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# MAEFISTO -

# Multidisciplinary Aircraft Design in the Industrial Environment

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**19. Kurzfassung**

Multidisciplinary aircraft design in an industrial environment can be managed with a methodology called **Multidisciplinary Design Optimisation** (MDO). MDO helps to systematically structure design parameters and helps to eliminate as many feedback loops from the traditional design process as possible. Moreover, MDO maintains an overview of the design by global **system sensitivity derivatives**. The global system sensitivity derivatives are calculated from partial derivatives. Each partial derivative requires only input from one discipline. MDO keeps the human expert in the global design loop: Guided by system sensitivity derivatives, improvements to the design are discussed in a central team. Through **continuous improvements**, the team guides the design to a practical **optimum**.

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# 1    Introduction

In mathematical terms, **the aircraft design problem** can be stated as a general constrained multi-objective optimisation task. The problem is to minimise one or more objective functions[1]

( 1 )    $f_k(\mathbf{z})$    $k = 1$ to $q$ , the number of objectives .

**Fig. 1** provides some insight into optimisation goals in aircraft design. Optimisation goals can be summarised by: "maximise manufacturer's profit at committed Direct Operating Costs (DOC's)".

Evidently, certain constraints have to be met:

( 2 )    $g_j(\mathbf{z}) \leq 0$    $j = 1$ to $p$ , the number of constraints .

The design variables have to stay within certain limits. These limits are nothing more than additional constraints which are specifically expressed as lower and upper bounds:

(3)    $z_i^l \ \leq \ z_i \ \leq \ z_i^u \ i = 1$ to $o$ , the number of design variables.

It is convenient to distinguish between given (known, independent or input) design variables - denoted here as $x$ - and those (unknown, dependent or output) design variables which are determined during the design process - denoted here as $y$ . Both types - independent and dependent - design variables are elements of the vector $\mathbf{z}$ that includes all parameters:

( 4 )    $\mathbf{z} = \left[ y_1, y_2, ..., y_n, x_1, x_2, ..., x_m \right]^T$ .

$m$ design variables are considered known (or independent), whereas $n$ design variables are considered unknown (or dependent): $o = m + n$ .

## 1.1    The Traditional Iterative Approach to Aircraft Design

The traditional approach to aircraft design follows a "manual" iterative procedure (see e.g. [TORENBEEK 82], [ROSKAM 89] and [RAYMER 92]). This traditional approach to aircraft design is illustrated in **Fig. 2**. Fig. 2 was inspired by the general design procedure as proposed by [TORENBEEK 82] (Fig. 5-18). Note, that Fig. 2 takes a very rough view on the aircraft design process. Therefore, only the outer iteration loops are shown. The traditional iterative design procedure is also a hierarchic design procedure because each main tasks can be broken down into several smaller sub-tasks. Traditionally, main tasks are taken care of one after the other. Fig. 2 lists the input parameters $x$ to aircraft design, output parameters $y$ follow from subsequent calculations.

The independent design variables $x$ can be grouped into
- requirements,
- technology dependent parameters and physical parameters,
- freely chosen parameters.

---

[1]    Notation: Matrices are indicated by upper-case bold letters, vectors by lower-case bold letters and scalars by plain lower-case letters.

The most fundamental **requirements** are payload and range. **Technology dependent parameters** (e.g. from aerodynamics, propulsion and materials) and physical parameters (e.g. air density) are input values to almost all calculations. They can only be influenced within a very limited interval. The designer has to make decisions as e.g. for or against a canard configuration versus a conventional tail aft configuration. These design decisions outside the iteration loops of the traditional design process are characterised by **freely chosen parameters**.

Still in the 1960th, [CORNING 64] considered it impossible to succeed any different in aircraft design than by iterative procedures:

> *In the design of the airplane, it would be convenient and simple if an exact solution could be reached with the first estimate. However, due to the interrelation of different components of the airplane to each other and to the airplane as a unit, this is* <u>impossible</u>.

With the advent of the computer, the traditional iterative aircraft design approach was strongly supported by computer tools. Examples of commercial tools of this sort are *AAA* [DARCORP] and *PIANO* [SIMOS 96]. However, these tools were written for aircraft design departments as potential users and not for a fully multidisciplinary design environment.

## 1.2 Other Approaches to Aircraft Design

Furthermore, expectations had been high to automate the aircraft design process with extended computer usage. There are two main approaches to automated aircraft design:

- Formal **optimisation procedures** have been applied to solve aircraft design problems (see e.g. [DOVI 90], [VAN DER VELDEN 94]). Optimisation procedures aim at finding a minimum to a single objective function or to a weighted sum of objective functions. A single objective function could e.g. calculate Direct Operation Costs (DOC). The optimisation procedure will then aim at minimising the calculated Direct Operation Costs.

- Also **expert systems** have been applied to generate an aircraft design. ADROIT (Aircraft Design by Regulation of Independent Tasks) is an example of an expert system which concentrates on wing design and works with a mixture of ordinary procedural calculations and rule based reasoning [ALSINA 87].

Both computerised approaches show certain **deficiencies**:

- Formal optimisation - used on its own - does not sufficiently recognise the role of the human mind as the leading force in the design process.

- Expert systems, as generally implemented with current techniques, have very limited means of knowledge representation, and therefore cannot achieve the requirements of the aircraft design process.

The main problem with most automated aircraft design methods is, however, that they have a "monolithic" structure. They assume all aircraft design knowledge to be gathered in form of source code in one program. This assumption is unrealistic in an industrial environment, where the multidisciplinary and distributed nature of the aircraft design process has to be acknowledged.

## 1.3    Multidisciplinary Design Optimisation (MDO)

This paper favours an approach known as Multidisciplinary Design Optimisation (MDO). MDO coordinates the couplings between specialised disciplines during the aircraft design process in a very systematic way. The specialised disciplines have a very detailed view on their subject and maintain locally their own specialised computer tools. At the same time the specialised disciplines provide that bit of information to the MDO which is relevant in a general aircraft design context. The information provided, is backed up by the entire disciplinary knowledge including experience from other aircraft projects and proven computing capabilities.

The theoretical background of MDO is promoted by a Technical Committee (TC) of the American Institute of Aeronautics and Astronautics (AIAA) [AIAA 91]. The MDO methodology was originally developed and published in several papers by *J. Sobieski*, working at NASA Langley (see e.g. [SOBIESKI 90a]).   *J. Sobieski* is also chairman of the AIAA TC on MDO. An industrialised version of the MDO-approach is presently being developed at Daimler-Benz Aerospace Airbus.  The industrialised MDO-approach is called MAEFISTO: **M**ultidisciplinary **Ae**roelastics **F**light Dynamics & **I**ndustrialised **St**ructural Dynamics **O**ptimisation.

# 2    Fundamentals of MDO

Aircraft design is based on answering "what if" questions. Mathematically, a variation $\Delta x$ of one specific independent design variable $x$ is fed into the design process. After re-evaluating the entire design - traditionally with all its iterative loops - the consequences of the design change (given as $\Delta y$) on dependent design variables $y$ can be found. This approach requires an enormous computational effort - especially if the aircraft design is highly coupled. MDO helps to reduce the computational effort of a series of "what if" questions by introducing the *global sensitivity equation* for the system under discussion.

## 2.1 Deriving the Global Sensitivity Equation for a Set of Three Independent Design Variables

For the sake of simplicity we will (for the moment) only consider a small system with one independent design variable $x$ and with three dependent design variables $y_1$, $y_2$ and $y_3$. **Independent design variables** $x$ follow directly from design requirements, technology dependent parameters or freely chosen aircraft design parameters. **Dependent design variables** $y$ follow rigidly from calculations based on independent design variables $x$. Dependent design variables specify further aircraft parameters that do not allow for any deviation from their calculated value, if the aircraft shall perform as intended. Due to the fact that we treat systems in this context as being coupled, we assume $y_1$ to be a function of all the other independent and dependent design variables. The same is assumed for $y_2$ and $y_3$. Hence

$$y_1 = f_1(x, y_2, y_3)$$

( **5** )
$$y_2 = f_2(x, y_1, y_3)$$

$$y_3 = f_3(x, y_1, y_2) \quad .$$

$f_1$, $f_2$ and $f_3$ can be equations of any form (nonlinear, transcendental, ...). These functions may be linearised in the neighbourhood (denoted by $x$, $y_1$, $y_2$, $y_3$) of the solution for a possible system design (denoted by $_0x$, $_0y_1$, $_0y_2$, $_0y_3$). Using a Taylor series abridged to its linear part yields

$$y_1 = {_0y_1} - \frac{\partial f_1}{\partial x}(x - {_0x}) - \frac{\partial f_1}{\partial y_2}(y_2 - {_0y_2}) - \frac{\partial f_1}{\partial y_3}(y_3 - {_0y_3})$$

$$y_2 = {_0y_2} - \frac{\partial f_2}{\partial x}(x - {_0x}) - \frac{\partial f_2}{\partial y_1}(y_1 - {_0y_1}) - \frac{\partial f_2}{\partial y_3}(y_3 - {_0y_3})$$

( **6** )

$$y_3 = {_0y_3} - \frac{\partial f_3}{\partial x}(x - {_0x}) - \frac{\partial f_3}{\partial y_1}(y_1 - {_0y_1}) - \frac{\partial f_3}{\partial y_2}(y_2 - {_0y_2}) \quad .$$

