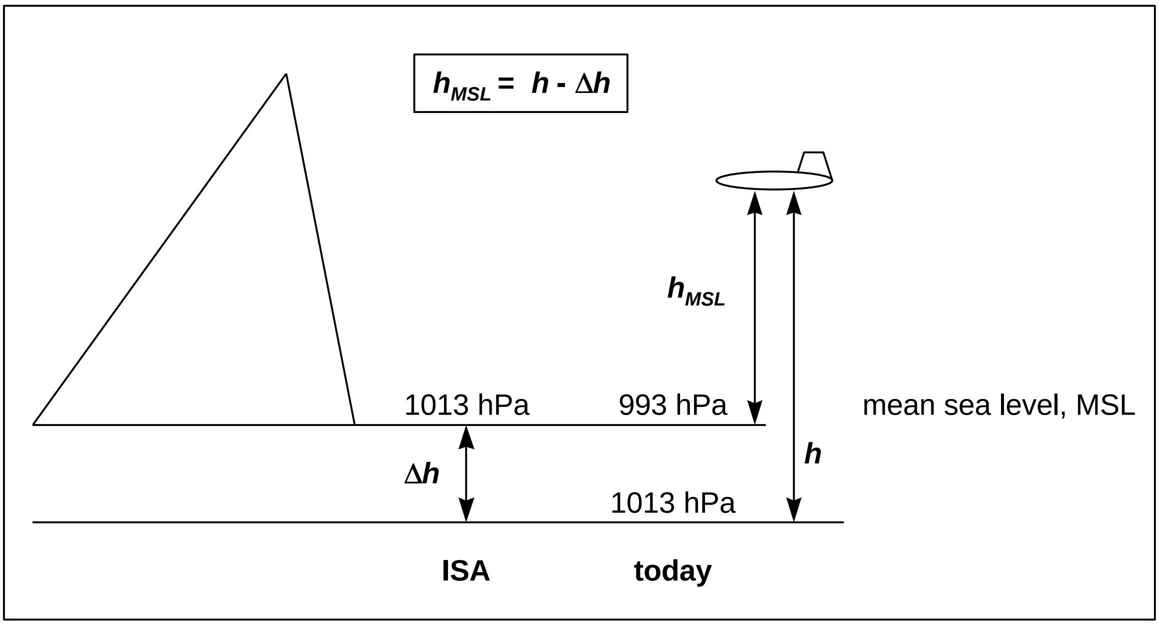
$$\Delta T = T - T_{ISA} = 273.15 \text{ K} - 280.22 \text{ K} = -7.08 \text{ K}$$

The temperature at the present day is 7 °C below ISA standard conditions.

- Calculating the geometric height of the airplane above mean sea level (MSL):

$$H = h_p \cdot \frac{T_0 + \Delta T}{T_0} = 10000 \text{ ft} \cdot \frac{288.15 \text{ K} - 7.08 \text{ K}}{288.15 \text{ K}} = 9754.5 \text{ ft}$$

$$h = \frac{r_{earth} \cdot H}{r_{earth} - H} = 9759 \text{ ft}$$ geometric height above a level with $p = 1013.25 \text{ hPa}$



On this present day:

$$\Delta h \approx \Delta H = H = \frac{T_0 + \Delta T}{L}\left(1 - \delta^{\frac{1}{5.25588}}\right)$$

$$= \frac{288.15 \text{ K} - 7.08 \text{K}}{1.9812 \cdot 10^{-3} \text{ K}} \text{ft} \cdot \left(1 - \left(\frac{993}{1013.25}\right)^{\frac{1}{5.25588}}\right) = 544 \text{ ft}$$

$$h_{MSL} = h - \Delta h = 9759 \text{ ft} - 544 \text{ ft} = 9215 \text{ ft}$$

Answer: The airplane is flying 10000 ft - 9215 ft = 785 ft below the mountain top.

1.3 On 01.02.99 the news group *sci.aeronautics.airliners* lists an article in which a rule of thumb is presented to calculate the true air speed:

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Example conditions:     Altitude 35,000',
                        IAS 280 knots
                        TAT -15°C

Altitude (1,000's) x 6  35 x 6 = 210
+IAS            210 + 280 = 490
+ TAT           490 + (-15) = 475 KTAS
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a) Calculate the true air speed for the above conditions from physical principles.

b) Compare your results with the true airspeed given by the rule of thumb. Calculate absolute and relative error.

c) Several more checks of the same nature would be necessary for different altitudes, air speeds and total air temperatures (TAT) to show the overall accuracy of the rule of thumb. Use the computer with a suitable software tool for this task and present your results in tabular and graphical form.

Solution

Given: $h = 35000\text{ ft}$

$T_T = 258.15\text{ K}$

$V_I = 280\text{ kt} \approx \text{V}_\text{C}$

This is an iterative solution. A good first value for M may be obtained from Fig. 1.5 (Compressibility correction to calibrated airspeed) in "Unterlagen zur Vorlesung Flugmechanik 1".

a) $$T = \frac{T_T}{1+0.2\cdot M^2} = \frac{258.15\text{K}}{1+0.2\cdot 0.82135^2} = 227.46\text{K}$$

$$T_{ISA}(35000\text{ ft}) = T_0 - LH = 288.15\text{K} - 1.9812\cdot 10^{-3}\frac{\text{K}}{\text{ft}}\cdot 35000\text{ ft} = 218.81\text{ ft}$$

$$\Delta T = T - T_{ISA} = 277.46\text{K} - 218.81\text{K} = 8.65\text{K}$$

For non-ISA conditions and approximating $h \approx H$:

$$\delta = \frac{p}{p_0} = \left(1 - \frac{L}{T_0 + \Delta T}\cdot H\right)^{5.25588} = \left(1 - \frac{1.9812\cdot 10^{-3}\text{K/ft}}{288.15\text{K} + 8.65\text{K}}\cdot 35000\text{ft}\right)^{5.25588} = 0.24696$$

$$M = \sqrt{5\left\{\left[\frac{1}{\delta}\left\{\left[1+0.2\left(\frac{V_C}{a_0}\right)^2\right]^{3.5} - 1\right\} + 1\right]^{\frac{1}{3.5}} - 1\right\}}$$

$$M = \sqrt{5 \cdot \left\{ \left[\frac{1}{0.24696} \left\{ \left[1 + 0.2 \left(\frac{280}{661.48} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right]^{\frac{1}{3.5}} - 1 \right\}} = 0.80441$$

$$a = \sqrt{\gamma R T} = \sqrt{1.4 \cdot 287.053 \frac{\mathrm{Nm}}{\mathrm{kg} \cdot \mathrm{K}} \cdot 227.46\ \mathrm{K}} = 302.34\ \mathrm{m/s} = 587.70\ \mathrm{kt}$$

$$V = M \cdot a = 0.80441 \cdot 587.70\ \mathrm{kt} = 472.7\ \mathrm{kt}$$

Answer: TAS = V = 473 kt .

b) Answer: The absolute error is 475 kt - 473 kt = + 2 kt. The relative error is +0.4 %
This is an amazingly good result for a rule of thumb.

c) The solution is left to the student.

2 Definitions and Aerodynamic Fundamentals

2.1 An aircraft is equipped with a wing of symmetrical airfoils. The lift curve slope of the total aircraft is estimated to be $\frac{\partial C_L}{\partial \alpha} = 0.8 \cdot 2\pi \frac{1}{\text{rad}}$. The stall angle of attack (AOA) is 12°. Wing area is 16 m². Use $g = g_0$ and $\rho = \rho_0$.

What is the aircraft's mass during a flight on which a stall speed of 50 kt was observed.

