DATA SHEET - Stability and Control

Equations of motion for a six degree of freedom rigid body:

$$X = m[\dot{U} + QW - RV + g\sin\Theta]$$

$$Y = m[\dot{V} + RU - PW - g\cos\Theta\sin\Phi]$$

$$Z = m[\dot{W} + PV - QU - g\cos\Theta\cos\Phi]$$

$$L = \dot{P}I_{xx} - I_{xz}(\dot{R} + PQ) + (I_{zz} - I_{yy})QR$$

$$M = \dot{Q}I_{yy} + I_{xz}(P^2 - R^2) + (I_{xx} - I_{zz})PR$$

$$N = \dot{R}I_{zz} - I_{xz}\dot{P} + PQ(I_{yy} - I_{xx}) + I_{xz}QR$$

Angular velocities and Euler angles:

$$P = \dot{\Phi} - \dot{\Psi}\sin\Theta$$

$$Q = \dot{\Theta}\cos\Phi + \dot{\Psi}\cos\Theta\sin\Phi$$

$$R = -\dot{\Theta}\sin\Phi + \dot{\Psi}\cos\Theta\cos\Phi$$

Velocity components along the body axes:

$$U = V_T \cos\beta \cos\alpha$$

$$V = V_T \sin\beta$$

$$W = V_T \cos\beta \sin\alpha$$

for small angles:

$$U = V_T$$

$$V = U\cdot\beta$$

$$W = U\cdot\alpha$$

Positive <u>control surface deflections:</u>

always in positive rotation about positive axis:

Elevator: down
Left Aileron: down
Rudder: left

<u>Equations of motion</u> for straight , symmetric, <u>perturbed flight</u> with wings level in stability axis system and some simplifications (e.g. $\gamma_0 = 0$):

$$\dot{u} = X_u \cdot u + X_w \cdot w - g \cdot \theta + X_{\delta_E} \cdot \delta_E$$

$$\dot{w} = Z_u \cdot u + Z_w \cdot w + U_0 \cdot q + Z_{\delta_E} \cdot \delta_E$$

$$\dot{q} = \tilde{M}_u \cdot u + \tilde{M}_w \cdot w + \tilde{M}_q \cdot q + \tilde{M}_{\delta_E} \cdot \delta_E$$

$$\dot{\theta} = q$$

$$\dot{\beta} = Y_v \cdot \beta - r + \frac{g}{U_0} \cdot \theta + Y^*_{\delta_A} \cdot \delta_A + Y^*_{\delta_R} \cdot \delta_R$$

$$\dot{p} = L'_\beta \cdot \beta + L'_p \cdot p + L'_r \cdot r + L'_{\delta_A} \cdot \delta_A + L'_{\delta_R} \cdot \delta_R$$

$$\dot{r} = N'_\beta \cdot \beta + N'_p \cdot p + N'_r \cdot r + N'_{\delta_A} \cdot \delta_A + N'_{\delta_R} \cdot \delta_R$$

$$\dot{\phi} = p$$

$$\dot{\psi} = r$$

Definition of "tilde" derivatives:

$$\tilde{M}_u = M_u + M_{\dot{w}} Z_u$$

$$\tilde{M}_w = M_w + M_{\dot{w}} Z_w$$

$$\tilde{M}_q = M_q + U_0 M_{\dot{w}}$$

$$\tilde{M}_\theta = -g M_{\dot{w}} \sin\gamma_0$$

$$\tilde{M}_{\delta_E} = M_{\delta_E} + M_{\dot{w}} Z_{\delta_E}$$

Definition of primed stability derivatives:

$$L'_{(\,)} = \frac{L_{(\,)} + \dfrac{I_{xz}}{I_{xx}} N_{(\,)}}{1 - \dfrac{I_{xz}^2}{I_{xx} \cdot I_{zz}}}$$

$$N'_{(\,)} = \frac{N_{(\,)} + \dfrac{I_{xz}}{I_{zz}} L_{(\,)}}{1 - \dfrac{I_{xz}^2}{I_{xx} \cdot I_{zz}}}$$

for simplification assume $I_{xz} << I_{xx}$, $I_{xz} << I_{yy}$, $I_{xz} << I_{zz}$

$$L'_{()} = L_{()} + \frac{I_{xz}}{I_{xx}} \cdot N_{()}$$

$$N'_{()} = N_{()} + \frac{I_{xz}}{I_{zz}} \cdot L_{()}$$

e.g. $$N'_\beta = N_\beta + \frac{I_{xz}}{I_{zz}} \cdot L_\beta$$

Table 4-3. Longitudinal dimensional stability derivatives (stability axis system).

Quantity	In terms of basic stability derivatives: Dimensional — Definitions	Dimensional — Unit	Nondimensional
X_u	$\frac{1}{m}\frac{\partial X}{\partial u}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{m}(-C_D - C_{D_u})$[a]
X_w	$\frac{1}{m}\frac{\partial X}{\partial w}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{2m}(C_L - C_{D_\alpha})$
X_δ	$\frac{1}{m}\frac{\partial X}{\partial \delta}$	$\frac{\text{m}}{\text{rad-sec}^2}$	$\frac{\rho SU^2}{2m}(-C_{D_\delta})$
Z_u	$\frac{1}{m}\frac{\partial Z}{\partial u}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{m}(-C_L - C_{L_u})$[c]
Z_w	$\frac{1}{m}\frac{\partial Z}{\partial w}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{2m}(-C_{L_\alpha} - C_D)$
$Z_{\dot{w}}$	$\frac{1}{m}\frac{\partial Z}{\partial \dot{w}}$	$\frac{1}{1}$	$\frac{\rho Sc}{4m}(-C_{L_{\dot{\alpha}}})$
Z_q	$\frac{1}{m}\frac{\partial Z}{\partial q}$	$\frac{\text{m}}{\text{rad-sec}}$	$\frac{\rho SUc}{4m}(-C_{L_q})$
Z_δ	$\frac{1}{m}\frac{\partial Z}{\partial \delta}$	$\frac{\text{m}}{\text{rad-sec}^2}$	$\frac{\rho SU^2}{2m}(-C_{L_\delta})$
M_u	$\frac{1}{I_y}\frac{\partial M}{\partial u}$	$\frac{1}{\text{m-sec}}$	$\frac{\rho SUc}{I_y}(C_M + C_{M_u})$
M_w	$\frac{1}{I_y}\frac{\partial M}{\partial w}$	$\frac{1}{\text{m-sec}}$	$\frac{\rho SUc}{2I_y}C_{M_\alpha}$
$M_{\dot{w}}$	$\frac{1}{I_y}\frac{\partial M}{\partial \dot{w}}$	$\frac{1}{\text{m}}$	$\frac{\rho Sc^2}{4I_y}C_{M_{\dot{\alpha}}}$
M_q	$\frac{1}{I_y}\frac{\partial M}{\partial q}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUc^2}{4I_y}C_{M_q}$
M_δ	$\frac{1}{I_y}\frac{\partial M}{\partial \delta}$	$\frac{1}{\text{rad-sec}^2}$	$\frac{\rho SU^2c}{2I_y}C_{M_\delta}$

[a] The thrust gradient terms are neglected here in the interests of symmetry and consistency.