To write equation (6) more compact, we define

( **7** )
$$\Delta x = x - {_0x}$$

and

$$\Delta y_1 = y_1 - {_0y_1}$$

( **8** )
$$\Delta y_2 = y_2 - {_0y_2}$$

$$\Delta y_3 = y_3 - {_0y_3} \quad .$$

Substituting equations (7) and (8) into (6) and rearranging the results leads to

$$\Delta y_1 - \frac{\partial f_1}{\partial y_2} \Delta y_2 - \frac{\partial f_1}{\partial y_3} \Delta y_3 = \frac{\partial f_1}{\partial x} \Delta x$$

( 9 )
$$-\frac{\partial f_2}{\partial y_1} \Delta y_1 + \Delta y_2 - \frac{\partial f_2}{\partial y_3} \Delta y_3 = \frac{\partial f_2}{\partial x} \Delta x$$

$$-\frac{\partial f_3}{\partial y_1} \Delta y_1 - \frac{\partial f_3}{\partial y_2} \Delta y_2 + \Delta y_3 = \frac{\partial f_3}{\partial x} \Delta x$$

This divided by $\Delta x$ and written in matrix form is

( 10 )
$$\begin{bmatrix} 1 & -\dfrac{\partial f_1}{\partial y_2} & -\dfrac{\partial f_1}{\partial y_3} \\[2mm] -\dfrac{\partial f_2}{\partial y_1} & 1 & -\dfrac{\partial f_2}{\partial y_3} \\[2mm] -\dfrac{\partial f_3}{\partial y_1} & -\dfrac{\partial f_3}{\partial y_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\Delta y_1}{\Delta x} \\[2mm] \dfrac{\Delta y_2}{\Delta x} \\[2mm] \dfrac{\Delta y_3}{\Delta x} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x} \\[2mm] \dfrac{\partial f_2}{\partial x} \\[2mm] \dfrac{\partial f_3}{\partial x} \end{bmatrix}.$$

Equation (10) is called *global sensitivity equation* and is of the form $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$. Matrix $\mathbf{A}$ is a Jacobian matrix and depends only on the coupling of the original equations (5) and contains the *sensitivity derivatives*. The *sensitivity derivatives* are partial derivatives. Each of them requires only knowledge of one system function $f_1$, $f_2$ **or** $f_3$ for its calculation.

The terms $\Delta y_i / \Delta x$ in vector $\mathbf{x}$ are called *system sensitivity derivatives*. Each of them accounts fully for all of the couplings in the system. **The *system sensitivity derivatives* answer the "what if" questions.**

The expressions $\partial f_i / \partial x$ in vector $\mathbf{b}$ show the sensitivity of a single dependent variable $y_i$ with respect to the independent variable $x$.

If equations $f_1$, $f_2$ and $f_3$ were uncoupled, all the partial derivatives in $\mathbf{A}$ were zero and the *system sensitivity derivatives* would be equal to the sensitivity derivatives in $\mathbf{b}$. The virtue of equation (10) lies in the fact that with only the knowledge of system functions $f_1$, $f_2$ and $f_3$ **separately** (and the use of **partial** derivatives), the **total** change of dependent system parameters ( $y$ ) with changes of the independent parameter ( $x$ ) can be determined - no matter how complex and coupled the considered system might be.

The specialised disciplines in the aircraft design process are expected to provide "their" *sensitivity derivatives* $- \partial f_i / \partial y_j$ in the Jacobian together with "their" *sensitivity derivatives* $\partial f_i / \partial x$ in vector $\mathbf{b}$.

A solver for sets of linear equations has to be applied to solve for **x** in equation (10). If the sensitivities due to changes of another independent variable are of interest, only vector **b** has to be re-evaluated and the global sensitivity equation (10) has to be solved again for **x**.

## 2.2 Deriving the Global Sensitivity Equation for *n* Independent Design Variables

Systems which can be described by three equations would not be considered "complex". The foregoing simplification was chosen for the sake of clarity but can now - without further discussion - be extended to any number of dependent variables *y*. Each dependency is established by one equation. Constraints as well as upper and lower limits to design variables are treated separately by each discipline. In general, the sensitivity matrix **A** is a square matrix of dimension *n* x *n*.

$$
( 11 ) \quad
\begin{bmatrix}
1 & -\dfrac{\partial f_1}{\partial y_2} & \cdots & -\dfrac{\partial f_1}{\partial y_n} \\[2ex]
-\dfrac{\partial f_2}{\partial y_1} & 1 & \cdots & -\dfrac{\partial f_2}{\partial y_n} \\[2ex]
\vdots & \vdots & \vdots & \vdots \\[2ex]
-\dfrac{\partial f_n}{\partial y_1} & -\dfrac{\partial f_n}{\partial y_2} & \cdots & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
\dfrac{\Delta y_1}{\Delta x} \\[2ex]
\dfrac{\Delta y_2}{\Delta x} \\[2ex]
\vdots \\[2ex]
\dfrac{\Delta y_n}{\Delta x}
\end{bmatrix}
=
\begin{bmatrix}
\dfrac{\partial f_1}{\partial x} \\[2ex]
\dfrac{\partial f_2}{\partial x} \\[2ex]
\vdots \\[2ex]
\dfrac{\partial f_n}{\partial x}
\end{bmatrix}
\quad .
$$

**Fig. 3** relates the aircraft design parameters from Fig. 2 to the parameters as given in the *global sensitivity equation* (11).

## 2.3 Obtaining Consistent Design Variables by Newton's Method

Before it is possible to look at sensitivities and design optimisation, consistent design variables have to be found. All dependent and independent design variables have to meet the system equations (as e.g. given with equations (5)). Any method capable of solving sets of general nonlinear equations may be applied. *Newton's Method* [HOFFMAN 92] can e.g. be applied to calculate dependent variables from given independent variables as proposed by [GROSSMAN 90]. For the application of *Newton's Method*, equations (5) are rewritten in the form:

$$f_1\big(x, y_1, y_2, y_3\big) = 0$$

$$( \, \mathbf{12} \, ) \qquad f_2\big(x, y_1, y_2, y_3\big) = 0$$

$$f_3\big(x, y_1, y_2, y_3\big) = 0 \quad .$$

*Newton's Method* states that starting from an initial guess of $^0\mathbf{y} = [^0y_1, \, ^0y_2, \, ^0y_3]^T$, the dependent design variables can be found by repetitively solving for $\Delta\mathbf{y}$ in $\mathbf{A} \cdot \Delta\mathbf{y} = \mathbf{f}$ and calculating values for iteration steep $t+1$ from the previous iteration step $t$ with

$$( \, \mathbf{13} \, ) \qquad ^t\Delta \, y \; = \; ^t\mathbf{A}^{-1} \cdot \, ^t\mathbf{f} \qquad \text{and}$$

$$( \, \mathbf{14} \, ) \qquad ^{t+1}\mathbf{y} = \, ^t\mathbf{y} + ^t\Delta\mathbf{y} \quad .$$

Combining equations (13) and (14) gives

$$( \, \mathbf{15} \, ) \qquad ^{t+1}\mathbf{y} = \, ^t\mathbf{y} + \, ^t\mathbf{A}^{-1} \cdot \, ^t\mathbf{f} \, .$$

Convergence is reached if $\left|^t\Delta y_i\right| \leq \varepsilon$ and/or $\left|^{t+1}f_i - \, ^tf_i\right| \leq \delta$. $\mathbf{A} \cdot \Delta\mathbf{y} = \mathbf{f}$ expands e.g. for three dependent design variables to

$$( \, \mathbf{16} \, ) \qquad \begin{bmatrix} 1 & -\dfrac{\partial f_1}{\partial y_2} & -\dfrac{\partial f_1}{\partial y_3} \\[2mm] -\dfrac{\partial f_2}{\partial y_1} & 1 & -\dfrac{\partial f_2}{\partial y_3} \\[2mm] -\dfrac{\partial f_3}{\partial y_1} & -\dfrac{\partial f_3}{\partial y_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta y_1 \\[2mm] \Delta y_2 \\[2mm] \Delta y_3 \end{bmatrix} = \begin{bmatrix} f_1 \\[2mm] f_2 \\[2mm] f_3 \end{bmatrix} .$$

*Newton's Method* as described above, is considered to be the standard approach for solving sets of general nonlinear equations. Other methods exist however, and might be beneficial in certain circumstances. **Fig. 4** shows, how the traditional hierarchic and iterative design process (compare with Fig. 2) has now become a non-hierarchic design process: All design tasks are taken care of at the same time.

## 2.4 Using Newton's Method and the Global Sensitivity Equation for Multidisciplinary Aircraft Design Optimisation

A systematic aircraft design procedure may use *Newton's Method* and the *global sensitivity equation.* The computerised, but human driven design procedure may be organised in the following steps as proposed by [SOBIESKI 90a]:

**STEP 1**   Use *Newton's Method* to obtain a set of consistent dependent design variables **y** from the independent design variables **x**.

**STEP 2**   Assemble and solve the *global sensitivity equation* to obtain the *system sensitivity derivatives* $\frac{\Delta y_i}{\Delta x_j}$

**STEP 3**   Use the *system sensitivity derivatives* to decide how **x** has to be changed in order
- to take care of constraints $g_j(\mathbf{z}) \leq 0$ that might still be violated at this stage,
- to improve system behaviour by minimising the objective function $f_k(\mathbf{z})$.