Solution

Given: $\frac{\partial C_L}{\partial \alpha} = 5.0265 \frac{1}{\text{rad}}$ $\alpha_{max} = 12° = 0.20944 \text{ rad}$

$V = 50 \text{ kt} = 92.6 \text{ km/h} = 25.72 \text{ m/s}$

$S_W = 16 \text{ m}^2$

$$C_{L,max} = \frac{\partial C_L}{\partial \alpha} \cdot \alpha_{max} = 5.0265 \frac{1}{\text{rad}} \cdot 0.20944 \text{ rad} = 1.05275$$

$$L = \frac{1}{2}\rho V^2 \cdot C_{L,max} \cdot S_W = \frac{1}{2}\rho_0 V^2 \cdot C_{L,max} \cdot S_W = W = m \cdot g = m \cdot g_0$$

$$m = \frac{\rho_0 \cdot V^2}{2 g_0} \cdot C_{L,max} \cdot S_W = \frac{1.225 \text{ kg} \cdot \text{s}^2 \cdot 25.72^2 \cdot \text{m}^2}{\text{m}^3 \cdot 2 \cdot 9.80665 \cdot \text{m} \cdot \text{s}^2} \cdot 1.05275 \cdot 16 \text{ m} = 696 \text{ kg}$$

Answer: The aircraft's mass is 696 kg.

5 Level, Climbing and Descending Flight

5.1 We consider two types of planes: a motor glider and a single engine piston prop general aviation aircraft. For each type, generic data is given as presented below.

	motor glider	single engine aircraft
P_S / *MTOW*	7 W/N	12 W/N
E	30	10

Consider a climb directly after take-off with MTOW at a speed of 100 km/h. Assume a propeller efficiency η_P = 0.8.

Calculate and compare the climb gradient, γ and the rate of climb, ROC of both airplane categories! Assume small angle approximation.

Solution

Small angle approximation: $L = W$

$$P_T = \eta_P \cdot P_S = T \cdot V \qquad \Rightarrow \qquad T = \frac{\eta_P \cdot P_S}{V}$$

$$\sin\gamma = \frac{T}{W} - \frac{D}{W} = \frac{T}{W} - \frac{D}{L} = \frac{T}{W} - \frac{1}{E} = \frac{\eta_P \cdot P}{V \cdot W} - \frac{1}{E}$$

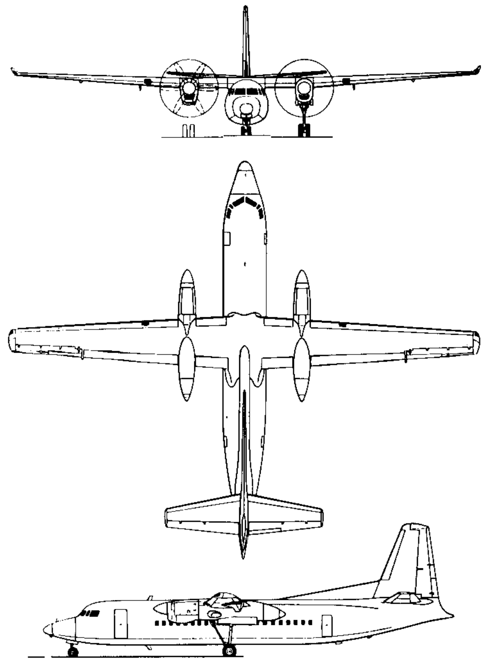
$$ROC = V \cdot \sin\gamma$$

	motor glider	single engine aircraft
γ	9.7°	14.2°
ROC	4.7 m/s = 920 ft/min	6.8 m/s = 1340 ft/min

Answer: The single engine aircraft shows a lift-to-drag ratio which is much less than that of the motor glider, however its power-to-weight ratio is higher. The combined effect reveals a steeper climb as well as a higher rate of climb for the single engine general aviation aircraft.

5.2 Jane's all the Worlds Aircraft from 1993 lists data of the Fokker 50 Series 100:

- wing span 29 m
- wing area 70 m²
- MTOW 19950 kg
- max. operating altitude 25000 ft
- power plant: two Pratt & Whitney Canada PW125B turboprops, each flat rated at 1864 kW (2500 shp) at S/L.



Fokker 50

Abbreviations:

- shp shaft horse power
- S/L sea level.

For cruise conditions, it can be assumed:

- maximum lift-to-drag ratio, E_{max} = 16
- Oswald efficiency factor, e = 0.85
- propeller efficiency, η_P = 0.8 .

a) Calculate the aspect ratio, A.

b) Calculate the drag coefficient at zero lift, C_{D0}.

c) Calculate the aircraft's equivalent power, P_e at seal level conditions. Assume the ratio of jet thrust to propeller thrust to be 0.15 .

d) Calculate the equivalent power at maximum operating altitude. Assume the variation of power with height is given by $P_e \propto \sigma^{0.5}$. σ is the relative density.

e) Derive an equation for level steady flight which can be used to calculate the maximum cruising speed.

f) Simplify the equation from task e) and calculate an approximation of the maximum cruising speed at max. operating altitude and MTOW.

g) Complete the table below and draw a graph of the data. Consider max. operating altitude and MTOW.

equivalent airspeed, V_E	trust, T	drag, D
15 m/s		
25 m/s		
60 m/s		
115 m/s		

h) Calculate the maximum cruising speed at max. operating altitude and MTOW.

i) Comment on the feasibility of flying on the "back side of the power curve" under given conditions.

Solution

a) $A = \frac{b^2}{S} = \frac{29^2 \text{ m}^2}{70 \text{ m}^2} = 12.01$

b) $E_{max} = \frac{1}{2}\sqrt{\frac{\pi A e}{C_{D0}}} \quad \Rightarrow \quad C_{D0} = \frac{\pi A e}{4\, E_{max}{}^2} = \frac{\pi \cdot 12.01 \cdot 0.85}{4 \cdot 16^2} = 0.031319$

c) Considering the aircraft's two engines (n_E = 2) at sea level conditions:

$$\left(P_e\right)_{S/L} = n_E \cdot P_S \cdot \left(1 + \frac{T_J}{T_P}\right) = 2 \cdot 1864 \text{ kW} \cdot (1 + 0.15) = 4287.2 \text{ kW}$$

d) with $\sqrt{\sigma(25000 \text{ ft})}$ = 0.66942 from the ISA table:

$$P_e(25000 \text{ ft}) = \left(P_e\right)_{S/L} \cdot \sqrt{\sigma(25000 \text{ ft})} = 4287.2 \text{ kW} \cdot 0.66942 = 2869.9 \text{ kW}$$

e) $$D = A_1 \cdot V_E{}^2 + B_1 \cdot V_E{}^{-2} \qquad \textbf{(1)}$$

with $A_1 = \frac{C_{D0} \cdot \rho_0 \cdot S}{2}$ = 1.3428 kg/m

and $B_1 = \frac{2 \cdot m^2 \cdot g^2}{\pi \cdot A \cdot e \cdot \rho_0 \cdot S}$ = 2.786 $\frac{\text{kg} \cdot \text{m}^3}{\text{s}^4}$

$$T = \frac{\eta_P \cdot P_e}{V} = \frac{\eta_P \cdot P_e \cdot \sqrt{\sigma}}{V_E} = 1.5369 \cdot 10^6 \, \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} \cdot \frac{1}{V_E} \qquad \textbf{(2)}$$

$$T = D \qquad \Rightarrow \qquad \frac{\eta_P \cdot P_e \cdot \sqrt{\sigma}}{V_E} = A_1 \cdot V_E{}^2 + B_1 \cdot V_E{}^{-2}$$

$$\eta_P \cdot P_e \cdot \sqrt{\sigma} = A_1 \cdot V_E{}^3 + B_1 \cdot V_E{}^{-1} \qquad \textbf{(3)}$$

$$A_1 \cdot V_E{}^4 - \eta_P \cdot P_e \cdot \sqrt{\sigma} \cdot V_E + B_1 = 0 \qquad \textbf{(4)}$$