[c] For $C_{L_u} = 0$, as in subsonic flight, and $C_L = W/(\rho U^2 S/2)$, as in trimmed flight for $\gamma_0 = 0$, $Z_u = -2g/U_0$.

Table 4-4. Lateral dimensional stability derivative parameters (stability or body axis systems).

Quantity[a]	In terms of basic stability derivatives: Dimensional — Definitions	Dimensional — Unit	Nondimensional
Y_v	$\frac{1}{mU}\frac{\partial Y}{\partial \beta}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{2m}C_{y_\beta}$
$Y_{\dot{v}}$	$\frac{1}{mU}\frac{\partial Y}{\partial \dot{\beta}}$	$\frac{1}{1}$	$\frac{\rho Sb}{4m}C_{y_{\dot{\beta}}}$
Y_r^*	$\frac{1}{mU}\frac{\partial Y}{\partial r}$	$\frac{1}{\text{rad}}$	$\frac{\rho Sb}{4m}C_{y_r}$
Y_p^*	$\frac{1}{mU}\frac{\partial Y}{\partial p}$	$\frac{1}{\text{rad}}$	$\frac{\rho Sb}{4m}C_{y_p}$
Y_δ^*	$\frac{1}{mU}\frac{\partial Y}{\partial \delta}$	$\frac{1}{\text{rad-sec}}$	$\frac{\rho SU}{2m}C_{y_\delta}$
N_β	$\frac{1}{I_z}\frac{\partial N}{\partial \beta}$	$\frac{1}{\text{sec}^2}$	$\frac{\rho SU^2b}{2I_z}C_{n_\beta}$
$N_{\dot{\beta}}$	$\frac{1}{I_z}\frac{\partial N}{\partial \dot{\beta}}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUb^2}{4I_z}C_{n_{\dot{\beta}}}$
N_r	$\frac{1}{I_z}\frac{\partial N}{\partial r}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUb^2}{4I_z}C_{n_r}$
N_p	$\frac{1}{I_z}\frac{\partial N}{\partial p}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUb^2}{4I_z}C_{n_p}$
N_δ	$\frac{1}{I_z}\frac{\partial N}{\partial \delta}$	$\frac{1}{\text{rad-sec}^2}$	$\frac{\rho SU^2b}{2I_z}C_{n_\delta}$
L_β	$\frac{1}{I_x}\frac{\partial L}{\partial \beta}$	$\frac{1}{\text{sec}^2}$	$\frac{\rho SU^2b}{2I_x}C_{l_\beta}$
$L_{\dot{\beta}}$	$\frac{1}{I_x}\frac{\partial L}{\partial \dot{\beta}}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUb^2}{4I_x}C_{l_{\dot{\beta}}}$
L_r	$\frac{1}{I_x}\frac{\partial L}{\partial r}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUb^2}{4I_x}C_{l_r}$
L_p	$\frac{1}{I_x}\frac{\partial L}{\partial p}$	$\frac{1}{\text{sec}}$	$\frac{\rho SUb^2}{4I_x}C_{l_p}$
L_δ	$\frac{1}{I_x}\frac{\partial L}{\partial \delta}$	$\frac{1}{\text{rad-sec}^2}$	$\frac{\rho SU^2b}{2I_x}C_{l_\delta}$

[a] The starred derivatives arise when β rather than v is used as the lateral motion parameter (see Chapter 6); in general, $Y_\lambda^* = Y_\lambda/U_0$.

Note: $I_x = I_{xx}$, $I_y = I_{yy}$, $I_z = I_{zz}$

Table 4-1. Longitudinal nondimensional stability derivatives (stability axis system).[a]

Basic nondimensional stability derivatives	
Total airframe	
Definitions	Unit
$C_D = \dfrac{\text{drag}}{qS}$	$\dfrac{1}{1}$
$C_{D_u} = \dfrac{U}{2}\dfrac{\partial C_D}{\partial U}$	$\dfrac{1}{1}$
$C_{D_\alpha} = \dfrac{\partial C_D}{\partial \alpha}$	$\dfrac{1}{\text{rad}}$
$C_{D_\delta} = \dfrac{\partial C_D}{\partial \delta}$	$\dfrac{1}{\text{rad}}$
$C_L = \dfrac{\text{lift}}{qS}$	$\dfrac{1}{1}$
$C_{L_u} = \dfrac{U}{2}\dfrac{\partial C_L}{\partial U}$	$\dfrac{1}{1}$
$C_{L_\alpha} = \dfrac{\partial C_L}{\partial \alpha}$	$\dfrac{1}{\text{rad}}$
$C_{L_{\dot\alpha}} = \dfrac{\partial C_L}{\partial(\dot\alpha c/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{L_q} = \dfrac{\partial C_L}{\partial(qc/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{L_\delta} = \dfrac{\partial C_L}{\partial \delta}$	$\dfrac{1}{\text{rad}}$
$C_M = \dfrac{M}{qSc}$	$\dfrac{1}{1}$
$C_{M_u} = \dfrac{U}{2}\dfrac{\partial C_M}{\partial U}$	$\dfrac{1}{1}$
$C_{M_\alpha} = \dfrac{\partial C_M}{\partial \alpha}$	$\dfrac{1}{\text{rad}}$
$C_{M_{\dot\alpha}} = \dfrac{\partial C_M}{\partial(\dot\alpha c/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{M_q} = \dfrac{\partial C_M}{\partial(qc/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{M_\delta} = \dfrac{\partial C_M}{\partial \delta}$	$\dfrac{1}{\text{rad}}$

[a] The symbol q, in addition to its normal use to designate pitching velocity, is also used in these tables to denote the dynamic pressure, $\rho U^2/2$, in accordance with long-established aeronautical practice. When particularized by the subscript h (or v), it signifies the *local* dynamic pressure at the horizontal (or vertical) tail. The local flow angles relative to free stream conditions are denoted by $-\epsilon$ (X, Z plane) and $-\sigma$ (X, Y plane).

Table 4-2. Lateral nondimensional stability derivatives (stability or body axis systems).