**STEP 4**   Repeat from STEP 1 with another change to the same or a different $x_j$ and continue in this way until satisfactory system behaviour is attained.

**Fig. 5** (adapted from [SOBIESKI 90b], Fig.9) shows how the design procedure given above translates into every day engineering life: The disciplinary specialists provide an input to overall aircraft optimisation in form of "their" sensitivity derivatives to the Jacobean matrix **A** in equations (10), (11) and (16) and to vector **b** in equation (10) and (11). After calculating consistent design variables with *Newton's Method*, the *system sensitivity derivatives* are calculated. They can be used as principal means of communication among the disciplinary specialists regarding the improvement of the design. [SOBIESKI 90b] describes the **communication process based on** *system sensitivity derivatives*:

> *The specialists are called upon to generate information in their disciplines and to augment it with the partial sensitivity derivatives of their outputs with respect to inputs and to the design variables. After the partial sensitivity derivatives are used in the global sensitivity equation to calculate the system sensitivity derivatives, the specialists are being called upon again to decide on the design modifications, using system sensitivity derivatives with the aid of formal optimization, and/or judgementally including due consideration to the nonquantitative aspects of design.*

# 3   Numerical Examples to MDO

The approach to multidisciplinary aircraft design as presented above will now be demonstrated with two simple numerical examples.

## 3.1   Getting Started with MDO

A first insight into the mathematics presented above can most easily be obtained with a very simple example. We consider two coupled equations

**( 17 )**        $y_1 = x_1 \cdot y_2^2$

**( 18 )**        $y_2 = x_2 \cdot y_1$ .

This is a set of coupled nonlinear equations (because one equation is nonlinear). This set of equations can be solved quite readily which allows us to check our further calculations based on the derivations above: We obtain a trivial solution $y_1 = 0$; $y_2 = 0$ and a nontrivial solution

$$y_1 = \frac{1}{x_1 \cdot x_2^2} \quad \text{and} \quad y_2 = \frac{1}{x_1 \cdot x_2}.$$

**STEP 1**:     For the application of ***Newton's Method*** we write

( **19** )     $f_1(x_1, x_2, y_1, y_2) = x_1 \cdot y_2^2 - y_1 = 0$

( **20** )     $f_2(x_1, x_2, y_1, y_2) = x_2 \cdot y_1 - y_2 = 0$

( **21** )     $- \dfrac{\partial f_1}{\partial y_2} = -2 \cdot x_1 \cdot y_2$

( **22** )     $- \dfrac{\partial f_2}{\partial y_1} = -x_2$

Equations (19) through (22) can now be combined to $\mathbf{A} \cdot \Delta \, \mathbf{y} = \mathbf{f}$ which is

( **23** )     $\begin{bmatrix} 1 & -2 \cdot x_1 \cdot y_2 \\ -x_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta \, y_1 \\ \Delta \, y_2 \end{bmatrix} = \begin{bmatrix} x_1 \cdot y_2^2 - y_1 \\ x_2 \cdot y_1 - y_2 \end{bmatrix}.$

With

( **24** )     $\mathbf{A}^{-1} = \begin{bmatrix} \dfrac{1}{1 - 2 \cdot x_1 \cdot x_2 \cdot y_2} & \dfrac{2 \cdot x_1 \cdot y_2}{1 - 2 \cdot x_1 \cdot x_2 \cdot y_2} \\ \dfrac{x_2}{1 - 2 \cdot x_1 \cdot x_2 \cdot y_2} & \dfrac{1}{1 - 2 \cdot x_1 \cdot x_2 \cdot y_2} \end{bmatrix}$

and from equations (15)

( **25** )     $^{t+1}\mathbf{y} = \, ^{t}\mathbf{y} + \, ^{t}\mathbf{A}^{-1} \cdot \, ^{t}\mathbf{f}$   .

For given independent variables $x_1 = x_2 = 2$ and an initial guess $\mathbf{y} = [1; 1]^{\mathrm{T}}$ the iteration yields results presented in **Table 1**.

**Table 1**:     Solving Equations (17) and (18)
by Newton's Method with $x_1 = x_2 = 2$

| $t$ | $y_1$ | $y_2$ |
|---|---|---|
| 0 | 1,0000 | 1,0000 |
| 1 | 0,2857 | 0,5714 |
| 2 | 0,1828 | 0,3657 |
| ... | ... | ... |
| 5 | 0,1250 | 0,2500 |

The results form Table 1 are in agreement with the nontrivial solution calculated above.

Lets now assume an objective function $f = f(x_1, x_2, y_1, y_2) = y_1 + y_2$. The question is in which direction do we have to change $x_1$ and $x_2$ in order to minimise our objective function.

One way to obtain this answer would be to apply *Newton's Method* again with a different input to $x_1$ and $x_2$. Let's choose $\Delta x_1 = +0,1$ and $\Delta x_2 = +0,1$ (one modification at a time). Following the iteration, new values for $y_1$ and $y_2$ are obtained. In addition, exact values of the *system sensitivity derivatives* $\Delta y_1 / \Delta x_j$ and $\Delta y_2 / \Delta x_j$ can be calculated. The results of such an approach are given in **Table 2**.

SOBIESKI proposes another more direct procedure to calculate *system sensitivity derivatives*. Following **STEP 2** from Chapter 2.4 we need - in addition to the Jacobean **A** - the partial sensitivity derivatives of functions *f* with respect to an independent parameter *x*. For a variation of $x_1$ we need

( **26** )         $\dfrac{\partial f_1}{\partial x_1} = y_2^2$     and     $\dfrac{\partial f_2}{\partial x_1} = 0$       which yields    $\mathbf{b}_1 = \begin{bmatrix} y_2^2 \\ 0 \end{bmatrix}$.

For a variation of $x_2$ we need

( **27** )         $\dfrac{\partial f_1}{\partial x_2} = 0$     and     $\dfrac{\partial f_2}{\partial x_2} = y_1$     which yields    $\mathbf{b}_2 = \begin{bmatrix} 0 \\ y_1 \end{bmatrix}$.

In comparison to *Newton's Method*, the additional effort to calculate these derivatives is probably more than compensated by not having to iterate towards a solution: Following SOBIESKI, we get an approximation of $\Delta y_i / \Delta x_j$ from a single step

( **28** )         $\dfrac{\Delta \mathbf{y}}{\Delta x_1} = \mathbf{A}^{-1} \cdot \mathbf{b}_1$         respectively    $\dfrac{\Delta \mathbf{y}}{\Delta x_2} = \mathbf{A}^{-1} \cdot \mathbf{b}_2$.

If required, also approximations to $\Delta \mathbf{y}$ and $\mathbf{y}$ may be calculated. This was done for comparison with *Newton's Method* and is presented in **Table 2**.

**Table 2**: SOBIESKI'S Sensitivity Calculation Compared with Newton's Method

| | $y_1$ | $y_2$ | $\Delta y_1$ | $\Delta y_2$ | $\Delta y_1 / \Delta x_1$ | $\Delta y_2 / \Delta x_1$ |
|---|---|---|---|---|---|---|
| | $x_1 = 2,1$ | $x_2 = 2,0$ | $\Delta x_1 = +0,1$ | | | |
| Newton's Method (exact) | 0,11905 | 0,23810 | -0,00595 | -0,01190 | -0,0595 | -0,1190 |
| SOBIESKI'S Method | 0,11875 | 0,23750 | -0,00625 | -0,01250 | -0,0625 | -0,1250 |
| relative error | -0,25 % | -0,25 % | | | | |
| | $x_1 = 2,0$ | $x_2 = 2,1$ | $\Delta x_2 = +0,1$ | | $\Delta y_1 / \Delta x_2$ | $\Delta y_2 / \Delta x_2$ |
| | $y_1$ | $y_2$ | $\Delta y_1$ | $\Delta y_2$ | | |
| Newton's Method (exact) | 0,11338 | 0,23810 | -0,01162 | -0,01190 | -0,1162 | -0,1190 |
| SOBIESKI'S Method | 0,11250 | 0,23750 | -0,01250 | -0,01250 | -0,1250 | -0,1250 |
| relative error | -0,77 % | -0,25 % | | | | |

Table 2 shows that we have to use positive $\Delta x$ to obtain negative $\Delta y$ which minimise our objective function. Changes to $x_1$ and $x_2$ have the same impact on $y_2$. Changes to $x_2$ have a higher impact on $y_1$ than equal changes to $x_1$. Therefore, striving for a minimisation of the objective function, it will be "less expensive" to change $x_2$ than $x_1$. This discussion shows in which way **sensitivities can be used in decision making** related to aircraft design.

The application of *Newton's Method* for sensitivities, **has the advantage of better accuracy at the cost of higher computational effort**. Sensitivities are only correct within small step sizes of the independent variables. In our example for a step size $\Delta x_2 = +0,5$, the relative error on $y_1$ would already be as high as -21%.

**Fig. 6** shows the two simple example function. The functions intersect at two points. The intersection at the origin [0; 0] is the trivial solution. The other intersection is the nontrivial solution. With increasing $x_1$ and $x_2$ both $y_1$ and $y_2$ decrease. The partial derivatives with respect to $y_1$, $y_2$ and $x$ are indicated. They are used to determine the global sensitivities $\Delta \mathbf{y} / \Delta x_j$.