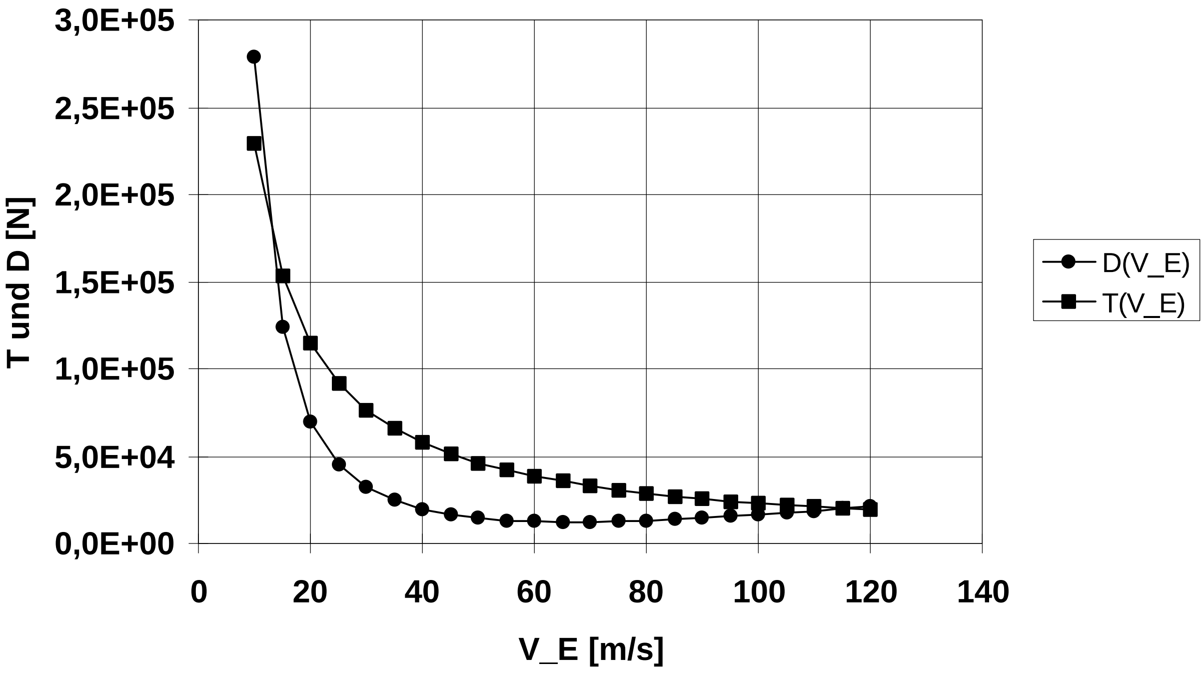
f) Dropping the last term of Equation (3):

$$V_E = \sqrt[3]{\frac{\eta_P \cdot P_e \cdot \sqrt{\sigma}}{A_1}} = \sqrt[3]{\frac{2 \cdot \eta_P \cdot P_e \cdot \sqrt{\sigma}}{C_{D0} \cdot \rho_0 \cdot S}} = \sqrt[3]{\frac{2 \cdot 0.8 \cdot 2869.9 \cdot 10^3 \text{ kg} \cdot \text{m}^2 \cdot \text{m}^3 \cdot 0.66942}{\text{s}^3 \cdot 0.031319 \cdot 1.225 \text{ kg} \cdot 70 \text{ m}^2}} = 104.6 \, \frac{\text{m}}{\text{s}}$$

$$V_E = 203.3 \text{ kt} \qquad V = \frac{V_E}{\sqrt{\sigma}} = \frac{203.3 \text{ kt}}{0.66942} = 303.7 \text{ kt}$$

g)

V_E [m/s]	$D(V_E)$ [N]	$T(V_E)$ [N]	V [kt]
10	2,7869E+05	2,2959E+05	29
15	**1,2410E+05**	**1,5306E+05**	**44**
20	7,0175E+04	1,1480E+05	58
25	**4,5407E+04**	**9,1838E+04**	**73**
30	3,2158E+04	7,6532E+04	87
35	2,4383E+04	6,5599E+04	102
40	1,9557E+04	5,7399E+04	116
45	1,6473E+04	5,1021E+04	131
50	1,4497E+04	4,5919E+04	145
55	1,3268E+04	4,1745E+04	160
60	**1,2569E+04**	**3,8266E+04**	**174**
65	1,2263E+04	3,5322E+04	189
70	1,2261E+04	3,2799E+04	203
75	1,2501E+04	3,0613E+04	218
80	1,2941E+04	2,8699E+04	232
85	1,3551E+04	2,7011E+04	247
90	1,4309E+04	2,5511E+04	261
95	1,5198E+04	2,4168E+04	276
100	1,6206E+04	2,2959E+04	290
105	**1,7322E+04**	**2,1866E+04**	**305**
110	1,8540E+04	2,0872E+04	319
115	1,9854E+04	1,9965E+04	334
120	2,1259E+04	1,9133E+04	348



h) Newton iteration of Equation (4): $(V_E)_{n+1} = (V_E)_n - \frac{f((V_E)_n)}{f'((V_E)_n)}$

V_E [m/s]	f (V_E) [kg*m³/s4]	f ' (V_E) [kg*m²/s³]	V [kt]
100,000	-6,75397E+07	3,07205E+06	290,4
121,985	4,49372E+07	7,44798E+06	354,2
115,952	4,21963E+06	6,07249E+06	336,7
115,257	5,20640E+04	5,92294E+06	334,7
115,248	8,26451E+00	5,92106E+06	334,7
115,248	1,93715E-07	5,92106E+06	334,7

Answer: The **maximum cruising speed** at max. operating altitude and MTOW is **334 kt**.

Comment: Jane's all the Worlds Aircraft states: "Typical cruising speed 282 kt".

i) The graph above shows a second solution of Equation (4) at about $V_E = 15\frac{\text{m}}{\text{s}}$.

Flight at this speed would require a lift coefficient of

$$C_L = \frac{2 \cdot m \cdot g}{\rho_0 \cdot {V_E}^2 \cdot S} = \frac{2 \cdot 19500\text{kg} \cdot 9.81\text{m} \cdot \text{m}^3 \cdot \text{s}^2}{\text{s}^2 \cdot 1.225\text{kg} \cdot 15^2 \cdot \text{m}^2 \cdot 70\text{m}^2} = 19.8$$

Such lift coefficient is apparently not possible.

Answer:
Under given conditions, this power setting results in (only) one aircraft speed of 334 kt.

5.3 A jet performs a steady climb. Aircraft data is given as follows:

- drag coefficient at zero lift, C_{D0} = 0.02 ,
- wing area, S = 100 m^2 ,
- wing loading, $\frac{m}{S}$ = 700 kg/m² ,
- aspect ratio, A = 10 ,
- Oswald efficiency factor, e = 0.85 ,
- thrust to weight ratio, $\frac{T}{m \cdot g}$ = 0.2 .

a) Calculate mass and thrust of the aircraft.
b) For standard sea level conditions, plot rate of climb, ROC and climb gradient γ versus equivalent airspeed. Assume a parabolic drag polar.
c) Determine **graphically** for standard sea level conditions:
- maximum rate of climb and equivalent airspeed for the maximum rate of climb,
- maximum climb angle and equivalent airspeed for the maximum climb angle.

d) Determine numerically for standard sea level conditions:
- maximum rate of climb and equivalent airspeed for the maximum rate of climb,
- maximum climb angle and equivalent airspeed for the maximum climb angle.