Basic nondimensional stability derivatives	
Total airframe	
Definitions	Unit
$C_{y_\beta} = \dfrac{\partial C_y}{\partial \beta}$	$\dfrac{1}{\text{rad}}$
$C_{y_{\dot\beta}} = \dfrac{\partial C_y}{\partial(\dot\beta b/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{y_r} = \dfrac{\partial C_y}{\partial(rb/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{y_p} = \dfrac{\partial C_y}{\partial(pb/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{y_\delta} = \dfrac{\partial C_y}{\partial \delta}$	$\dfrac{1}{\text{rad}}$
$C_{n_\beta} = \dfrac{\partial C_n}{\partial \beta}$	$\dfrac{1}{\text{rad}}$
$C_{n_{\dot\beta}} = \dfrac{\partial C_n}{\partial(\dot\beta b/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{n_r} = \dfrac{\partial C_n}{\partial(rb/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{n_p} = \dfrac{\partial C_n}{\partial(pb/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{n_\delta} = \dfrac{\partial C_n}{\partial \delta}$	$\dfrac{1}{\text{rad}}$
$C_{l_\beta} = \dfrac{\partial C_l}{\partial \beta}$	$\dfrac{1}{\text{rad}}$
$C_{l_{\dot\beta}} = \dfrac{\partial C_l}{\partial(\dot\beta b/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{l_r} = \dfrac{\partial C_l}{\partial(rb/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{l_p} = \dfrac{\partial C_l}{\partial(pb/2U)}$	$\dfrac{1}{\text{rad}}$
$C_{l_\delta} = \dfrac{\partial C_l}{\partial \delta}$	$\dfrac{1}{\text{rad}}$

Tables taken from: McRuer, Ashkenas and Graham. 1973. Aircraft Dynamics and Automatic Control. Princeton University Press.

– adapted to our text –

Definition of Y_δ^* :

$$Y^*_{\delta_A} = \frac{Y_{\delta_A}}{U_0}$$

$$Y^*_{\delta_R} = \frac{Y_{\delta_R}}{U_0}$$

Conversion from w- into α-stability derivatives:

$$(\ )_\alpha = (\ )_w \, U_0$$

$$e.g. \qquad Z_\alpha = Z_w \, U_0$$

Normal acceleration:

$$a_{z_{cg}} = \dot{w} - U_0 q$$

$$n_{z_{cg}} = \frac{a_{z_{cg}}}{g}$$

$$a_{z_x} = \dot{w} - U_0 q - l_x \cdot \dot{q}$$

a_{zx} is normal acceleration on the fuselage centre line a distance l_x forward of the cg.

$$\ddot{h}_{cg} = -a_{z_{cg}}$$

Lateral acceleration:

$$a_{y_{cg}} = \dot{v} - g\theta + U_0 r$$

Heading:

$$\lambda = \psi + \beta$$

Aircraft Dynamics in State and Output Equation

2 Linear Simulation of Aircraft Dynamics

The linear simulation assumes small perturbations about an equilibrium or trimmed condition. It can be shown that for this case, longitudinal and lateral motions are uncoupled .

2.1 Longitudinal Aircraft Dynamics

The State Equation

A linear system can be represented in state space notation. **x** is the state vector with perturbations (deviation from the steady state or equilibrium condition) of the state variables. **u** is the control vector. The control vector consists of control inputs from elevator and flaps. In this case also disturbances from gust inputs are built into the control vector. **A** is called system matrix or state coefficient matrix and **B** is called control matrix.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\ \boldsymbol{x} + \boldsymbol{B}\ \boldsymbol{u}$$

$$\boldsymbol{x} = \begin{bmatrix} u \\ w \\ q \\ \Theta \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \delta_E \\ \delta_F \\ u_g \\ w_g \\ q_g \end{bmatrix}$$

$$A = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & U_0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} X_{\delta_E} & X_{\delta_F} & -X_u & -X_w & 0 \\ Z_{\delta_E} & Z_{\delta_F} & -Z_u & -Z_w & -U_0 \\ \tilde{M}_{\delta_E} & \tilde{M}_{\delta_F} & -\tilde{M}_u & -\tilde{M}_w & -\tilde{M}_q \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

The matrices A and B consist mainly of stability and control derivatives. The American notation is applied here which uses dimensional stability derivative parameters. These stability derivative parameters lead to very compact equations. U_0 is the steady state forward speed of the aircraft. The "tilde"-stability derivatives are defined as follows:

$$\tilde{M}_u = M_u + M_{\dot{w}} Z_u$$

$$\tilde{M}_w = M_w + M_{\dot{w}} Z_w$$

$$\tilde{M}_q = M_q + U_0 M_{\dot{w}}$$

$$\tilde{M}_{\delta_E} = M_{\delta_E} + M_{\dot{w}} Z_{\delta_E}$$

The Output Equation

If other values than the state variables are of interest, they can be calculated by use of the output equation. Values considered here are the angle of attack α, the flight path angle γ, the acceleration in z-direction at the center of gravity a_{zcg} and the corresponding load factor n_{zcg}.

$$\alpha = \frac{w}{U_0} \qquad \gamma = \theta - \alpha \qquad a_{z_{cg}} = \dot{w} - U_0 \cdot q$$

$$n_{z_{cg}} = \frac{a_{z_{cg}}}{g}$$

The output equation can be written as

$$\boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{x} + \boldsymbol{D}\,\boldsymbol{u}$$

$$\boldsymbol{y} = \begin{bmatrix} \alpha \\ \gamma \\ a_{z_{cg}} \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \delta_E \\ \delta_F \\ u_g \\ w_g \\ q_g \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & \frac{1}{U_0} & 0 & 0 \\ 0 & -\frac{1}{U_0} & 0 & 1 \\ Z_u & Z_w & 0 & 0 \end{bmatrix} \qquad \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ Z_{\delta_E} & Z_{\delta_F} & -Z_u & -Z_w & -U_0 \end{bmatrix}$$

A change in aircraft height can be obtained from an integration of c.g.–acceleration in the z–direction.

$$\ddot{h} = -a_{z_{cg}} \qquad \dot{h} = \int \ddot{h}\, dt \qquad h = \int \dot{h}\, dt$$

2.2 Lateral Aircraft Dynamics

Lateral aircraft dynamics follow the same mathematical approach as described for longitudinal aircraft dynamics. Refer to the List of Symbols for queries related to the notation. Again, the American notation using dimensional stability derivative parameters is used.