## 3.2 MDO Applied to the *Breguet* Range Equation

The **independent parameters** consist of **requirements**
- to carry a payload $PL = 33000$ kg
- over a distance $R = 13500$ km,

**technology related parameters** (valid for cruising flight):
- lift coefficient $C_L = 0.5$,
- drag coefficient $C_D = 0.0277$,
- thrust specific fuel consumption $SFC = 17$ mg / N / s,
- ratio: operating weight empty, *OWE* to *MTOW*: $OWE / MTOW = 0.497$,

and physical parameters experienced in the chosen cruising altitude:
- earth acceleration $g = 9.80$ m / s$^2$,
- air density $\rho = 0.3$ kg / m$^3$,

as well as **"freely chosen" parameters**:
- wing loading $W / S = 700$ kg / m$^2$.

Fuel reserves are considered to be included in the payload. The task is to find three **dependent parameters**:
- maximum take-off weight, *MTOW*,
- landing weight, *LW*,
- wing area, *S*.

The problem will be solved based on a form of the **Breguet Range Equation** as presented e.g. in [ANDERSON 89]. This equation assumes constant lift and drag coefficients and decreasing airspeed as fuel is being burned during flight:

( **29** ) $$R = 2\sqrt{\frac{2}{\rho g S}} \cdot \frac{\sqrt{C_L}}{C_D} \cdot \frac{1}{SFC} \cdot \sqrt{MTOW} \cdot \left(1 - \sqrt{\frac{LW}{MTOW}}\right) .$$

Three equations are needed (compare to equations (5) ) to calculate the three dependent parameters:

( **30** ) $$y_1 = MTOW = \left(k_R \cdot \sqrt{S} + \sqrt{LW}\right)^2 \quad,$$

( **31** ) $$y_2 = LW = k_W \cdot MTOW + PL \quad,$$

( **32** ) $$y_3 = S = k_S \cdot MTOW \quad.$$

These equations include also the independent parameters which are combined to four parameters named

( **33** ) $$x_1 = k_R = R \cdot \sqrt{0.125 \cdot \rho\, g} \cdot \frac{C_D}{\sqrt{C_L}} \cdot SFC \quad,$$

( **34** ) $$x_2 = k_W = OWE\, /\, MTOW \quad,$$

( **35** ) $$x_3 = k_S = \frac{1}{W\,/\,S} \quad,$$

( **36** ) $$x_4 = PL \quad.$$

For the application of *Newton's Method*, equations (12) are rewritten:

( **37** ) $$f_1 = \left(k_R \cdot \sqrt{S} + \sqrt{LW}\right)^2 - MTOW = 0 \quad,$$

( **38** ) $$f_2 = k_W \cdot MTOW + PL - LW = 0 \quad,$$

( **39** ) $$f_3 = k_S \cdot MTOW - S = 0 \quad.$$

and differentiated

( **40** ) $$-\frac{\partial f_1}{\partial y_2} = 1 - k_R \cdot \sqrt{\frac{S}{LW}} \quad,$$

( **41** ) $$-\frac{\partial f_1}{\partial y_3} = -k_R\left(k_R + \sqrt{\frac{LW}{S}}\right) \quad,$$

( **42** ) $$-\frac{\partial f_2}{\partial y_1} = -k_W \quad,$$

( **43** )    $-\dfrac{\partial f_2}{\partial y_3} = 0$   ,

( **44** )    $-\dfrac{\partial f_3}{\partial y_1} = -k_S$   ,

( **45** )    $-\dfrac{\partial f_3}{\partial y_2} = 0$   .

These partial derivatives are entered into equation (16) which then reads

$$
( \mathbf{46} ) \quad
\begin{bmatrix}
1 & 1 - k_R\sqrt{\dfrac{S}{LW}} & -k_R\left(k_R + \sqrt{\dfrac{LW}{S}}\right) \\[2ex]
-k_W & 1 & 0 \\[2ex]
-k_S & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
\Delta y_1 \\[1ex] \Delta y_2 \\[1ex] \Delta y_3
\end{bmatrix}
=
\begin{bmatrix}
\left(k_R \cdot \sqrt{S} + \sqrt{LW}\right)^2 - MTOW \\[2ex]
k_W \cdot MTOW + PL - LW \\[2ex]
k_S \cdot MTOW - S
\end{bmatrix}
$$

**Table 3** shows the iterative solution of $y_1$, $y_2$ and $y_3$ with *Newton's Method*. 40 iterations were necessary to solve for *MTOW* and *LW* with an accuracy of a few kg.

**Table 3:**    Iterative Solution with Newton's Method

| $t$ | MTOW [kg] | LW [kg] | S [m²] |
|---|---|---|---|
| 0 | 125000 | 80000 | 270 |
| 1 | 118298 | 91794 | 168 |
| 2 | 136983 | 101080 | 195 |
| 3 | 153018 | 109050 | 218 |
| 10 | 216552 | 140626 | 309 |
| 20 | 242134 | 153340 | 345 |
| 40 | 248699 | 156603 | 355 |

*System sensitivity derivatives* could now be calculated as shown in Chapter 3.1.

# 4    The Coupling-Matrix of Disciplines

Aircraft design activities consist of various contributing analysis (CA) usually associated with a particular specialised engineering discipline, or a distinct physical part (a subsystem) of the system, or both. Partitioning of the system analysis into separate but coupled CA's amounts to a **system decomposition**. In aircraft design, like in most engineering problems, both types of decomposition are used simultaneously to break the large task into smaller ones. The mathematics presented here applies equally to both types of decomposition.

## 4.1 Deriving the Coupling-Matrix of Disciplines

The coupling-matrix of disciplines - also called *N*-square Matrix - can be derived from the global sensitivity equation (10). For this purpose, the global sensitivity equation is divided into various subsets. The principle of the **decomposition and composition** can be shown conveniently with two partitions called $\alpha$ and $\beta$. The principle can then easily be generalised to arbitrarily large numbers of partitions.

We now assume, as example, a system having four independent variables $y_1$, $y_2$, $y_3$, $y_4$. Additionally, we recognise that there exists more than just one independent design variable $x$, but rather a vector **x** including all the independent design variables. From the independent design variables in $\mathbf{x} = \left[ x_1, x_1, ..., x_k, ..., x_m \right]^T$ may only one at a time (denoted by $x_k$) be taken for a sensitivity analysis. Introducing $x_k$ into equation (11) yields

$$( 47 ) \quad
\begin{bmatrix}
1 & -\dfrac{\partial f_1}{\partial y_2} & -\dfrac{\partial f_1}{\partial y_3} & -\dfrac{\partial f_1}{\partial y_4} \\[2mm]
-\dfrac{\partial f_2}{\partial y_1} & 1 & -\dfrac{\partial f_2}{\partial y_3} & -\dfrac{\partial f_2}{\partial y_4} \\[2mm]
-\dfrac{\partial f_3}{\partial y_1} & -\dfrac{\partial f_3}{\partial y_2} & 1 & -\dfrac{\partial f_3}{\partial y_4} \\[2mm]
-\dfrac{\partial f_4}{\partial y_1} & -\dfrac{\partial f_4}{\partial y_2} & -\dfrac{\partial f_4}{\partial y_3} & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
\dfrac{\Delta y_1}{\Delta x_k} \\[2mm]
\dfrac{\Delta y_2}{\Delta x_k} \\[2mm]
\dfrac{\Delta y_3}{\Delta x_k} \\[2mm]
\dfrac{\Delta y_4}{\Delta x_k}
\end{bmatrix}
=
\begin{bmatrix}
\dfrac{\partial f_1}{\partial x_k} \\[2mm]
\dfrac{\partial f_2}{\partial x_k} \\[2mm]
\dfrac{\partial f_3}{\partial x_k} \\[2mm]
\dfrac{\partial f_4}{\partial x_k}
\end{bmatrix}
.
$$

Furthermore, we assume $y_1$ and $y_2$ to belong to the first subset (or contributing analysis, CA) "$\alpha$" and $y_3$ and $y_4$ to belong to the second subset "$\beta$". For a **composition** step, subsets are treated as black boxes with abbreviations like

$$( 48 ) \quad -\frac{\partial \mathbf{f}_\alpha}{\partial \mathbf{y}_\alpha} =
\begin{bmatrix}
1 & -\dfrac{\partial f_1}{\partial y_2} \\[2mm]
-\dfrac{\partial f_2}{\partial y_1} & 1
\end{bmatrix}
$$

used for sensitivity derivatives on the main diagonal of **A** and abbreviations like

$$( 49 ) \qquad - \frac{\partial \mathbf{f}_\alpha}{\partial \mathbf{y}_\beta} = \begin{bmatrix} - \dfrac{\partial f_1}{\partial y_3} & - \dfrac{\partial f_1}{\partial y_4} \\ \\ - \dfrac{\partial f_2}{\partial y_3} & - \dfrac{\partial f_2}{\partial y_4} \end{bmatrix}$$

used for sensitivity derivatives belonging to off-diagonal elements.