Solution

a) $$m = \frac{m}{S} \cdot S = 700 \, \frac{\text{kg}}{\text{m}^2} \cdot 100 \text{ m}^2 = 70000 \text{ kg}$$

$$T = \frac{T}{m \cdot g} \cdot m \cdot g = 0.2 \cdot 70000 \text{ kg} \cdot 9.81 \, \frac{\text{N}}{\text{kg}} = 137340 \text{ N} .$$

b) $$D = A_1 \cdot V_E{}^2 + B_1 \cdot V_E{}^{-2}$$

$$\sin\gamma = \frac{T}{W} - \frac{D}{W} = \frac{T}{W} - \frac{A_1}{W} \cdot V_E{}^2 - \frac{B_1}{W} \cdot V_E{}^{-2}$$

$$\gamma = \arcsin\left(\frac{T}{W} - \frac{A_1}{W} \cdot V_E{}^2 - \frac{B_1}{W} \cdot V_E{}^{-2} \right)$$

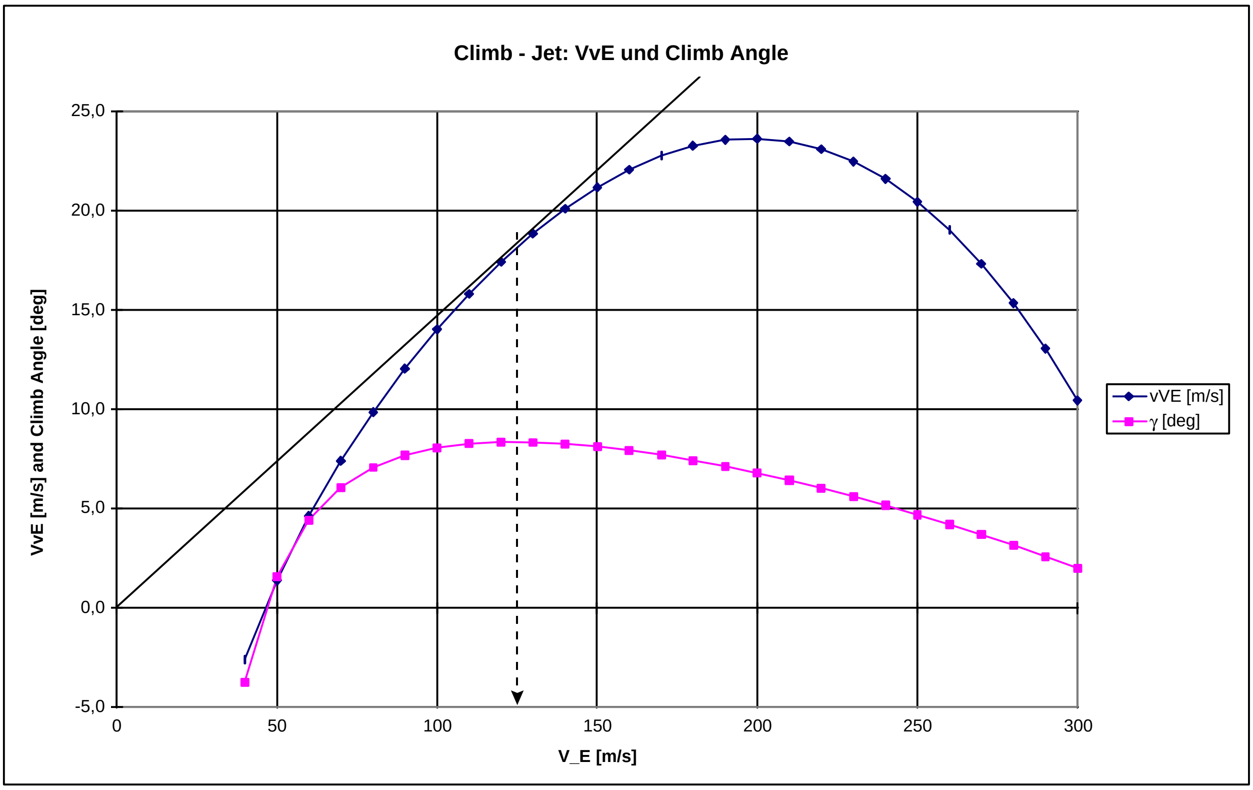
$$V_{v_E} = V_E \cdot \sin\gamma = \frac{T}{W} \cdot V_E - \frac{A_1}{W} \cdot V_E{}^3 - \frac{B_1}{W} \cdot V_E{}^{-1}$$

$$ROC = V \cdot \sin\gamma = \frac{V_E}{\sqrt{\sigma}} \cdot \sin\gamma = \frac{V_{v_E}}{\sqrt{\sigma}}$$ **for sea level conditions**: $\boxed{ROC = V_{v_E}}$

$$A_1 = \frac{C_{D0} \cdot \rho_0 \cdot S}{2} = \frac{0.02 \cdot 1.225 \text{ kg} \cdot 100 \text{ m}^2}{\text{m}^3 \cdot 2} = 1.225 \, \frac{\text{kg}}{\text{m}}$$

$$B_1 = \frac{2 \cdot m^2 \cdot g^2}{\pi \cdot A \cdot e \cdot \rho_0 \cdot S} = \frac{2 \cdot 70000^2 \text{ kg}^2 \cdot 9.81^2 \text{ N}^2 \cdot \text{m}^3}{\text{kg}^2 \cdot \pi \cdot 10 \cdot 0.85 \cdot 1.225 \text{ kg} \cdot 100 \text{ m}^2} = 2.8831 \cdot 10^8 \, \frac{\text{Nm}^2}{\text{s}^2}$$

V_E [m/s]	T [N]	V_{vE} [m/s]	γ [deg]
40	137340	-2,6	-3,7
60	137340	4,6	4,4
80	137340	9,8	7,1
100	137340	14,0	8,1
120	137340	17,4	8,3
140	137340	20,1	8,3
160	137340	22,1	7,9
180	137340	23,3	7,4
200	137340	23,6	6,8
220	137340	23,1	6,0
240	137340	21,6	5,2
260	137340	19,0	4,2
280	137340	15,3	3,1
300	137340	10,4	2,0



c)
- maximum rate of climb: 23.6 m/s = 4650 ft/min ,
- equivalent airspeed for the maximum rate of climb: 200 m/s = 389 kt ,
- maximum climb angle: 8.4 ° ,
- equivalent airspeed for the maximum climb angle: 125 m/s = 243 kt .

d) For a jet, the equivalent airspeed for the maximum climb angle is the minimum drag speed $V_{E,md}$:

$$V_{E,md} = \left[\frac{B_1}{A_1}\right]^{1/4} = \left[\frac{2.8831 \cdot 10^8 \, \text{N m}^2 \, \text{m}}{\text{s}^2 \cdot 1.225 \text{kg}}\right]^{1/4} = 123.86 \frac{\text{m}}{\text{s}}$$

At this speed with equations form b):

$$V_{v_E} = 17.99 \frac{\text{m}}{\text{s}}$$

$$g = 8.35°$$

For a jet, the equivalent airspeed for the **maximum rate of climb** is calculated from:

$$V_E = \left[\frac{1}{6A_1}\left(T \pm \sqrt{T^2 + 12 \, A_1 \, B_1}\right)\right]^{1/2}$$

$$= \left[\frac{1 \text{m}}{6 \cdot 1.225 \text{kg}}\left(137340 \text{ N} + \sqrt{137340^2 \text{ N}^2 + 12 \cdot 1.225 \frac{\text{kg}}{\text{m}} \cdot 2.8831 \cdot 10^8 \frac{\text{Nm}^2}{\text{s}^2}}\right)\right]^{1/2} = 198.40 \frac{\text{m}}{\text{s}}$$

At this speed with equations form b):

$$V_{v_E} = 23.63 \frac{\text{m}}{\text{s}}$$

$$g = 6.84°$$

5.4 A piston-prop performs a steady climb. Aircraft data is given as follows:

- maximum lift-to-drag ratio, $E_{max} = 10$,
- wing area, $S = 16.3 \text{ m}^2$,
- aircraft mass, $m = 1200$ kg ,
- aspect ratio, $A = 7.4$,
- Oswald efficiency factor, $e = 0.74$,
- propeller efficiency, $\eta_P = 0.8$,
- shaft power, $P_S = 150$ hp = 111855 W .

a) Calculate the zero lift drag coefficient.

b) For standard sea level conditions, plot rate of climb, ROC and climb gradient γ versus equivalent airspeed. Assume a parabolic drag polar and constant propeller efficiency.

c) Determine graphically for standard sea level conditions:
   - maximum rate of climb and equivalent airspeed for the maximum rate of climb,
   - maximum climb angle and equivalent airspeed for the maximum climb angle.

d) Determine numerically for standard sea level conditions:
   - maximum rate of climb and equivalent airspeed for the maximum rate of climb,
   - maximum climb angle and equivalent airspeed for the maximum climb angle.