The State Equation

Aileron and rudder control the lateral dynamics of the aircraft. Gust inputs are again built into the control vector.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{u}$$

$$\boldsymbol{x} = \begin{bmatrix} \beta \\ p \\ r \\ \Phi \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \delta_A \\ \delta_R \\ \beta_g \\ p_g \\ r_g \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} Y_v & 0 & -1 & g/U_0 \\ L_\beta' & L_p' & L_r' & 0 \\ N_\beta' & N_p' & N_r' & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} Y_{\delta_A}{}^* & Y_{\delta_R}{}^* & -Y_v & 0 & 1 \\ L_{\delta_A}' & L_{\delta_R}' & -L_\beta' & -L_p' & -L_r' \\ N_{\delta_A}' & N_{\delta_R}' & -N_\beta' & -N_p' & -N_r' \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The 'primed' and 'stared' stability derivatives are defined as follows:

$$L_\beta' = L_\beta + \frac{I_{XZ}}{I_X} N_\beta \qquad N_\beta' = N_\beta + \frac{I_{XZ}}{I_Z} L_\beta$$

$$L_p' = L_p + \frac{I_{XZ}}{I_X} N_p \qquad N_p' = N_p + \frac{I_{XZ}}{I_Z} L_p$$

$$L_r' = L_r + \frac{I_{XZ}}{I_X} N_r \qquad N_r' = N_r + \frac{I_{XZ}}{I_Z} L_r$$

$$L_{\delta_A}' = L_{\delta_A} + \frac{I_{XZ}}{I_X} N_{\delta_A} \qquad N_{\delta_A}' = N_{\delta_A} + \frac{I_{XZ}}{I_Z} L_{\delta_A}$$

$$L_{\delta_R}' = L_{\delta_R} + \frac{I_{XZ}}{I_X} N_{\delta_R} \qquad N_{\delta_R}' = N_{\delta_R} + \frac{I_{XZ}}{I_Z} L_{\delta_R}$$

$$Y_{\delta_R}* = \frac{Y_{\delta_R}}{U_0} \qquad Y_{\delta_A}* = \frac{Y_{\delta_A}}{U_0}$$

The Output Equation

Further values that could be of interest are the sideslip velocity v and the side–acceleration at the centre of gravity a_{ycg}. They can be calculated by use of the output equation. The yaw angle Ψ and the load factor in y–direction can be calculated from yaw rate and side–acceleration as given below.

$$v = \beta \cdot U_0 \qquad a_{y_{cg}} = \dot{v} - g \cdot \phi + U_0 \cdot r$$

$$\boldsymbol{y} = \boldsymbol{C}\ \boldsymbol{x} + \boldsymbol{D}\ \boldsymbol{u}$$

$$\boldsymbol{y} = \begin{bmatrix} v \\ a_{y_{cg}} \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} U_0 & 0 & 0 & 0 \\ Y_v \cdot U_0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ Y_{\delta_A}* \cdot U_0 & Y_{\delta_R}* \cdot U_0 & -Y_v \cdot U_0 & 0 & U_0 \end{bmatrix}$$

$$\psi = \int r\ dt \qquad n_{y_{cg}} = \frac{a_{y_{cg}}}{g}$$

Input functions - an overview:

Input form	control input	internal disturbance	external disturbance	
			vehicle induced	environment
step function, pulse function, cutoff ramp	control surface movement	errors in control surface movements	thrust eccentricities, c.g. shifts, store release	gust, wind shear
initial conditions		automatic control engagement		
(1 - cos) gust				gust
periodic input	some terrain variations		dynamic unbalance, vibration, flutter	

Gust input:

If there is a gust input λ_g in the direction of a certain state variable λ, substitute this state variable in the equations of motion by $\lambda - \lambda_g$.

(1 - cos) gust:

$$\lambda_g(t) = k/T \cdot (1 - \cos (2\pi/T) \cdot t)$$

Vertical wind gradient:

$v_{wind} = k\, h^{n}$ h = height k = constant

n = 0.40 over city
n = 0.28 over country side
n = 0.16 over sea side.

Table 5-1. Longitudinal control-input transfer function coefficients.

	A	B	C	D	E
Δ	1	$-M_q - M_{\dot\alpha} - Z_w - X_u$	$Z_w M_q - M_\alpha - X_w Z_u$ $+ X_u(M_q + M_{\dot\alpha} + Z_w)$	$-X_u(Z_w M_q - M_\alpha)$ $+ Z_u(X_w M_q + g M_{\dot w})$ $- M_u(X_\alpha - g)$	$g(Z_u M_w - M_u Z_w)$
N_δ^θ	$M_\delta + Z_\delta M_{\dot w}$	$X_\delta(Z_u M_{\dot w} + M_u)$ $+ Z_\delta(M_w - X_u M_{\dot w})$ $- M_\delta(X_u + Z_w)$	$X_\delta(Z_u M_w - Z_w M_u)$ $+ Z_\delta(M_u X_w - M_w X_u)$ $+ M_\delta(Z_w X_u - X_w Z_u)$		
N_δ^w	Z_δ	$X_\delta Z_u - Z_\delta(X_u + M_q) + M_\delta U_0$	$X_\delta(U_0 M_u - Z_u M_q)$ $+ Z_\delta X_u M_q - U_0 M_\delta X_u$	$g(Z_\delta M_u - M_\delta Z_u)$	
N_δ^u	X_δ	$-X_\delta(Z_w + M_q + M_{\dot\alpha}) + Z_\delta X_w$	$X_\delta(Z_w M_q - M_\alpha)$ $- Z_\delta(X_w M_q + g M_{\dot w})$ $+ M_\delta(X_\alpha - g)$	$g(M_\delta Z_w - Z_\delta M_w)$	
sN_δ^h	$-Z_\delta$	$-X_\delta Z_u + Z_\delta(M_q + M_{\dot\alpha} + X_u)$	$X_\delta Z_u(M_q + M_\alpha)$ $- Z_\delta[X_u(M_q + M_{\dot\alpha}) - M_\alpha]$ $- M_\delta Z_\alpha$	$-X_\delta(Z_\alpha M_u - M_\alpha Z_u)$ $+ Z_\delta[M_u(X_\alpha - g) - M_\alpha X_u]$ $+ M_\delta[Z_\alpha X_u - Z_u(X_\alpha - g)]$	

Table 6-1. Lateral control-input transfer function coefficients.