$$( 50 ) \qquad \frac{\Delta \mathbf{y}_\alpha}{\Delta \mathbf{x}_k} = \begin{bmatrix} \dfrac{\Delta y_1}{\Delta x_k} \\ \\ \dfrac{\Delta y_2}{\Delta x_k} \end{bmatrix}$$

is an example of an abbreviated element of $\mathbf{x}$ and

$$( 51 ) \qquad \frac{\Delta \mathbf{f}_\alpha}{\Delta x_k} = \begin{bmatrix} \dfrac{\Delta f_1}{\Delta x_k} \\ \\ \dfrac{\Delta f_2}{\Delta x_k} \end{bmatrix}$$

an example for an abbreviated element of $\mathbf{b}$. Introducing these abbreviations into equation (45) reduces that very equation to

$$( 52 ) \qquad \begin{bmatrix} - \dfrac{\partial \mathbf{f}_\alpha}{\partial \mathbf{y}_\alpha} & - \dfrac{\partial \mathbf{f}_\alpha}{\partial \mathbf{y}_\beta} \\ \\ - \dfrac{\partial \mathbf{f}_\beta}{\partial \mathbf{y}_\alpha} & - \dfrac{\partial \mathbf{f}_\beta}{\partial \mathbf{y}_\beta} \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\Delta \mathbf{y}_\alpha}{\Delta x_k} \\ \\ \dfrac{\Delta \mathbf{y}_\beta}{\Delta x_k} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{f}_\alpha}{\partial x_k} \\ \\ \dfrac{\partial \mathbf{f}_\beta}{\partial x_k} \end{bmatrix} \cdot$$

Equations (48) through (51) only show **abbreviations**. A calculation with these abbreviations is not possible. Before starting a calculation, the original matrix has to be regenerated by **decomposition** and back-substitution and of the original terms. In contrast to Fig. 3 which shows the Jacobean matrix with indications of preliminary aircraft design parameters, **Fig. 7** shows a much more detailed Jacobean taking care of the industrial dimensions of the multidisciplinary aircraft design problem. Named are the various subsets (i.e. disciplines and contributing analysis activities) as relevant to MAEFISTO. The disciplines of interest for the multidisciplinary aircraft design process are: aeroelastics, aerodynamics, loads, handling qualities, landing and ground loads, control systems, structures.

## 4.2   Organising the Design Process with the Coupling-Matrix of Disciplines

In addition to the mathematical backbone of *Newtons's Method* and sensitivity calculations, the Jacobean matrix in equations (10), (11) and (16), know as **N-square Matrix** or Coupling-Matrix of Disciplines, helps to **organise the design process**. Each contributing analysis (or: module) covers a block of space on the main diagonal of the *N*-square Matrix. Each block is capable of accepting **input horizontally** from the right and from the left and of transmitting **output vertically** up and down (Fig. 7). The data flow from one module to the other is represented by a dot at the intersection of rows and columns of the two blocks considered. The absence of a dot means that no data is being transmitted. A dot indicates only that data transmission occurs in form of partial derivatives. However, the dot does not define precisely *what* partial derivatives are of interest. To see that, we would need to zoom in and decompose the *N*-square Matrix again. It is possible that the same output data items are being sent form one module to several others. On the contrary, an input datum for a module must be coming from only one, and no more than one, source module.

To establish an *N*-square Matrix, one begins with the modules placed on the diagonal in the best guess order. The data flow among each pair of modules is systematically investigated. If more than one source was found for any data item required, a choice must be made. If data is missing, it must come from outside - or another module needs to be added.

In a traditional design environment, after defining the data flow, attention shifts to determining the best sequence of execution for the modules. In the convention that assumes execution order along the diagonal from the upper left corner, each dot in the lower left half of the matrix marks an instance of the data passed forward (**feedforward**). Conversely, a dot in the upper right half marks the instance of a **feedback**. Each instance of a feedback implies an iteration in the traditional design process. The number of feedback instances and the number of associated iterations may be reduced by reordering the modules on the main diagonal. If the *N*-square Matrix is stored on the computer, a program like DeMAID [ROGERS 92] can search for an iteration-minimising pattern.

Note: The interpretation of elements in the Jacobean matrix as providing "feedforward" or "feedback" information relates back to the traditional iterative design process. When a system is solved by Newton's Method, all equations in the set are tackled simultaneously. Hence no flow of information can be addressed as being "feedforward" or "feedback".

# 5    A Note on Formal Optimisation

If the design process shall be further automated, **formal optimisation** may be applied. Taking the human experience further out of the optimisation loop requires the application of global optimisation algorithms. Global optimisation, however, allows only a limited number (50 ... 100) of independent design variables that may be considered simultaneously. This limits the use of global opti-

misation to preliminary aircraft design. For <u>local optimisation</u>, on the other hand, many well proven algorithms exist. They could be used if it becomes possible to keep the human expert in the optimisation loop. The human expert will then guide the *local* optimisation to the *global* optimum.

Due to the coupling of equations in aircraft design, a simple straight forward automated optimisation is not possible. During a first step, a consistent set of dependent design parameters has to be found as explained in Chapter 2.4 and 3.1. Only then, obtained parameters can be formally optimised. Hence, the automated design takes the form of continuous switching between consolidation and optimisation.

# 6     Conclusions

Two paths have been taken in aircraft design in the past.
1. The traditional aircraft design process is a tedious, predominantly manual and iterative procedure with many feedback loops.
2. The computerised approach has been either dealt with formal optimisation or expert systems.

A certain danger has been experienced with taking the designer out of the loop, as might be case with computer approaches. **Multidisciplinary Aircraft Design Optimisation**, MDO, takes an approach intermediate to 1. and 2.:
- The design is structured formally by use of the **Coupling-Matrix of Disciplines** also known as **N-square Matrix**. Input- and output data is checked and recorded properly. Feedback loops will be reduced as much as possible which simplifies the traditional design process.
- Consistent design variables are found using *Newton's Method*.
- The design is optimised "manually" by the introduction of the *system sensitivity derivatives*. They allow a qualitative, judgmental change of independent design variables. In addition also quantitative improvements can be made according to information from the *system sensitivity derivatives*. These step by step improvements will finally guide the design to a practical optimum.