Solution

a) $E_{max} = \frac{1}{2}\sqrt{\frac{\pi \cdot A \cdot e}{C_{D0}}} \quad \Rightarrow \quad C_{D0} = \frac{\pi \cdot A \cdot e}{4 \cdot E_{max}{}^2} = \frac{\pi \cdot 7.4 \cdot 0.74}{4 \cdot 10^2} = 0.043$

b) $D = A_1 \cdot V_E{}^2 + B_1 \cdot V_E{}^{-2}$

$$\sin\gamma = \frac{T}{W} - \frac{D}{W} = \frac{T}{W} - \frac{A_1}{W} \cdot V_E{}^2 - \frac{B_1}{W} \cdot V_E{}^{-2}$$

$$V_{v_E} = V_E \cdot \sin\gamma = \frac{T}{W} \cdot V_E - \frac{A_1}{W} \cdot V_E{}^3 - \frac{B_1}{W} \cdot V_E{}^{-1}$$

$$T = \frac{\eta_P \cdot P_S \cdot \sqrt{\sigma}}{V_E} \qquad T \cdot V_E = \eta_P \cdot P_S \cdot \sqrt{\sigma}$$

$$V_{v_E} = \frac{\eta_P \cdot P_S \cdot \sqrt{\sigma}}{W} - \frac{A_1}{W} \cdot V_E{}^3 - \frac{B_1}{W} \cdot V_E{}^{-1}$$

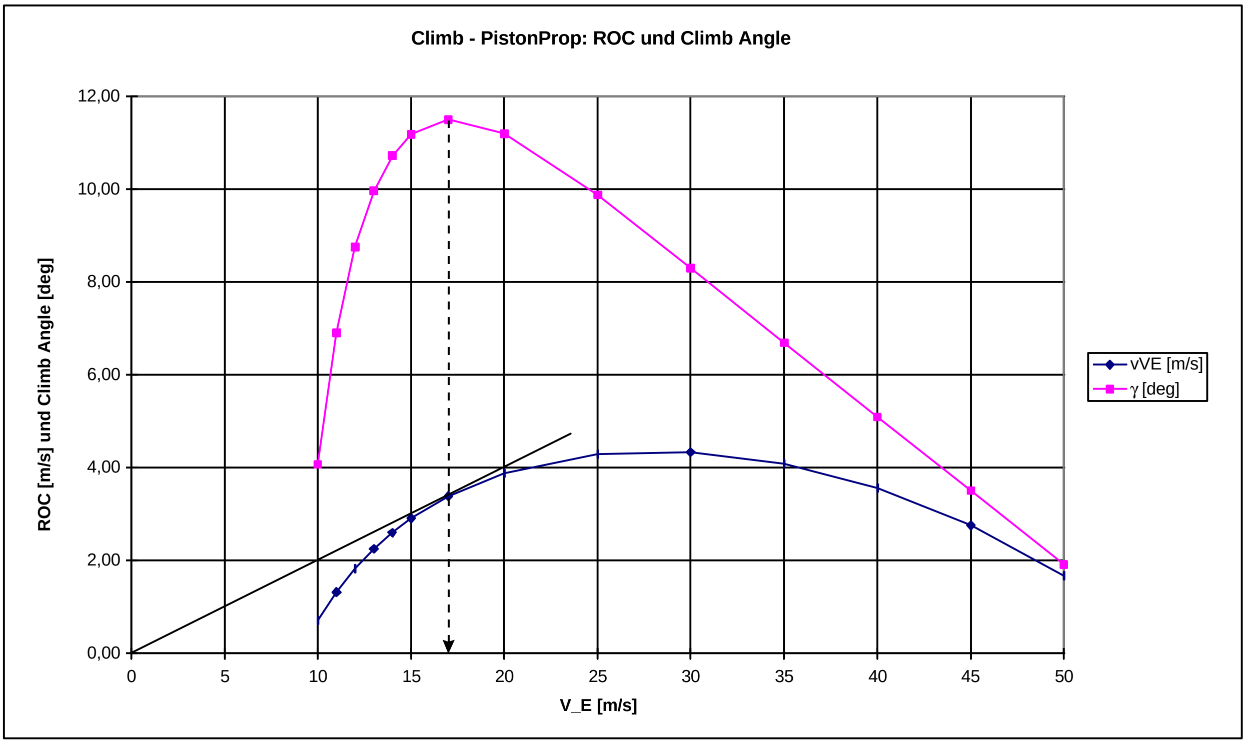
$$\gamma = \arcsin\left(\frac{\eta_P \cdot P_S \cdot \sqrt{\sigma}}{W} \cdot V_E{}^{-1} - \frac{A_1}{W} \cdot V_E{}^2 - \frac{B_1}{W} \cdot V_E{}^{-2} \right)$$

$ROC = V \cdot \sin\gamma = \frac{V_E}{\sqrt{\sigma}} \cdot \sin\gamma = \frac{V_{v_E}}{\sqrt{\sigma}}$ **for sea level conditions**: $\boxed{ROC = V_{v_E}}$

$$A_1 = \frac{C_{D0} \cdot \rho_0 \cdot S}{2} = \frac{0.043 \cdot 1.225\ \text{kg} \cdot 16.3\ \text{m}^2}{\text{m}^3 \cdot 2} = 0.429\ \frac{\text{kg}}{\text{m}}$$

$$B_1 = \frac{2 \cdot W^2}{\pi \cdot A \cdot e \cdot \rho_0 \cdot S} = \frac{2 \cdot 1200^2\ \text{kg}^2 \cdot 9.81^2\ \text{N}^2 \cdot \text{m}^3}{\text{kg}^2 \cdot \pi \cdot 7.4 \cdot 0.74 \cdot 1.225\ \text{kg} \cdot 16.3\ \text{m}^2} = 8.0685 \cdot 10^5\ \frac{\text{N m}^2}{\text{s}^2}$$

V_E [m/s]	T [N]	V_{vE} [m/s]	g [deg]
10	8948	0,71	4,1
11	8135	1,32	6,9
12	7457	1,83	8,8
13	6883	2,25	10,0
14	6392	2,61	10,7
15	5966	2,91	11,2
17	5264	3,39	11,5
20	4474	3,88	11,2
25	3579	4,29	9,9
30	2983	4,33	8,3
35	2557	4,08	6,7
40	2237	3,55	5,1
45	1989	2,75	3,5
50	1790	1,67	1,9



c)
- maximum rate of climb: 4.3 m/s = 850 ft/min ,
- equivalent airspeed for the maximum rate of climb: 30 m/s = 58 kt
- maximum climb angle: 11.5 ° ,
- equivalent airspeed for the maximum climb angle: 17 m/s = 33 kt .

  At the very low speed associated with this flight condition (33 kt), the accuracy of both the drag and power idealisations, is reduced. The aircraft will be operating near to the stall, introducing discrepancies between the parabolic drag polar and the actual drag polar. Furthermore, the assumption that shaft power and propeller efficiency do not change with speed yields a very poor approximation at this low speed condition.

d) For a piston-prop, the equivalent airspeed for the **maximum rate of climb** is the minimum drag power speed $V_{E,mp}$:

$$V_{E,mp} = \left[\frac{B_1}{3A_1}\right]^{1/4} = \left[\frac{8.0685 \cdot 10^5 \, \mathrm{N\,m^2\,m}}{\mathrm{s^2} \; 3 \cdot 0.429 \, \mathrm{kg}}\right]^{1/4} = 28.14 \frac{\mathrm{m}}{\mathrm{s}}$$

At this speed with equations form b):

$$V_{v_E} = 4.35 \frac{\mathrm{m}}{\mathrm{s}}$$

$$g = 8.9°$$

For a piston-prop, the equivalent airspeed for the **maximum climb angle** is calculated from:

$$\frac{d \sin g}{dV_E} = \frac{-h_P P_S \sqrt{s}}{W} V_E^{-2} - \frac{2A_1}{W} V_E + \frac{2B_1}{W} V_E^{-3} = 0$$

This solved iteratively (e.g. with Newton Iteration) yields: $V_E = 17.19\,\frac{\text{m}}{\text{s}}$

At this speed with equations form b):

$V_{v_E} = 3.43\,\frac{\text{m}}{\text{s}}$

$g = 11.5°$

5.5 A jet climbs with constant equivalent airspeed, V_E = 150 m/s. Aircraft data is given as in Question 5.3:

- drag coefficient at zero lift, C_{D0} = 0.02 ,
- wing area, S = 100 m^2 ,
- mass, m = 70000 kg ,
- aspect ratio, A = 10 ,
- Oswald efficiency factor, e = 0.85 ,
- thrust at sea level, T_0 = 135900 N .

a) Calculate the variation of thrust with altitude in the troposphere.
b) Calculate and plot the variation of true airspeed V with altitude in the troposphere.
c) Calculate the variation of Mach number with altitude in the troposphere.
d) Calculate and plot the variation of rate of climb, ROC and climb angle, γ .
e) Calculate the time to climb to an altitude of 8 km.
f) Calculate the absolute ceiling.