	A	B	C	D	E
Δ	$1 - \frac{I_{xz}^2}{I_x I_z}$	$-Y_v\left(1 - \frac{I_{xz}^2}{I_x I_z}\right) - L_p - N_r - \frac{I_{xz}}{I_x} N_p - \frac{I_{xz}}{I_z} L_r$	$N_\beta + L_p(Y_v + N_r) + N_p\left(\frac{I_{xz}}{I_x} Y_v - L_r\right) + Y_v\left(\frac{I_{xz}}{I_z} L_r + N_r\right) + \frac{I_{xz}}{I_z} L_\beta$	$-N_\beta L_p + Y_v(N_p L_r - L_p N_r) + N_p L_\beta - \frac{g}{U_0}\left(L_\beta + \frac{I_{xz}}{I_x} N_\beta\right)$	$\frac{g}{U_0}(L_\beta N_r - N_\beta L_r)$
N^β	$Y^*_\delta\left(1 - \frac{I_{xz}^2}{I_x I_z}\right)$	$-Y^*_\delta\left[L_p + N_r + \frac{I_{xz}}{I_x} N_p + \frac{I_{xz}}{I_z} L_r\right] - \frac{I_{xz}}{I_z} L_\delta - N_\delta$	$Y^*_\delta(L_p N_r - N_p L_r) + N_\delta L_p - L_\delta N_p + \frac{g}{U_0}\left(L_\delta + \frac{I_{xz}}{I_x} N_\beta\right)$	$\frac{g}{U_0}(N_\delta L_r - L_\delta N_r)$	
N^φ_δ	$L_\delta + \frac{I_{xz}}{I_x} N_\delta$	$Y^*_\delta\left(L_\beta + \frac{I_{xz}}{I_x} N_\beta\right) - L_\delta(N_r + Y_v) + N_\delta\left(L_r - \frac{I_{xz}}{I_x} Y_v\right)$	$Y^*_\delta(L_r N_\beta - L_\beta N_r) + L_\delta(Y_v N_r + N_\beta) - N_\delta(L_\beta + Y_v L_r)$		
N^r_δ	$N_\delta + \frac{I_{xz}}{I_z} L_\delta$	$Y^*_\delta\left(N_\beta + \frac{I_{xz}}{I_z} L_\beta\right) + L_\delta\left(N_p - \frac{I_{xz}}{I_z} Y_v\right) - N_\delta(Y_v + L_p)$	$Y^*_\delta(L_\beta N_p - N_\beta L_p) - L_\delta Y_v N_p + N_\delta Y_v L_p$	$\frac{g}{U_0}(L_\delta N_\beta - N_\delta L_\beta)$	
$N^{a_y\,c.g.}_\delta$	$Y_\delta\left(1 - \frac{I_{xz}^2}{I_x I_z}\right)$	$-Y_\delta\left(L_p + N_r + \frac{I_{xz}}{I_x} N_p + \frac{I_{xz}}{I_z} L_r\right)$	$Y_\delta\left(N_\beta + L_p N_r - N_p L_r + \frac{I_{xz}}{I_z} L_\beta\right) - U_0 Y_v\left(\frac{I_{xz}}{I_z} L_\delta + N_\delta\right)$	$Y_\delta\left[N_p L_\beta - \frac{g L_\beta}{U_0} - N_\beta\left(L_p + \frac{g I_{xz}}{U_0 I_x}\right)\right] + U_0 Y_v\left[L_\delta\left(\frac{g}{U_0} - N_p\right) + N_\delta\left(\frac{g I_{xz}}{I_x U_0} + L_p\right)\right]$	$g[Y^*_\beta(L_\beta N_r - N_\beta L_r) + Y_v(N_\delta L_r - L_\delta N_r)]$

Note: To convert to primed derivatives, eliminate all I_{xz} terms and substitute L' and N' for L and N, respectively.

Longitudinal control-input transfer function coefficients

for the truncated short period and phugoid equations of motion

3 D phugoid			
	A	B	C
Δ_{ph_3}	$-M_\alpha$	$M_\alpha X_u - M_u(X_\alpha - g)$	$M_w Z_u g - M_u Z_w g$
N_δ^u	$(X_\alpha - g) M_\delta - M_\alpha X_\delta$	$g(M_\delta Z_w - Z_\delta M_w)$	
N_δ^w	$U_0 M_\delta$	$U_0 M_u X_\delta - U_0 X_u M_\delta$	$g(Z_\delta M_u - M_\delta Z_u)$
N_δ^θ	M_δ	$M_u X_\delta + M_w Z_\delta$ $- (X_u + Z_w) M_\delta$	$(Z_u M_w - Z_w M_u)\ X_\delta$ $+ (M_u X_w - M_w X_u)\ Z_\delta$ $+ (Z_w X_u - X_w Z_u)\ M_\delta$

classical (2 D) phugoid			
	A	B	C
Δ_{ph_2}	$-U_0$	$U_0 X_u$	$Z_u \cdot g$
Δ_{ph_2} *further simplified*	$-U_0$	$-2 \cdot g \cdot \frac{C_D}{C_L}$	$-2 \cdot \frac{g^2}{U_0}$
N_δ^u	$Z_\delta \cdot g$		
N_δ^θ	Z_δ	$- X_u Z_\delta$	

2 D short period			
	A	B	C
Δ_{sp}	1	$-(Z_w + M_q + M_{\dot{w}} U_0)$	$Z_w M_q - U_0 M_w$
N_δ^w	Z_δ	$U_0 M_\delta - Z_\delta M_q$	
N_δ^q	$M_\delta + Z_\delta M_{\dot{w}}$	$Z_\delta M_w - M_\delta Z_w$	

Lateral control-input transfer function coefficients for the truncated modes

3 D dutch roll				
	A	B	C	D
Δ_{d_3}	1	$-(Y_v + N_r' + L_p')$	$L_p'(Y_v + N_r') + N_\beta' + Y_v N_r'$	$-L_p'(N_\beta' + Y_v N_r')$
N_δ^β	Y_δ^*	$-[(L_p' + N_r')Y_\delta^* + N_\delta']$	$L_p' N_r' Y_\delta^* + L_p' N_\delta'$	
N_δ^p	L_δ'	$Y_\delta^* L_\beta' - Y_v L_\delta' - N_r L_\delta'$	$Y_v N_r' L_\delta' + N_\beta' L_\delta' - \frac{L_\beta' N_\delta'}{N_\beta'} - Y_\delta^* L_\beta' N_r'$	
N_δ^r	N_δ'	$N_\beta' Y_\delta^* - N_\delta'(L_p' + Y_v)$	$L_p'(Y_v N_\delta' - N_\beta' Y_\delta^*)$	

Notes: 1.) In the denominator Δ_{d3} the term $(s-L_p')$ can be factored out.

2.) In the transfer function $r(s)/\delta(s)$ the numerator cancels the roll subsidence mode $(s - L_p')$ in the denominator.