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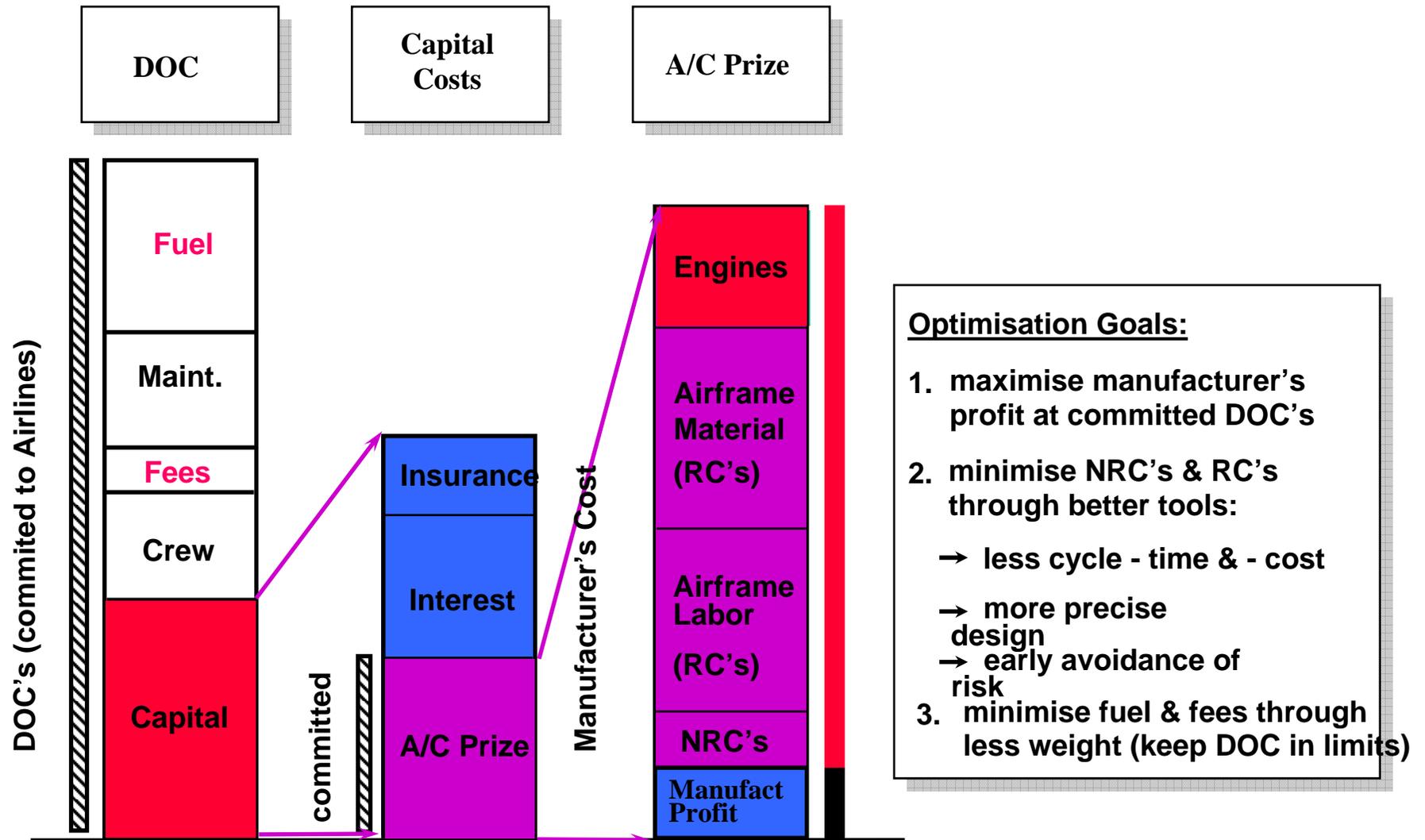
**Fig. 1:** Optimisation Goals in Aircraft Design
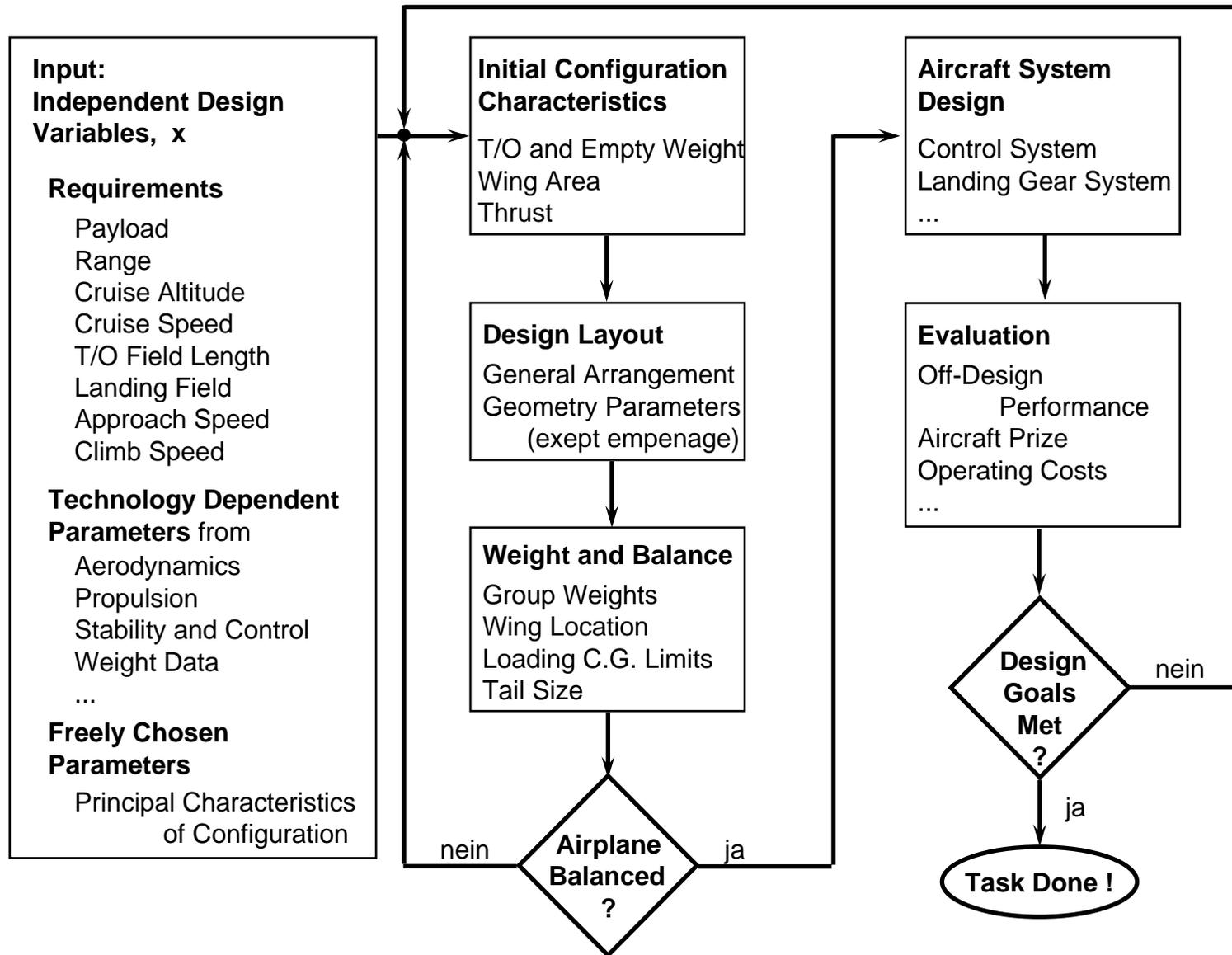
**Fig. 2:** Hierarchic and Iterative Aircraft Design

$$\frac{\partial f}{\partial y}$$

with respect to
parameters related to

Initial Configuration Characteristics

Design Layout

Weight and Balance

Aircraft System Design

Evaluation

$$\frac{\Delta y}{\Delta x}$$

$$\frac{\partial f}{\partial x}$$

**partial derivatives of
parameters related to**

**system sensitivity
derivatives related to**

**partial derivatives of
parameters related to**

Initial Configuration Characteristics
Design Layout
Weight and Balance
Aircraft System Design
Evaluation

$$\begin{bmatrix} 1 & ... & ... & ... & ... \\ ... & 1 & ... & ... & ... \\ ... & ... & 1 & ... & .... \\ ... & ... & ... & 1 & ... \\ ... & ... & ... & ... & 1 \end{bmatrix} \bullet \begin{bmatrix} \text{Initial Configuration Characteristics} \\ \text{Design Layout} \\ \text{Weight and Balance} \\ \text{Aircraft System Design} \\ \text{Evaluation} \end{bmatrix} = \begin{bmatrix} \text{Initial Configuration Characteristics} \\ \text{Design Layout} \\ \text{Weight and Balance} \\ \text{Aircraft System Design} \\ \text{Evaluation} \end{bmatrix}$$