Assume:

- an ideal jet engine,
- constant aircraft mass,
- a parabolic drag polar,
- a linear variation of the rate of climb within intervals Δh = 1000 m.

Solution

a) In the troposphere, thrust is roughly proportional to density. Neglecting the difference between geopotential height and geometric height:

$$T = T_0 \cdot s = T \cdot \left(1 - 0.022558 \frac{1}{\text{km}} \cdot h\right)^{4.25588}$$

Results are given in the Table below.

b) $$V = \frac{V_E}{\sqrt{s}} = \frac{V_E}{\left(1 - 0.022558 \frac{1}{\text{km}} \cdot h\right)^{4.25588/2}}$$

Results are given in the Table below.

c) $$M = \frac{V}{a} = \frac{V}{20.0468 \frac{1}{\sqrt{\text{K}}} \cdot \frac{\text{m}}{\text{s}} \cdot \sqrt{288.15\text{K} - 6.5 \frac{\text{K}}{\text{km}} \cdot h}}$$

Results are given in the Table below.

d) Compare with solution to Question 5.3:

$$\gamma = \arcsin\left(\frac{T}{W} - \frac{A_1}{W} \cdot V_E^{\,2} - \frac{B_1}{W} \cdot V_E^{\,-2} \right)$$

$$V_{v_E} = V_E \cdot \sin\gamma = \frac{T}{W} \cdot V_E - \frac{A_1}{W} \cdot V_E^{\,3} - \frac{B_1}{W} \cdot V_E^{\,-1}$$

$$ROC = V \cdot \sin\gamma = \frac{V_E}{\sqrt{\sigma}} \cdot \sin\gamma = \frac{V_{v_E}}{\sqrt{\sigma}}$$

$$A_1 = \frac{C_{D0} \cdot \rho_0 \cdot S}{2} = \frac{0.02 \cdot 1.225\,\text{kg} \cdot 100\,\text{m}^2}{\text{m}^3 \cdot 2} = 1.225\,\frac{\text{kg}}{\text{m}}$$

$$B_1 = \frac{2 \cdot m^2 \cdot g^2}{\pi \cdot A \cdot e \cdot \rho_0 \cdot S} = \frac{2 \cdot 70000^2\ \text{kg}^2 \cdot 9.81^2\ \text{N}^2 \cdot \text{m}^3}{\text{kg}^2 \cdot \pi \cdot 10 \cdot 0.85 \cdot 1.225\,\text{kg} \cdot 100\,\text{m}^2} = 2.8831 \cdot 10^8\,\frac{\text{Nm}^2}{\text{s}^2}$$

Results are given in the Table below.

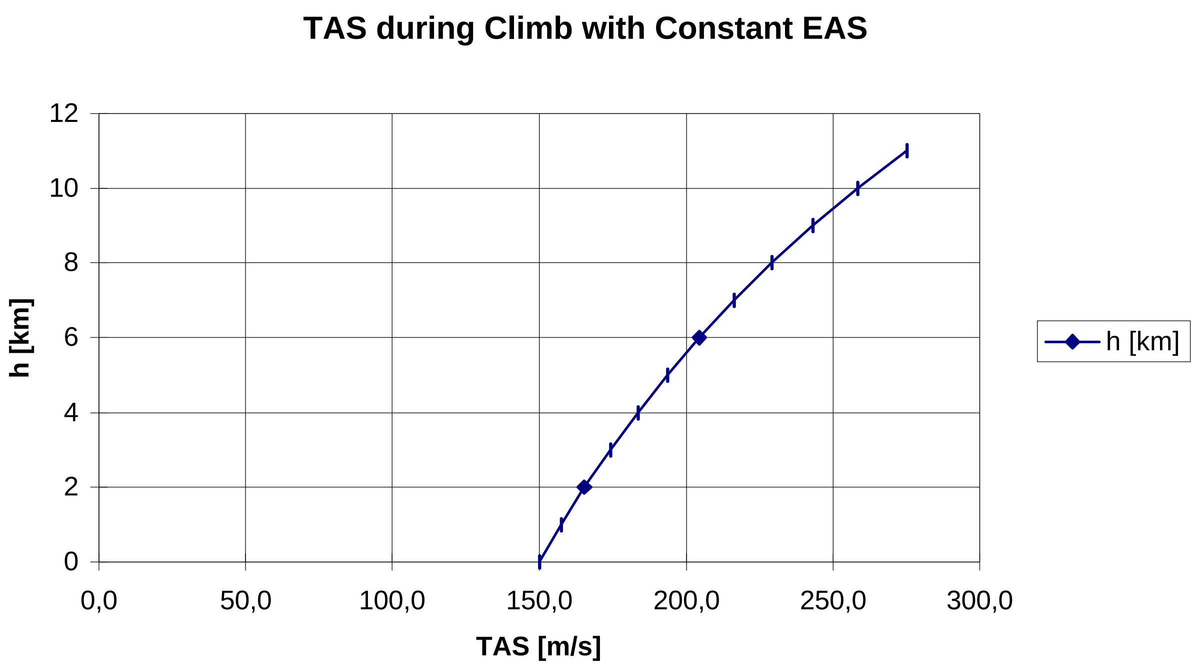
e) $$A_i = \frac{V_{v_{i+1}} - V_{v_i}}{\Delta h} \qquad \Delta t_i = \frac{1}{A_i} \cdot \ln\left(\frac{V_{v_{i+1}}}{V_{v_i}} \right)$$

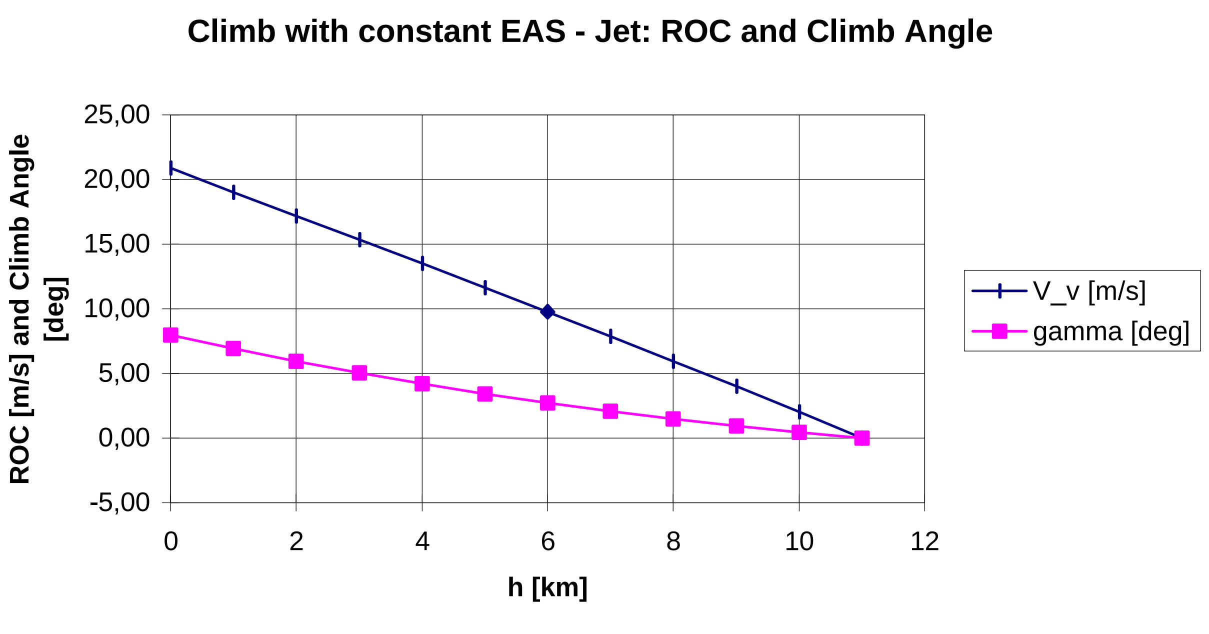
The time to climb to an altitude $h = j \cdot \Delta h$ is $t = \sum_{i=1}^{j} \Delta t_i$.