3 D spiral and roll subsidence			
	A	B	C
Δ_{SR}	1	$-L_p' + \frac{L_\beta'}{N_\beta'}\left(N_p' - \frac{g}{U_0}\right)$	$\frac{g}{U_0}\left(\frac{L_\beta' N_r'}{N_\beta'} - L_r'\right)$
N_δ^β	$\left(N_\delta^\beta\right)_{SR} = \left(N_\delta^\beta\right)_{lat} \cdot \frac{1}{N_\beta'}$		
N_δ^ϕ	$\frac{Y_\delta^* L_\beta'}{N_\beta'}$	$Y_\delta^* L_r' - \frac{N_r' L_\beta' Y_\delta^*}{N_\beta'} + L_\delta' - \frac{N_\delta' L_\beta'}{N_\beta'}$	
N_δ^r	y_δ^*	$Y_\delta^*\left(\frac{N_p' L_\beta'}{N_\beta'} - L_p'\right)$	$Y_\delta^*\left(\frac{g \cdot L_\delta'}{Y_\delta} - \frac{L_\beta' N_\delta'}{N_\beta'}\right)$

Note: For $(N_\delta^\beta)_{lat}$ see McRuer, Table 6-1.

2 D dutch roll			
	A	B	C
Δ_{d_2}	1	$-Y_v - N_r'$	$N_\beta' + Y_v N_r'$
N_δ^β	Y_δ^*	$-Y_\delta^* N_r' - N_\delta'$	
N_δ^r	N_δ'	$Y_\delta^* N_\beta' - Y_v N_\delta'$	

1 D roll subsidence		
	A	B
Δ_{r_1}	1	$-L_p'$
$N_{\delta_A}^p$	L_{δ_A}'	

Table 5-2. Longitudinal gust-input transfer function numerator coefficients.

	A	B	C	D
$\frac{N^{\theta}_{u_g}}{s}$	$-(M_u + Z_u M_{\dot{w}})$	$Z_w M_u - M_w Z_u$		
$\frac{N^{w}_{u_g}}{s^2}$	$-Z_u$	$Z_u M_q - U_0 M_u$		
$N^{u}_{u_g}$	$-X_u$	$X_u(Z_w + M_q + M_{\dot{\alpha}}) - Z_u X_w$	$-X_u(Z_w M_q - M_\alpha) + Z_u(X_w M_q + g M_{\dot{w}})$ $-M_u(X_\alpha - g)$	$g(Z_u M_w - M_u Z_w)$
$N^{h}_{u_g}$	Z_u	$-Z_u(M_q + M_{\dot{\alpha}})$	$-(Z_u M_\alpha - M_u Z_\alpha)$	
$\frac{U_0 N^{\theta}_{w_g}}{s}$	$M_q - M_{\dot{\alpha}}$	$-M_\alpha - Z_w M_q - X_u(M_q - M_{\dot{\alpha}})$	$X_u(M_\alpha + Z_w M_q) - X_w(U_0 M_u + Z_u M_q)$	
$N^{w}_{w_g}$	$-(Z_w - M_q + M_{\dot{\alpha}})$	$X_u(Z_w - M_q + M_{\dot{\alpha}}) - X_w Z_u - M_\alpha + Z_w M_q$	$X_u(M_\alpha - Z_w M_q) - X_w(U_0 M_u - Z_u M_q)$ $+ g Z_u \left(M_{\dot{w}} - \frac{M_q}{U_0}\right)$	$g(Z_u M_w - M_u Z_w)$
$\frac{U_0 N^{u}_{w_g}}{s}$	$-X_\alpha$	$2X_\alpha M_q + g(M_{\dot{\alpha}} - M_q)$	$g(M_\alpha + Z_w M_q)$	
$s N^{h}_{w_g}$	Z_w	$-2Z_w M_q + X_w Z_u - Z_w X_u$	$-2M_q(X_w Z_u - Z_w X_u) - g Z_u\left(M_{\dot{w}} - \frac{M_q}{U_0}\right)$	$-g(Z_u M_w - M_u Z_w)$

The denominators are the same as those for the control-input transfer functions

Table 5-3. Longitudinal gust-input transfer function numerator coefficients for the truncated short period and phugoid equations of motion.

	Phugoid (3D)		
	A	B	C
$\frac{N^{\theta}_{u_g}}{s}$	$-M_u$	$Z_w M_u - M_w Z_u$	
$\frac{N^{w}_{u_g}}{s^2}$	$-U_0 M_u$		
$N^{u}_{u_g}$	$X_u M_\alpha - M_u(X_\alpha - g)$	$g(Z_u M_w - M_u Z_w)$	
$\frac{U_0 N^{\theta}_{w_g}}{s}$	$-M_\alpha$	$(X_u M_\alpha - M_u X_\alpha)$	
$N^{w}_{w_g}$	$-M_\alpha$	$(X_u M_\alpha - M_u X_\alpha)$	$g(Z_u M_w - M_u Z_w)$
$\frac{U_0 N^{u}_{w_g}}{s}$	$g M_\alpha$		

	Short period (2D)	
$U_0 N^{\theta}_{w_g}$	$(M_q - M_{\dot{\alpha}})$	$-M_\alpha - Z_w M_q$
$\frac{N^{w}_{w_g}}{s}$	$-(Z_w - M_q + M_{\dot{\alpha}})$	$-M_\alpha + Z_w M_q$

Table 6-2. Lateral gust-input transfer function numerators.

	A	B	C	D
$N^{\beta}_{v_g}$	$N_p + \frac{I_{xz}}{I_z} L_p$	$\frac{g}{U_0}\left(-L_p - \frac{I_{xz}}{I_x} N_p\right)$	$\frac{g}{U_0}(L_p N_r - N_p L_r)$	
$N^{\varphi}_{v_g}$	$-L_p - \frac{I_{xz}}{I_x} N_p$	$L_p N_r - N_p L_r + Y_v\left(L_p + \frac{I_{xz}}{I_x} N_p\right)$	$L_\beta N_p - N_\beta L_p$ $+ Y_v(N_p L_r - L_p N_r)$	
$N^{r}_{v_g}$	$-N_p - \frac{I_{xz}}{I_z} L_p$	$Y_v\left(N_p + \frac{I_{xz}}{I_z} L_p\right)$	0	$\frac{g}{U_0}(L_\beta N_p - N_\beta L_p)$
$N^{\beta}_{\beta_g}$	$-Y_v\left(1 - \frac{I_{xz}^2}{I_x I_z}\right) + (N_r)_g$ $+ \frac{I_{xz}}{I_z}(L_r)_g$	$N_\beta + \frac{I_{xz}}{I_z} L_\beta - (N_r)_g\left(L_p + \frac{I_{xz}}{I_x}\frac{g}{U_0}\right)$ $+ Y_v\left(L_p + \frac{I_{xz}}{I_x} N_p + N_r\right.$ $\left.+ \frac{I_{xz}}{I_z} L_r\right) + (L_r)_g\left(N_p - \frac{g}{U_0}\right)$	$-Y_v(L_p N_r - L_r N_p)$ $- \frac{g}{U_0}\left(L_\beta + \frac{I_{xz}}{I_x} N_\beta\right) + L_\beta N_p$ $- N_\beta L_p - \frac{g}{U_0}[(N_r)_g L_r - (L_r)_g N_r]$	$\frac{g}{U_0}(L_\beta N_r - N_\beta L_r)$
$\frac{N^{\varphi}_{\beta_g}}{s}$	$-(L_r)_g - \frac{I_{xz}}{I_x}(N_r)_g$	$Y_v\left[(L_r)_g + \frac{I_{xz}}{I_x}(N_r)_g\right] + N_r(L_r)_g$ $- L_r(N_r)_g - L_\beta - \frac{I_{xz}}{I_x} N_\beta$	$Y_v[L_r(N_r)_g - N_r(L_r)_g]$ $+ L_\beta[N_r + (N_r)_g] - N_\beta[L_r + (L_r)_g]$	
$\frac{N^{r}_{\beta_g}}{s}$	$-(N_r)_g - \frac{I_{xz}}{I_z}(L_r)_g$	$Y_v\left[(N_r)_g + \frac{I_{xz}}{I_z}(L_r)_g\right] + L_p(N_r)_g$ $-N_p(L_r)_g - N_\beta - \frac{I_{xz}}{I_z} L_\beta$	$N_\beta L_p - L_\beta N_p$ $- Y_v[L_p(N_r)_g - N_p(L_r)_g]$	$\frac{g}{U_0}[L_\beta(N_r)_g - N_\beta(L_r)_g]$