***Global Sensitivity Equation***

**with respect to <u>one</u>
independent design
variable, x**

**with respect to <u>one</u>
independent design
variable, x**

**Fig. 3:** Non-Hierarchic Aircraft Design Translated into *Global Sensitivity Equation*
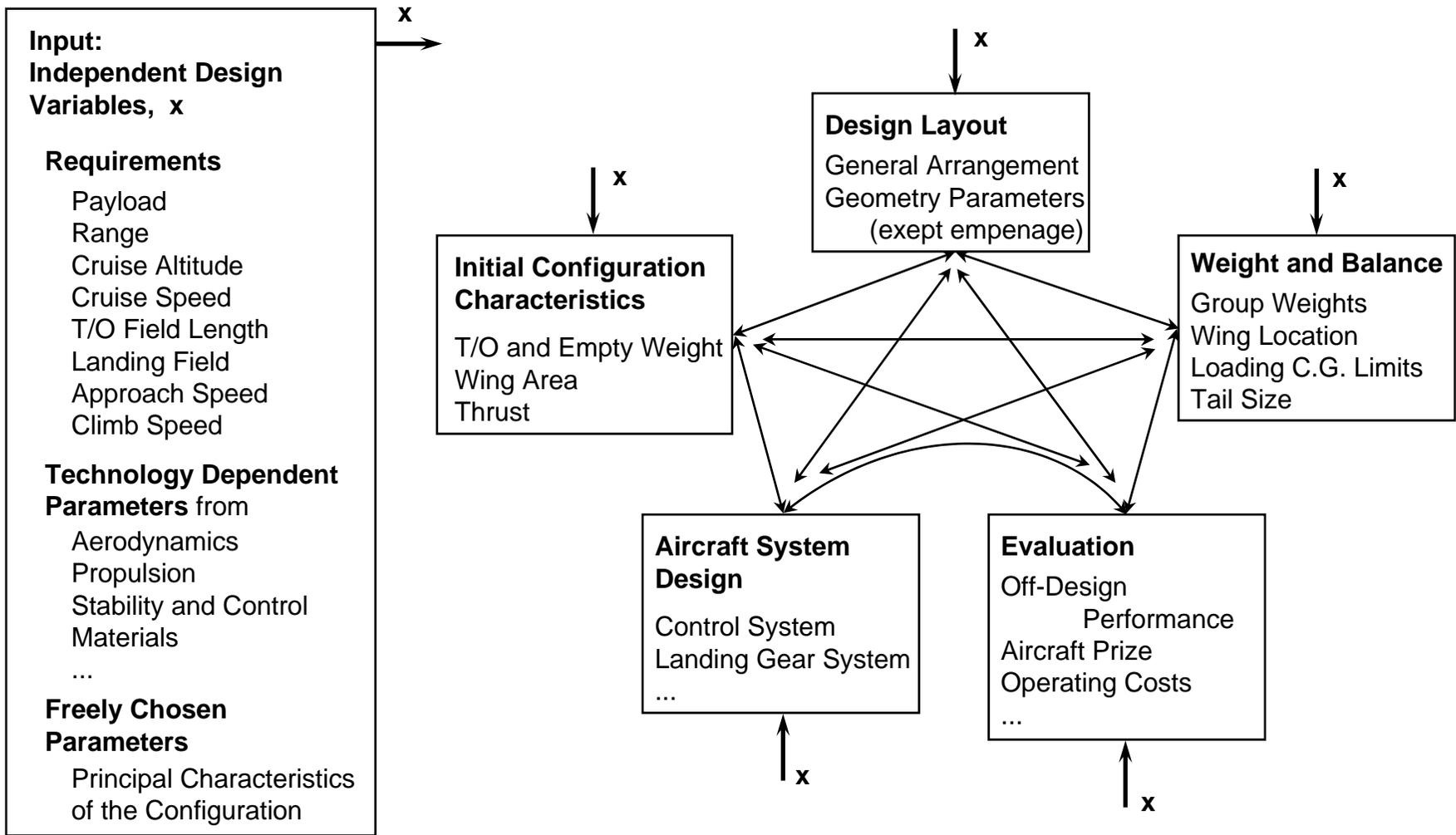
**Fig. 4:** Non-Hierarchic Aircraft Design
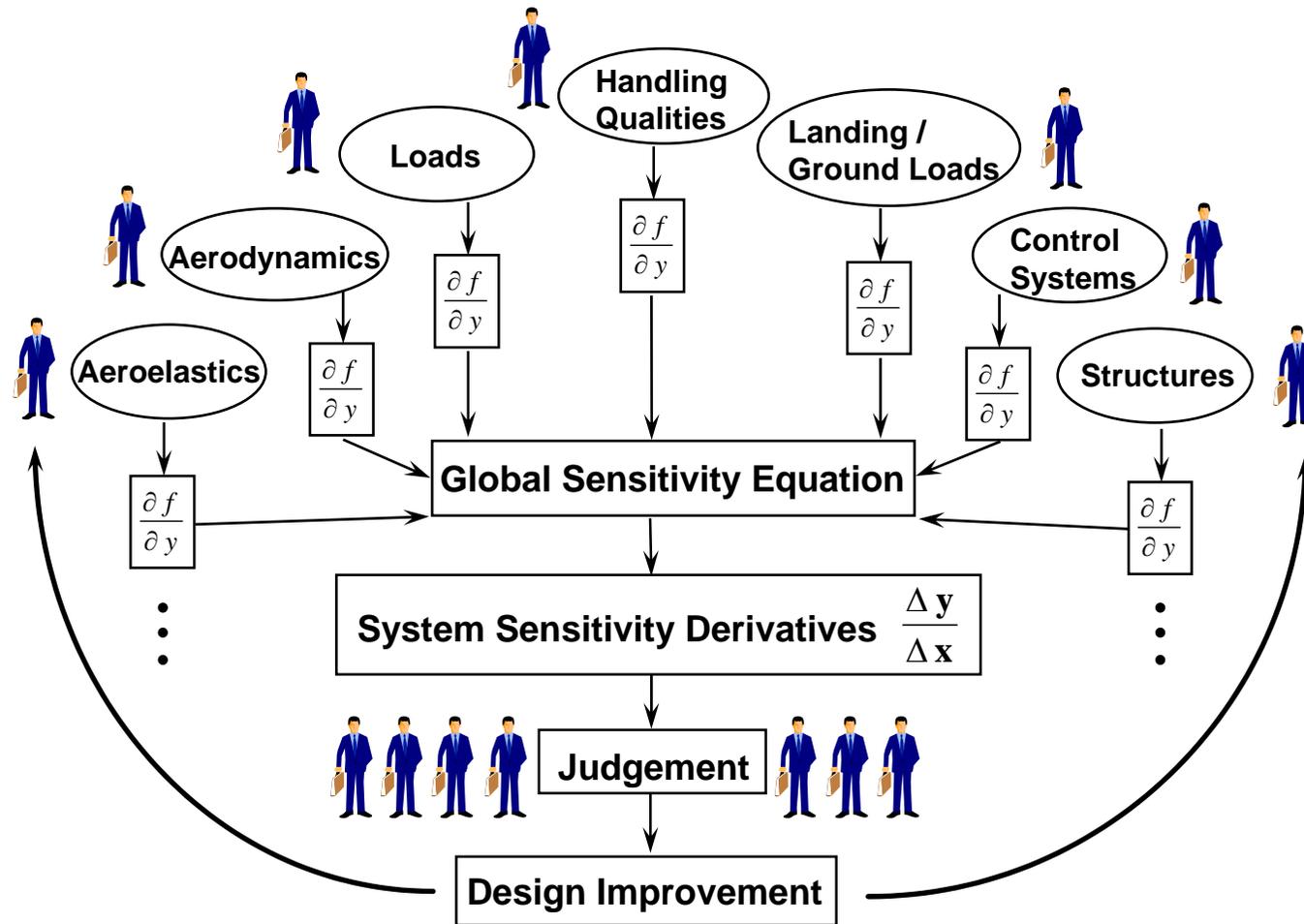
**Fig. 5:** Multidisciplinary Aircraft Design Optimisation - Human Interfaces in the Industrial Environment
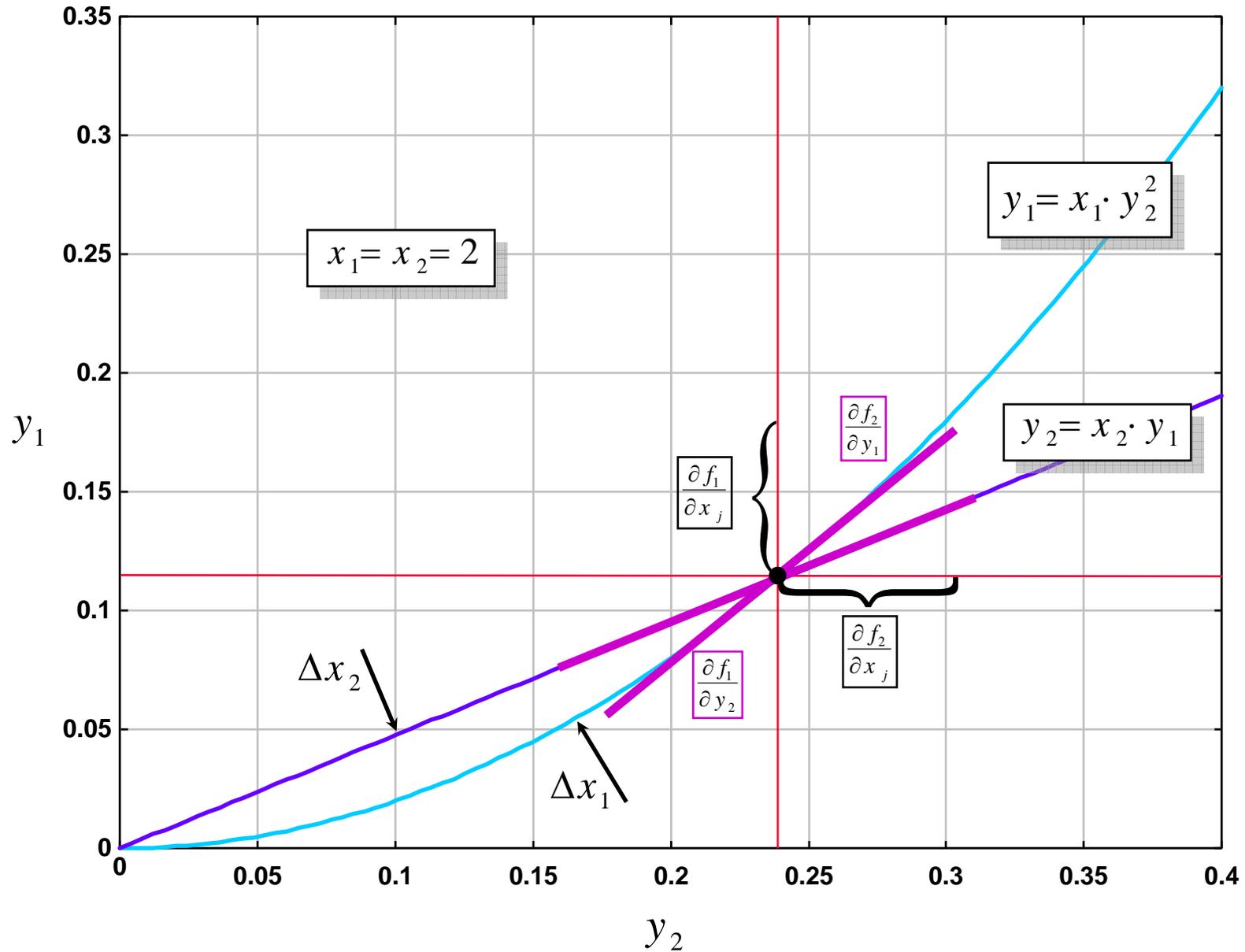
**Fig. 6:** Two Example Functions Used to Demonstrate the Various Partial Derivatives Required for Sensitivity Calculations
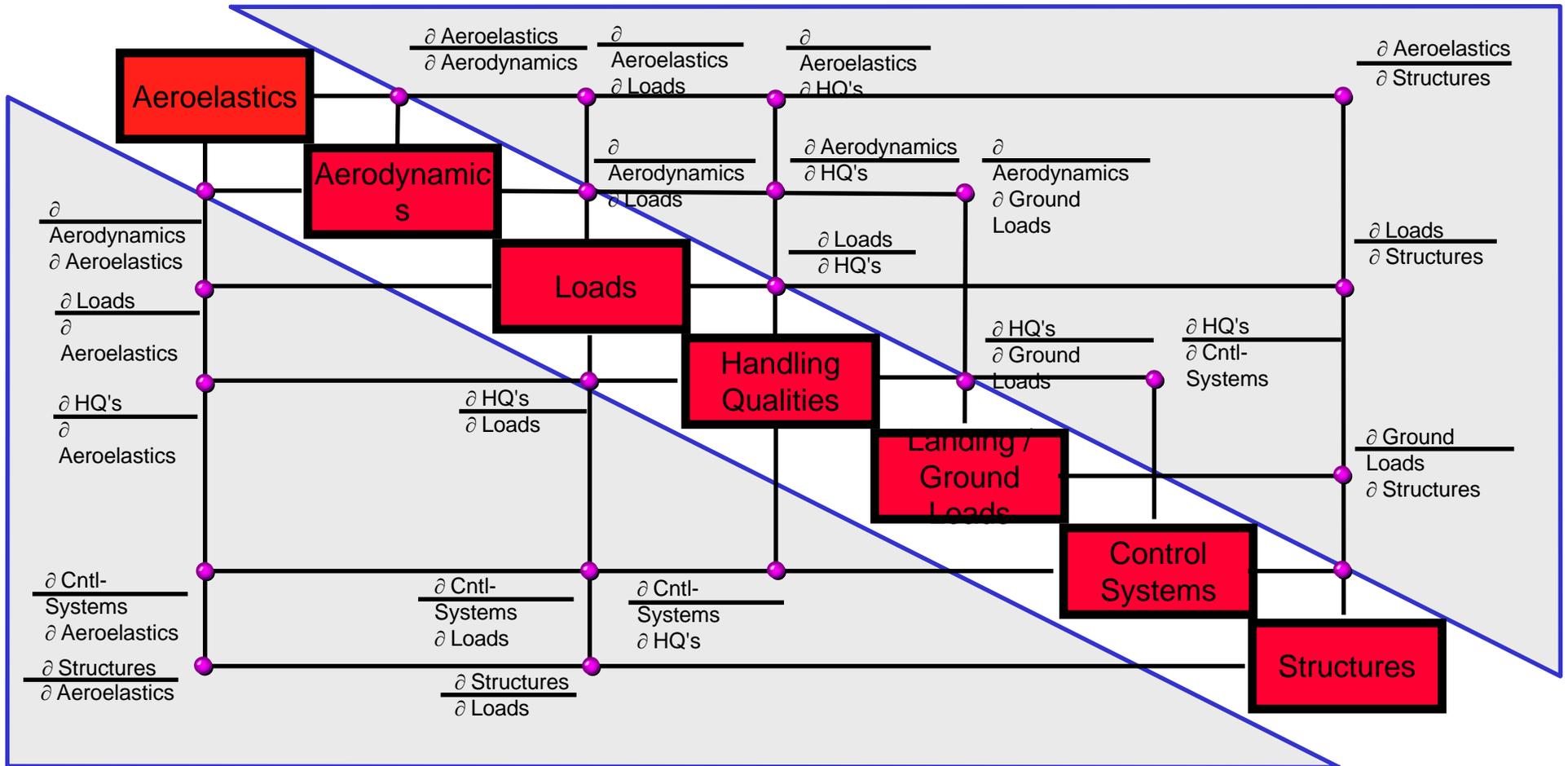
**Fig. 7:** Coupling - Matrix of Disciplines