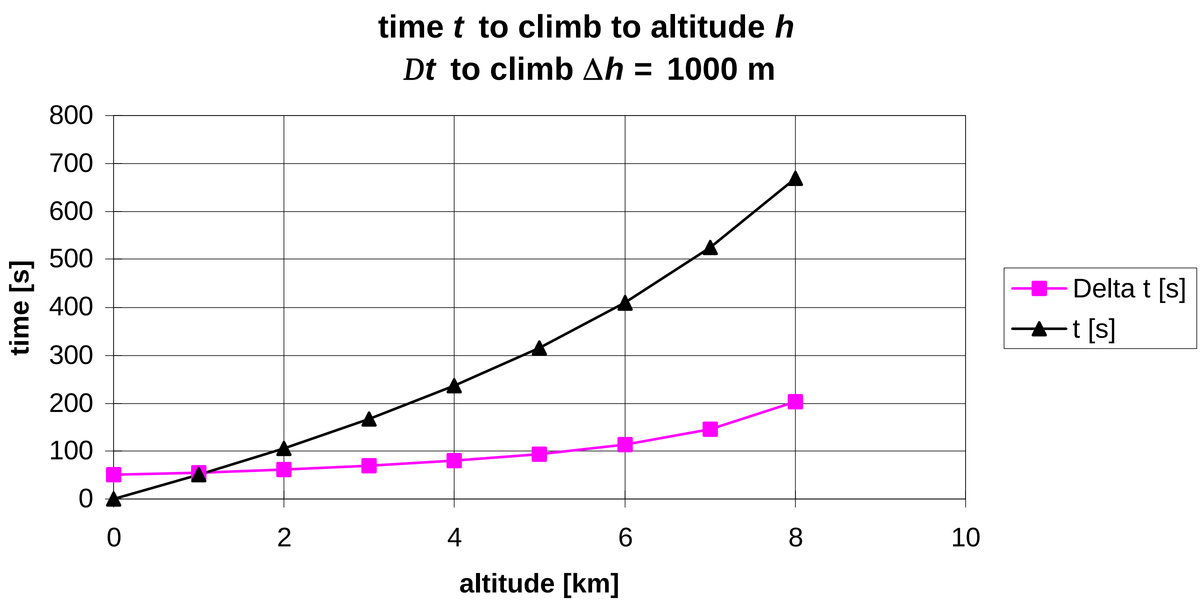
h [km]	T [N]	V [m/s]	M [-]	V_{vE} [m/s]	V_v [m/s]	γ [deg]	A_i [1/s]	Δt [s]	t [s]
0	135900	150,0	0,44	20,87	20,87	8,0	-0,001846	50	0
1	123324	157,5	0,47	18,12	19,02	6,9	-0,001842	55	50
2	111658	165,5	0,50	15,57	17,18	6,0	-0,001842	62	105
3	100856	174,1	0,53	13,21	15,34	5,1	-0,001846	69	167
4	90873	183,4	0,57	11,03	13,49	4,2	-0,001855	80	237
5	81663	193,5	0,60	9,02	11,63	3,4	-0,001868	94	316
6	73185	204,4	0,65	7,17	9,77	2,7	-0,001887	114	410
7	65398	216,2	0,69	5,47	7,88	2,1	-0,001912	145	524
8	58261	229,1	0,74	3,91	5,97	1,5	-0,001945	203	**669**
9	51735	243,1	0,80	2,48	4,02	0,9	-0,001986	343	872
10	45784	258,4	0,86	1,18	2,04	0,5	-0,002037		
11	40372	275,2	0,93	0,00	0,00	0,0	0,000002		

Answer:

The jet needs 669 s = 11 min 9 s (under given assumptions) as time to climb to an altitude of 8 km.


TAS during Climb with Constant EAS
h [km]
12
10
8
6
4
2
0
0,0
50,0
100,0
150,0
200,0
250,0
300,0
TAS [m/s]
h [km]


Climb with constant EAS - Jet: ROC and Climb Angle
ROC [m/s] and Climb Angle [deg]
25,00
20,00
15,00
10,00
5,00
0,00
-5,00
0
2
4
6
8
10
12
h [km]
V_v [m/s]
gamma [deg]



f) The table above yields: vertical speed $V_v = 0$ m/s at altitude 11 km and vertical speed. The absolute ceiling is 11 km.

5.6 A jet with an absolute ceiling of 11000 m climbs to an altitude of 8000 m. At sea level, the rate of climb ROC_0 is 20.87 m/s. Calculate the time to climb. Assume a linear variation of rate of climb with altitude during the whole manouver.

Solution

$$t_{CLB} = -\frac{h_{abs}}{ROC_0} \cdot \ln\left(1 - \frac{h}{h_{abs}}\right) = -\frac{11000\text{m s}}{20.87\text{m}} \cdot \ln\left(1 - \frac{8000}{11000}\right) = 685\text{s}$$

Compared to Question 5.5 (e), this approximate solution produces a relative error of 2.3%.

6 Stall, Speed Stability and Turning Performance

6.1 During an IFR flight, a passenger looks out the window while relaxing in his seat. He observes a turn and estimates the bank angle to be 30°. At the same time, the passenger observes the free surface of the orange juice in his glass: it is parallel to the tray.

a) The passenger assumes the turn being flown as a *rate one turn*. Explain the term *rate one turn*. Why is it correct to assume a *rate one turn*.

b) The passenger assumes the turn being flown as a *coordinated turn*. Explain the term *coordinated turn*. Why is it correct to assume a *coordinated turn*.

c) Calculate the aircraft's true airspeed.

Solution

a) A *rate one turn* is a turn with a heading change of 180° in 60 seconds. During IFR flights turns are performed as rate one turns.

b) In a *coordinated turn* (correctly banked turn)

- the lift force lies in the aircraft plane of symmetry,
- the ball in the turn and slip indicator is centered,
- there is no acceleration along the y-axis of the aircraft.

This phenomenon is also shown by the free surface of the orange juice in the glass which is parallel to the tray.

c) $\tan\Phi == \frac{V \cdot \Omega}{g}$

$$V = \frac{g}{\Omega} \cdot \tan\Phi \; = \; \frac{9.81\text{m} \cdot 60\text{s}}{\text{s}^2 \cdot \pi} \cdot \tan 30° = 108.2 \frac{\text{m}}{\text{s}} = 210\text{kt}$$

Answer: The aircraft's true airspeed is 210 kt.

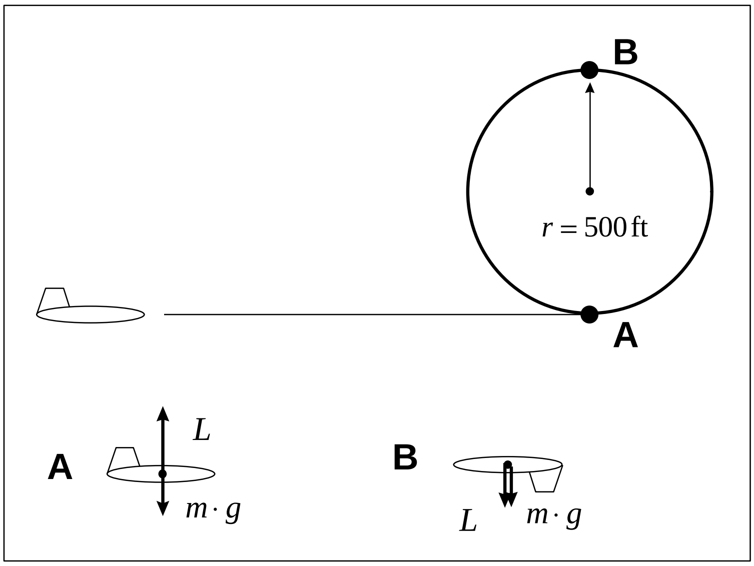
6.2 An aerobatic airplane flies a perfect circular looping. The pilot reads an altitude of 5000 ft at the highes point of the looping and an altitude of 4000 ft a the lowest point of the looping. The looping is flown at such a speed that the lowest load factor in the looping is $n = 0$. Assume that aircraft speed is nearly constant in the looping.

a) During which part of the looping does the airplane experience the highest load factor?

b) What is the aircraft's speed in the looping?

c) Calculate the highest load factor n during the looping.