Note: To convert to primed derivatives, eliminate all I_{xz} terms and substitute L' and N' for L and N, respectively.

Equations to calculate <u>flying qualities</u>:

First order characteristic equation:

$$T_p \cdot s + 1 = 0$$

For a negative time constant:

$$F(s) = \frac{1}{s - a} \qquad with \quad a > 0$$

$$t_{double} = \frac{\ln 2}{a}$$

Second order characteristic equation:

$$s^2 + 2\zeta\omega_n \cdot s + \omega_n^2 = 0$$

$$s_{1,2} = \sigma \pm j\,\omega_d$$

$$\sigma = -\zeta\,\omega_n$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{\omega_d}{\sigma}\right)^2}}$$

For a negative damping ratio calculate the time to double amplitude:

$$t_{double} = \frac{\ln 2}{-\zeta \cdot \omega_n}$$

For flying qualities from the Control Anticipation Parameter (CAP):

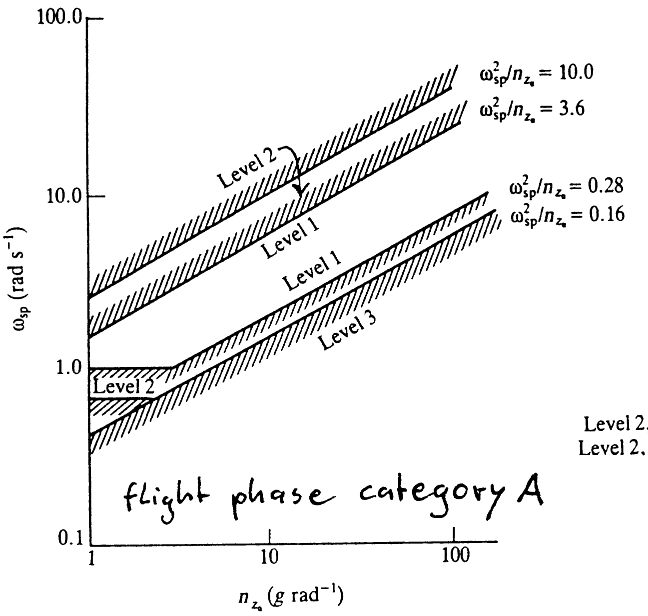
$$CAP = \frac{\omega_{sp}^2}{n_{z_\alpha}}$$

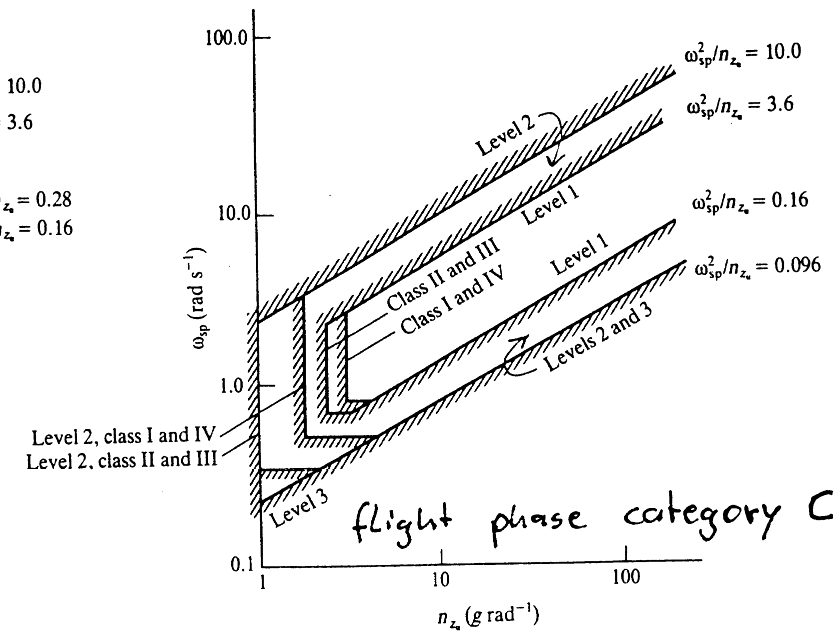
$$n_{z_\alpha} = -\frac{U_0}{g} \cdot Z_w$$

Dutch roll mode specification

Flight phase category	Class	Level 1 ζ_D rad/s	Level 1 $\zeta_D\,\omega_D$ rad/s	Level 1 ω_D	Level 2 ζ_D rad/s	Level 2 $\zeta_D\,\omega_D$ rad/s	Level 2 ω_D	Level 3 ζ_D rad/s	Level 3 $\zeta_D\,\omega_D$ rad/s	Level 3 ω_D
A	I, IV	0.19	0.35	1.0	0.02	0.05	0.4	0.02	-	0.4
A	I, III	0.19	0.35	0.4	0.02	0.05	0.4	0.02	-	0.4
B	all	0.08	0.15	0.4	0.02	0.05	0.4	0.02	-	0.4
C	I,IV	0.08	0.15	1.0	0.02	0.05	0.4	0.02	-	0.4
C	II, III	0.08	0.15	0.4	0.02	0.05	0.4	0.02	-	0.4

Note: Minimum values are specified. The governing damping requirement equals the largest value of ζ_D obtained from either column labelled ζ_D and $\zeta\omega_D$.