# Appendix
# Breguet's Range Equation

## List of Symbols

| | |
|---|---|
| $SFC$ | thrust specific fuel consumption, [kg/N/s] |
| $T$ | thrust, [N] |
| $m$ | mass, [kg] |
| $g$ | earth acceleration, [m/s²] |
| $L/D$ | lift over drag, [-] |
| $MTOW$ | maximum take-off weight, [kg] |
| $LW$ | landing weight, [kg] |
| $W_f$ | fuel weight, [kg] |
| $v$ | true air speed, [m/s] |
| $R$ | range, [m] |
| $C_L$ | lift coefficient, [-] |
| $\rho$ | air density, [kg/m³] |
| $S$ | wing area, [m²] |

## Derivation

### Standard Derivation

( **1** ) $$dm = -SFC \cdot T \cdot d\tau$$

( **2** ) $$d\tau = -\frac{1}{SFC \cdot T} dm$$

( **3** ) $$T = \frac{m \cdot g}{L/D}$$

( **4** ) $$d\tau = -\frac{L/D}{SFC \cdot g} \cdot \frac{1}{m} dm$$

with $dR = v \cdot d\tau$

( **5** ) $$dR = -\frac{L/D \cdot v}{SFC \cdot g} \cdot \frac{1}{m} dm$$

( **6** ) $$R = \int_0^R ds = -\frac{L/D \cdot v}{SFC \cdot g} \int_{m0}^{m1} \frac{1}{m} dm$$

( **7** ) $$R = -\frac{L/D \cdot v}{SFC \cdot g} \left[ \ln m_1 - \ln m_0 \right]$$

with $m_1 = LW$ and $m_0 = MTOW$

( 8 ) $$R = \frac{L/D \cdot v}{SFC \cdot g} \ln \frac{MTOW}{LW}$$

( 9 ) $$W_f = MTOW - LW$$

**Derivation as in [ANDERSON 89]**

( 10 ) $$dR = -\frac{L/D \cdot v}{SFC \cdot g} \cdot \frac{1}{m} dm$$

with $v = \sqrt{\dfrac{2 \cdot m \cdot g}{\rho \cdot S \cdot C_L}}$ and $\dfrac{C_L}{C_D} = L/D$

( 11 ) $$dR = -\sqrt{\frac{2}{\rho \cdot g \cdot S}} \cdot \frac{\sqrt{C_L}}{C_D} \cdot \frac{1}{SFC} \cdot m^{-\frac{1}{2}} dm$$

( 12 ) $$R = -\sqrt{\frac{2}{\rho \cdot g \cdot S}} \cdot \frac{\sqrt{C_L}}{C_D} \cdot \frac{1}{SFC} \int_{m0}^{m1} m^{-\frac{1}{2}} dm$$

( 13 ) $$R = 2 \cdot \sqrt{\frac{2}{\rho \cdot g \cdot S}} \cdot \frac{\sqrt{C_L}}{C_D} \cdot \frac{1}{SFC} \cdot \sqrt{MTOW} \cdot \left(1 - \sqrt{\frac{LW}{MTOW}}\right)$$

**Application and Comparison**

Both range equations show that the ratio *LW/MTOW* determines the characteristics of the range formula. For a comparison some data has been chosen and considered to be fixed:

| | |
|---|---|
| *SFC* | = 17 mg/N/s |
| *g* | = 9.80 m/s$^2$ |
| $C_L$ | = 0.5 |
| $C_D$ | = 0.025 |
| $\rho$ | = 0.3 kg/m$^3$ |
| *S* | = 362 m$^2$ |
| *v* | = 260 m/s |

With this data, range versus *LW/MTOW* has been plotted in **Fig. A-1**.
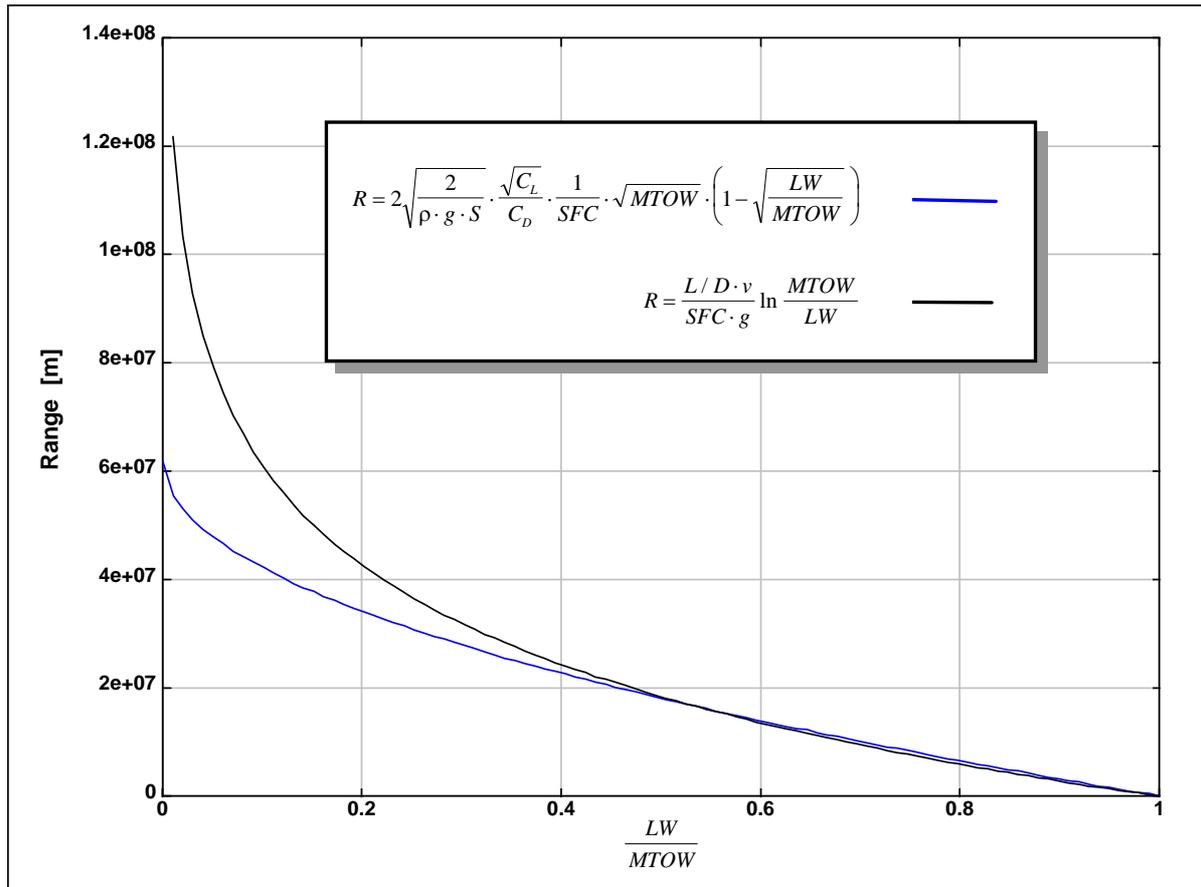
**Fig. A-1:** Comparison of Two Different Derivations of Breguet's Range Equation