Solution



Load factor:

$$n = \frac{L}{m \cdot g}$$

In point A:

(↑) $$L - m \cdot g = m \cdot a = m \cdot \frac{V^2}{r}$$

$$m \, \frac{V}{} \quad m \;\; g$$

$$n \quad \frac{V}{\cdot \, g} + 1$$

In point B:

↑) $$- L - m \cdot g = - \, m \cdot a = - \, m \cdot \frac{V}{r}$$ $$r = 500\text{ft} = 152.4 \text{ m}$$

$$L = m \cdot \frac{V^2}{r} - m \cdot g$$

$$n = \frac{V^2}{r \cdot g} - 1$$

a) Answer: The highest load factor is experienced in point A.

b) Point B: $0 = \frac{V^2}{r \cdot g} - 1 \quad \Rightarrow \quad \frac{V^2}{r \cdot g} = 1$

$$V = \sqrt{r \cdot g} = \sqrt{152.4 \text{ m} \cdot 9.81 \frac{\text{m}}{\text{s}^2}} = 38.67 \frac{\text{m}}{\text{s}} = 75.2 \text{ kt}$$

Answer: The aircraft speed is 75 kt.

c) Point A: $n = \frac{V^2}{r \cdot g} + 1 = \frac{38.67^2 \text{m}^2 \ \text{s}^2}{\text{s}^2 \ 152.4 \text{m} \cdot 9.81 \text{m}} + 1 = 2$

Answer: Under given assumptions (which are not conservative!), the highest load factor during the looping is $n = 2$.

7 Range and Endurance

7.1 A jet cruises with constant lift coefficient and constant Mach number of 0.82 in the stratosphere. The take-off mass with full fuel tanks is 271000 kg, the maximum fuel volume is 135000 liters, fuel density is 800 kg/m³. The specific fuel consumption is 16 mg/(Ns). The aircraft has a specified range of 7200 NM.

a) Write down the appropriate equation for these flight conditions from which the range can be calculated. How is this equation named?

b) Calculate the aircraft's mass without fuel.

c) On its fuel reserves, the aircraft could fly additional 920 NM plus further 30 min. How far could the aircraft fly in these 30 min. if we assume that the 30 min. are also flown at cruise speed? How far could the aircraft fly on its fuel reserves?

d) Calculate the aircraft's lift-to-drag ratio.

Solution

a) From the information given we know:

1.) jet aircraft

2.)

C_L = const.

M = const. $\Rightarrow$ V = const.

(even considering a shallow climb; flight in stratosphere)

$\Rightarrow$

2. flight schedule

$$R = \frac{V \cdot E}{c \cdot g} \cdot \ln \frac{m_1}{m_2}$$

this equation is known as *Breguet Range Equation*.

b) $m_1 = 271000 \text{ kg}$

$V_f = 135000 \text{ l} = 135 \text{ m}^3$ $\quad$ $m_f = V_f \cdot \rho_f = 135 \text{ m}^3 \cdot 800 \text{ kg/m}^3 = 108000 \text{ kg}$

$m_2 = m_1 - m_f = 271000 \text{ kg} - 108000 \text{ kg} = 163000 \text{ kg}$

c) In the stratosphere: $a = 295.07 \text{ m/s}$ (from ISA-Table)

$V = M \cdot a = 0.82 \cdot 295.07 \text{ m/s} = 241.96 \text{ m/s}$

$s_{30\text{min}} = 241.96 \text{ m/s} \cdot 1800 \text{ s} = 435.5 \text{ km} = 235.2 \text{ NM}$

$s_{reserve} = 920 \text{ NM} + 235.2 \text{ NM} = 1155.2 \text{ NM} = 2139.3 \text{ km}$

d) $R_{theory} = R = 7200 \text{ NM} \cdot 1.852 \text{ km/NM} + 2139.3 \text{ km} = 15474 \text{ NM}$

$$E = \frac{R \cdot c \cdot g}{V \cdot \ln\left(\frac{m_1}{m_2}\right)} = \frac{15474 \cdot 10^6 \text{ m} \cdot 16 \cdot 10^{-6} \text{ kg} \cdot 9.81 \text{ m} \cdot \text{s}}{\text{N} \cdot \text{s} \cdot \text{s}^2 \cdot 241.96 \text{ m} \cdot \ln\left(\frac{271000 \text{ kg}}{163000 \text{ kg}}\right)} = 19.7$$

Answer: From given data with given assumptions, a lift-to-drag ratio of 19.7 for cruise flight was calculated.

7.2 Derive an equation for the range of a jet aircraft in a (shallow) steady climb with constant airspeed and constant lift coefficient.

Solution

$$Q = c \cdot T$$

$$T = D + mg \cdot \sin\gamma$$

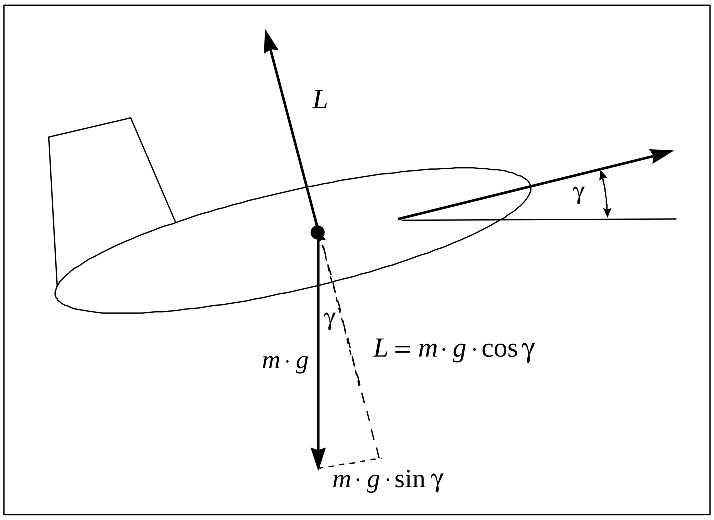
$$Q = c \cdot \left(\frac{D}{L} \cdot mg + mg \cdot \sin\gamma\right)$$

$$Q = c \cdot mg \cdot \left(\frac{1}{E} + \sin\gamma\right)$$

$$R = -\int_{m_1}^{m_2} \frac{V}{Q}\, dm$$

$$R = -\frac{V}{c \cdot g \cdot \left(\frac{1}{E} + \sin\gamma\right)} \int_{m_1}^{m_2} \frac{1}{m}\, dm$$

$$R = \frac{V \cdot \ln\left(\frac{m_1}{m_2}\right)}{c \cdot g \cdot \left(\frac{1}{E} + \sin\gamma\right)}$$



aircraft with lift and weight vector

18 Miscellaneous Questions

18.1 The category of effect of a failure is judged to be *hazardous*. Following *ACJ No. 1. to JAR 25.1309*

a) What is the largest permissible failure probability?

b) What is the mean time to failure *MTTF* ?

Solution

$F(t)$	probability of failure,
λ	failure rate
MTTF	mean time to failure
FH	flight hour

a) *hazardous* : $F(t = 1\,\mathrm{FH}) \leq 10^{-7}$

b) For small probabilities of failure: $\lambda \approx F / t = 10^{-7} \cdot \frac{1}{\mathrm{FH}}$

$$\mathrm{MTTF} = 1/\lambda = \frac{1}{10^{-7}}\,\mathrm{FH} = 10\,000\,000\,\mathrm{FH}$$

Answer: If a failure has a *hazardous* effect, the mean time to this failure may not be less than 10 000 000 FH.