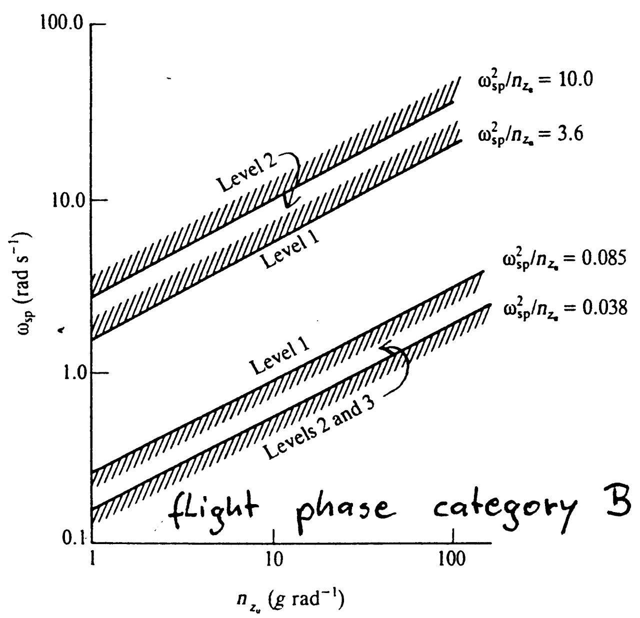


TABLE 4.7
Classification of airplanes

Class I	Small, light airplanes, such as light utility, primary trainer, and light observation craft
Class II	Medium-weight, low-to-medium maneuverability airplanes, such as heavy utility/search and rescue, light or medium transport/cargo/tanker, reconnaissance, tactical bomber, heavy attack and trainer for Class II
Class III	Large, heavy, low-to-medium maneuverability airplanes, such as heavy transport/cargo/tanker, heavy bomber and trainer for Class III
Class IV	High-maneuverability airplanes, such as fighter/interceptor, attack, tactical reconnaissance, observation and trainer for Class IV

TABLE 4.8
Flight phase categories

Nonterminal flight phase	
Category A	Nonterminal flight phases that require rapid maneuvering, precision tracking, or precise flight-path control. Included in the category are air-to-air combat ground attack, weapon delivery/launch, aerial recovery, reconnaissance, in-flight refueling (receiver), terrain-following, antisubmarine search, and close-formation flying
Category B	Nonterminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in the category are climb, cruise, loiter, in-flight refueling (tanker), descent, emergency descent, emergency deceleration, and aerial delivery.
Terminal Flight Phases:	
Category C	Terminal flight phases are notmally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this category are takeoff, catapult takeoff, approach, wave-off/go-around and landing.

Phugoid mode flying qualities

Level	Damping ratio of phugoid mode
1	≥ 0.04
2	≥ 0.0
3	An undamped oscillatory mode with a time to double amplitude of at least 55 s

Short period damping ratio specification

Flight phase category	Level 1		Level 2		Level 3	
	min.	max.	min.	max.	min.	max.
A and C	0.35	1.3	0.25	2.0	0.15	-
B	0.3	2.0	0.2	2.0	0.15	-

Roll mode time constant specification

Flight phase category	Class	T_R [s]		
		Level 1	Level 2	Level 3
A	I,IV	1.0	1.4	10.0
A	II, III	1.4	3.0	10.0
B	all	1.4	3.0	10.0
C	I, IV	1.0	1.4	10.0
C	II, III	1.4	3.0	10.0

Bank angle specification

Class	Flight phase category	Bank angle in fixed time		
		Level 1	Level 2	Level 3
I	A	60° in 1.3s	60° in 1.7s	60° in 2.6s
	B	60° in 1.7s	60° in 2.5s	60° in 3.4s
	C	30° in 1.3s	30° in 1.8s	30° in 2.6s
II	A	45° in 1.4s	45° in 1.9s	45° in 2.8s
	B	45° in 1.9s	45° in 2.8s	45° in 3.8s
	C	30° in 1.8s	30° in 2.5s	30° in 3.6s
III	A	30° in 1.5s	30° in 2.0s	30° in 3.0s
	B	30° in 2.0s	30° in 3.3s	30° in 5.0s
	C	30° in 2.5s	30° in 4.0s	30° in 6.0s
IV	A	90° in 1.3s	90° in 1.7s	90° in 2.6s
	B	90° in 1.7s	90° in 2.5s	90° in 3.4s
	C	30° in 1.1s	30° in 1.3s	30° in 2.0s

The time required to reach the bank angle has to be <u>less</u> than the specified maximum value.

Spiral mode stability specification

Flight phase category	Level		
	1	2	3
A and C	12s	8s	4s
B	20s	8s	4s

Selected aircraft control systems

A/C control system	feedback	applied control surfaces
pitch rate and relaxed static stability	pitch rate q	elevator
C^* - law	pitch rate q, normal acceleration $a_{z,cg}$	elevator
yaw damper	yaw rate r	rudder
roll rate damper	roll rate p	aileron
spiral mode stabilization	yaw rate r	aileron
pitch attitude control system	pitch attitude θ	elevator
roll angle	roll angle ϕ	aileron
roll angle with roll rate inner loop	roll rate p and roll angle ϕ	aileron
wing leveller: same as roll angle control with $\phi_{comm} = 0$	- - -	- - -
side slip suppression	sideslip β and yaw rate r	rudder
height hold system I	speed u, height h	elevator
height hold system II	height h, pitch rate q, pitch attitude θ	elevator
airspeed control system	speed u, acceleration $\partial u/\partial t$	thrust
direction control	heading ψ, command: roll angle ϕ	aileron

$$k1 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot \frac{U_p}{U_m} \cdot r^2$$

$$k2 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{U_p}{U_m}\right)^2 \cdot r^2$$

$$k3 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot r^3$$

$$k4 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot \frac{U_p}{U_m} \cdot r^3$$

$$k5 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot \frac{U_p}{U_m} \cdot r$$

$$k6 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot r^2$$

$$k7 = \frac{m_m}{m_p} \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{U_p}{U_m}\right)^2 \cdot r$$

$( )_p$ / $( )_m$	factor	$( )_p$ / $( )_m$	factor
X_u	k1	Y_v	k1
X_w	k1	Y_r^*	k3
X_δ	k2	Y_p^*	k3
Z_u	k1	Y_δ^*	k1
Z_w	k1	N_β	k7
Z_w	k3	N_r	k1
Z_q	k4	N_p	k1
Z_δ	k2	N_δ	k7
M_u	k5	L_β	k7
M_w	k5	L_r	k1
M_w	k6	L_p	k1
M_q	k1	L_δ	k7
M_δ	k7		